Numerical Methods & Algebra

Assignment - 1

Q1. Actual length $= r_1 = 12.5\,cm$

Actual area $= \pi r_1^2$

$= \pi (12.5)^2$

$= 490.87385\ cm^2$

Approximate radius $= 12.65$

Approx. area $= \pi r^2$

$= \pi (12.65)^2$

$= 502.72551\ cm^2$

1. Absolute error $=$ Approx area $-$ actual area

$= 502.72551 - 490.87385$

$= 11.85166\ cm^2$

2. Relative error $= \dfrac{\text{Absolute error}}{\text{Actual area}}$

$= \dfrac{11.85166}{490.87385}$

$= 0.024144$

3. Percentage error $=$ (Relative error) $\times 100$

$= 2.4144\ \%$

PAGE NO.:
DATE: / /

Q2. (i) Using linear interpolation

$x_1 = 4$, $y_1 = 0.60206$
$x_2 = 6$, $y_2 = 0.7781513$

$(6-4) = (0.7781513 - 0.60206)$

$2 = 0.1760913$

1 causes a change of $= \left(\frac{0.1760913}{2}\right) \times 1$

~~$5 = \left(\frac{0.1760913}{2}\right) \times 5$~~

$1 = 0.08804565$

Thus, $x = 5$

$y = (0.60206) + 0.08804565$

$y = 0.69010565$

(ii) $x_1 = 4.5$, $y_1 = 0.6532125$
$x_2 = 5.5$, $y_2 = 0.7403627$

$(5.5 - 4.5) = (y_2 - y_1)$

Change caused by $1 = 0.0871502$
Change caused by $0.5 = 0.0435751$

Thus, $x = 5$

$y = (0.6532125 + 0.0435751)$

$y = 0.6967876$

PAGE NO.:
DATE: / /

Q3. $x^4 - x - 10 = 0$

	x	y
$x_0 =$	1.5	−6.4375
$x_1 =$	2	4

1. $x_2 = \frac{x_0 + x_1}{2}$
$= 1.75$
$y_2 = -2.37109$

2. $x_3 = \frac{x_1 + x_2}{2}$
$= 1.875$
$y_3 = 0.48462$

3. $x_4 = \frac{x_2 + x_3}{2}$
$= 1.8125$
$y_4 = -1.0202$

4. $x_5 = \frac{x_4 + x_3}{2}$
$= 1.84375$
$y_5 = -0.28773$

5. $x_6 = \frac{x_5 + x_3}{2}$
$= 1.859375$
$y_6 = 0.0933$

Q4. $x^3 - x - 1 = 0$

$f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \left(-\frac{1}{2}\right)$$

$$= 1.5$$

$y_1 = 0.875$

$f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \left(\frac{0.875}{5.75}\right)$$

$x_2 = 1.34783$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44994$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3252$$

$y_3 = 0.00205$

$$f'(x_3) = 4.26846$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.32472$$

$$y_4 = 0.0000087\,12$$

Q5. $A^2 = A$

$$7A - (I+A)^3 = 7A - \left(I^3 + A^3 + 3A^2 I + 3AI^2\right)$$

$$= 7A - \left(I + A^2(A) + 3A + 3A\right)$$

$$= 7A - \left(I + A^2 + 6A\right)$$

$$= \cancel{7A} - (I + \cancel{7A})$$

$$= -I$$

Q6. $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

Using 3^{rd} row,

$$|A| = 0 - 1(6-8) + (-1)(0+4)$$

$$= 2 - 4$$

$$= -2$$

~~adj (A) =~~ $\cancel{\begin{bmatrix} -6 & 6 \\ 4 & -1 \\ 4 & -1 \end{bmatrix}}$

$$\operatorname{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \operatorname{adj}(A)$$

$$= -\frac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -\frac{1}{2} & 3 \\ -2 & \frac{1}{2} & -1 \\ -2 & \frac{1}{2} & -2 \end{bmatrix}$$

PAGE NO.:
DATE: / /

Q7. $\vec{a} = 8\hat{i} - 9\hat{j}$
$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

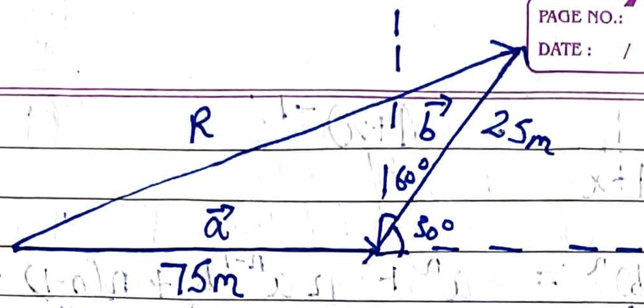
$\vec{a} \cdot \vec{b} = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$
$= 56 + 81$
$= 137$

$a = \sqrt{64+81}$
$\cancel{= 12.04159} = \sqrt{145}$
$b = \sqrt{49+81}$
$= \sqrt{130}$

$\vec{a} \cdot \vec{b} = ab\cos\theta$
$\cos\theta = \dfrac{137}{\sqrt{130} \times \sqrt{145}}$
$\cos\theta = 0.997849146$

$\theta = 3.75856^\circ$

Q8.



Displacement vector = $\vec{R}$

$\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30°$

$R^2 = 75^2 + 25^2 + 2(75)(25)\cos 30°$

$R^2 = 9497.595264$

$R = 97.4556$ metres

Thus, displacement = 97.4556 metres

Q9. Common difference, $d = 3$

$A.P = 101, 104, 107, \ldots, 998$

$a = 101$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$n = 300$

$S_n = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(998 + 101)$

$= 150 \times 1099$

$S_n = 1,64,850$

Q10. $a_6 = 32$

$ar^5 = 32$

$a_8 = 128$

$ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$ or -2

rejected as gives negative values

$r = 2$

Q11. $(3x+4)^5$

Using Pascal's Triangle :

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$\cancel{(a+b)}$ a
$\cancel{(a+b)^2}$ $(a+b)$
$\cancel{(a+b)^3}$ $(a+b)^2$
$\cancel{(a+b)^4}$ $(a+b)^3$
$(a+b)^4$
$(a+b)^5$

$$(3x+4)^5 = (3x)^5 + (5\times(3x)^4\times 4) + (10\times(3x)^3\times 4^2)$$
$$+(10\times(3x^2)\times 4^3) + (5\times 3x\times 4^4) + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2$$
$$+ 3840x + 1024$$

$$(3x+5)^4 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2$$
$$+ 3840x + 1024.$$

Q12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$|A-\lambda I|$

Expanding using 3^{rd} row

$$|A-\lambda I| = 0 - 0 + (3-\lambda)[(2-\lambda)(-5-\lambda)+6]$$

$$= (3-\lambda)[-10-2\lambda+5\lambda+\lambda^2+6]$$

$$= (3-\lambda)[\lambda^2+3\lambda-4]$$

$$= \cancel{3\lambda^2}+9\lambda-12-\lambda^3 - \cancel{3\lambda^2}+4\lambda$$

$$\cancel{= -\lambda^3 + 13\lambda - 12}$$

~~Comparing coefficients~~

$$\cancel{a+b+c+d = -1+0+13-12}$$

$$\cancel{=0}$$

~~Thus one root is 1~~

$$\cancel{\begin{array}{c|cccc} 1 & -1 & 0 & 13 & -12 \\ & & -1 & -2 & \\ \hline & -2 & -2 & & \end{array}}$$

$$|A-\lambda I| = -\lambda^3 + 13\lambda - 12$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

Comparing coefficients,

$$a+b+c+d = 1-13+12$$

$$= 0$$

Thus, one root $= 1$

PAGE NO.:
DATE: / /

1	1	0	−13	12
		1	1	−12
	1	1	−12	0

$$\lambda^2 + \lambda - 12 = 0$$
$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$
$$\lambda(\lambda+4) - 3(\lambda+4) = 0$$
$$(\lambda-3)(\lambda+4) = 0$$
$$\lambda = 3 \text{ or } -4$$

Eigen values = −4, 1 and 3.

Q13. $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13$

$f'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 5 - \left(\frac{13}{-13}\right)$$
$$= 6$$

$f(x_1) = 9$

$f'(x_1) = 32$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - \frac{9}{32}$$

$$x_2 = 5.71875$$
$$f(x_2) = 0.84787$$

Q14. $(x^2 - x + 1)^4 = ((x-1)^2 + x)^4$

Using Pascal's triangle

1	
1 1	$(a+b)$
1 2 1	$(a+b)^2$
1 3 3 1	$(a+b)^3$
1 4 6 4 1	$(a+b)^4$

$$[(x-1)^2 + x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$

$$(x^2 - x + 1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4.$$

PAGE NO.:
DATE: / /

Q15. $e^{-x} = 3\log(x)$

$3\log(x) + e^{-x} = 0$

x	y
0.1	−2.09516
0.2	−1.27818
0.3	−0.82782
0.4	−0.5235
0.5	−0.29656
0.6	−0.11673
0.7	0.03188

$x_0 = 0.6 \quad\quad y_0 = -0.11673$

$x_1 = 0.7 \quad\quad y_1 = 0.03188$

1. $x_2 = \frac{x_0 + x_1}{2}$

$= 0.65$

$y_2 = -0.03921$

2. $x_3 = \frac{x_2 + x_1}{2}$

$= 0.675$

$y_3 = -0.00293$

3. $x_4 = \frac{x_3 + x_1}{2}$

$= 0.6875$

$y_4 = 0.01465$

PAGE NO.:
DATE: / /

4. $$x_5 = \frac{x_4 + x_3}{2}$$

$$= 0.68125$$

$$y_5 = 0.0059$$