Numerical Methods and Algebra. Rishika Kudav (17)

Assignment 1.

1) Find the value of $\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

Let x be $\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

$x = \sqrt{7+x}$

$x^2 = 7+x$

$x^2 - x - 7 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{1 \pm \sqrt{29}}{2}$ $\quad x = \frac{1+\sqrt{29}}{2}, \frac{1-\sqrt{29}}{2}$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$.

$18x^2 = 2y^2$

$\frac{x^2}{y^2} = \frac{2}{18} = \frac{1}{9}$

$\frac{x}{y} = \pm \frac{1}{3}$

3) $x + 3y = 4$ $\quad x = 4 - 3y$

$x^2 + y^2 - 4y = 12$ $\quad x^2 + y^2 - 4y = 12$

$(4-3y)^2 + y^2 - 4y = 12$

$16^2 + 9y^2 - 24y + y^2 - 4y = 12$

$10y^2 - 28y + 4 = 0$

$5y^2 - 14y + 2 = 0$

$y = \frac{7+\sqrt{39}}{5}$ $\quad y = \frac{7-\sqrt{39}}{5}$

hence, $x = 4 - 3\left(\frac{7+\sqrt{39}}{5}\right)$

OR

$x = 4 - 3\left(\frac{7-\sqrt{39}}{5}\right)$

49) $d = 2r^2 - 16r + 34$

$d = 2(3.5) - 16(3.5) + 34$

$d = 7 - 56 + 34 = -15$ millions of particles / cubic foot.

b) $d' = 4r - 16 = 0$

$4r = 16$

$r = 4$

$2(4)^2 - 16(4) + 34 = 2$ particles / ft^3

c) $d = 100$ m/ft 3.

$100 = 2r^2 - 16r + 34$

$50 = r^2 - 8r + 17$

$r^2 - 8r - 33 = 0$

$(r+3)(r-11) = 0$.

r cant be negative.

speed of the engine = 11,000 rph.

renting a car.

5a) Let C_1 be the total cost of company 1.
Let n be the number of km travelled by the car

$C_1 = 1500 + 25n$

b) Let C_2 be the total cost of renting a car from company 2.
Let n be the number of km travelled by the car

$C_2 = 2100 + 22n$

c) $2100 + 22n < 1500 + 25n$

d) $2100 - 1500 < 25n - 22n$

$600 < 3n$

$600/3 < n$

$200 < n$

6) $f(x) = x^3 - 7x^2 + 14x - 6 = 0$

a) $[0, 1]$

a	b	f(a)	f(b)
0	1	-6	2
0.5	1	-0.625	2
0.5	0.75	-0.625	0.984375
0.5	0.625	-0.625	0.259766
0.5625	0.625	-0.10187	0.259766
0.578125	0.59375	-0.10187	0.054047
0.57125	0.585938	-0.005262	0.001031
0.582031	0.585938	-0.02572	0.001031

$$\frac{0.582031 + 0.585938}{2} = 0.583984$$

b) $[1, 3.2]$

a	b	f(a)	f(b)
1	3.2	2	-0.112
2.1	3.2	1.791	-0.112
2.65	3.2	0.55125	-0.112
2.925	3.2	0.085828	-0.112
3.5625	3.2	0.248291	-0.112
3.56225	3.38125	-0.248291	-0.035132
3.471875	3.471875		

7) $F(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$.

Midpoint of $[-1, 0]$ $P_0 = -\frac{1}{2}$

Midpoint of $[0, 1]$ $P_0 = \frac{1}{2}$

$[-1, 0]$ $P_4 = -0.040659$

$[0, 1]$ $P_6 = -0.040659$

~~for~~

8) a) $\frac{70+86+81+83+x}{5} = \frac{320+x}{5}$

b)

$\frac{320+x}{5} \geq 80$

$320+x \geq 400$

$x \geq 80$

$\frac{320+x}{5} < 90$

$320+x < 450$

$x < 130$

Hence $80 \leq x < 130$

Q9)

x	y
4%	8.54
20%	-11.81

$$x = x_1 + (y - y_1)\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$$

$$= 4\% + (+8.54)\left(\frac{16\%}{+20.35}\right)$$

$$x = 10.71\%.$$

Q10) $R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

Profit = $R(x) - C(x)$.

Profit = $20 - 0.21x^2 - 195$

Differentiate

$$20 - 0.42x = 0$$

$$20 = 0.42x$$

$$x = \frac{20}{0.42}$$

$x = 47.62$ units.