

Homework

Name $\Rightarrow$ Little Jain

Q1

x	7	8	9	10
f(x)=y	3	1	1	9

$$x = 9.5$$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$f(9.5) = \frac{1.5 \times 0.5 \times (-0.5) \times 3}{-1 \times -2 \times -3} + \frac{2.5 \times 0.5 \times (-0.5)}{1 \times -1 \times -2} +$$

$$\frac{2.5 \times 1.5 \times (-0.5)}{2 \times 1 \times (-1)} + \frac{2.5 \times 1.5 \times 0.5 \times 9}{3 \times 2 \times 1}$$

$$f(9.5) = \frac{3}{16} - \frac{5}{16} + \frac{15}{16} + \frac{45}{16} = \frac{58}{16} = \underline{\underline{3.625}}$$

Ques 2 $\Rightarrow$

x	3	7	9	10
f(x)	168	120	72	63

$$\begin{aligned}
 f(6) = & \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 \\
 & + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 \\
 & + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 \\
 & + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63
 \end{aligned}$$

$$\begin{aligned}
 f(6) = & \left(\frac{+1 \times -3 \times -4}{+4 \times -6 \times -7} \times 168 \right) + \frac{3 \times -3 \times -4}{4 \times -2 \times -3} \times 120 \\
 & + \frac{3 \times -1 \times (-4)}{6 \times 2 \times -1} \times 72 + \frac{3 \times -1 \times (-3)}{7 \times 3 \times 1} \times 63
 \end{aligned}$$

$$f(6) = 12 + 180 - 72 + 27 = \underline{\underline{147}}$$

Ques 3 ⇒

x	1	2	7	8
f(x)	4	5	5	4

$$f(6) = - \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4$$

$$+ \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$f(6) = \frac{4 \times (-1) \times (-2) \times 4}{(-1) \times (-6) \times (-7)} + \frac{5 \times (-1) \times (-2) \times 5}{1 \times (-5) \times (-6)} +$$

$$\frac{5 \times 4 \times (-2)}{6 \times 5 \times (-1)} \times 5 + \frac{5 \times 4 \times (-1) \times 4}{7 \times 6 \times 1}$$

$$= -\frac{16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{40}{21} = \frac{17}{3} = 5.667$$

Ques 4 →

X	1	2	3	4	7
f(x)	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

$$+ \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$f(5) = \frac{3 \times 2 \times 1 \times (-2) \times 2}{-1 \times -2 \times -3 \times -6} + \frac{4 \times 2 \times 1 \times (-2) \times 4}{-1 \times -2 \times -5} + \frac{4 \times 3 \times 2 \times 8}{2 \times (-1) \times (-4)}$$

$$+ \frac{4 \times 3 \times 2 \times (-2) \times 16}{3 \times 2 \times 1 \times (-3)} + \frac{4 \times 3 \times 2 \times 1 \times 128}{6 \times 5 \times 4 \times 3}$$

$$f(5) = -\frac{2}{3} + \frac{32}{5} + -24 + \frac{128}{3} + \frac{128}{15} = 32.9333$$

Ques 5 ⇒

X	10	20	14	16
f(x)	0.25	0.2	0.15	0.10

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25$$

$$+ \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.20$$

$$+ \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15$$

$$+ \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$f(15) = \frac{-5 \times 1 \times -1}{-10 \times -4 \times -6} \times 0.25 + \frac{5 \times 1 \times -1}{10 \times 6 \times 4} \times 0.20$$

$$+ \frac{5 \times (-5) \times (-1)}{4 \times (-6) \times (-2)} \times 0.15 + \frac{5 \times (-5) \times 1}{6 \times (-4) \times 2} \times 0.10$$

$$= \frac{-1}{192} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96} = \frac{29}{240}$$

$$= 0.120833$$

Homework - 2

Regula Falsi Method

$$1 \Rightarrow x^3 + x - 1 = 0$$

$$f(x) = x^3 + x - 1$$

x	y
0.5	-0.375
0.6	-0.184
0.7	0.043

$$x_1 = 0.6$$

$$x_2 = 0.7$$

$$y_1 = -0.184$$

$$y_2 = 0.043$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$x_n = 0.681057$$

$$y_n = -0.003042$$

Now

$$x_1 = 0.681057$$

$$x_2 = 0.7$$

$$y_1 = -0.003042$$

$$y_2 = 0.043$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.682309$$

$$y'_n = -0.000045$$

Thus, the root of $x^3 + x - 1 = 0$ is 0.682309

Ques 2 $\Rightarrow x^3 - 2x + 0.5 = 0$ b/w 0.81

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073

$$x_1 = 0.2$$

$$y_1 = 0.108$$

$$x_2 = 0.3$$

$$y_2 = -0.073$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.259669$$

$$y_n = -0.001829$$

Now

$$x_2 = 0.259669$$

$$y_2 = -0.001829$$

$$x_1 = 0.2$$

$$y_1 = 0.108$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$x'_n = 0.258675$$

$$y'_n = -0.000041$$

Thus root of $x^3 - 2x + 0.5$ is 0.258675

Ques 3 $x^3 + 2x - 20 = 0$ b/w 2 & 2.5

Function is $y = x^3 + 2x - 20$

x	y
2.3	-3.233
2.4	-1.376
2.5	0.625

$$x_1 = 2.4$$

$$x_2 = 2.5$$

$$y_1 = -1.376$$

$$y_2 = 0.625$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.468766$$

$$y_n = -0.015819$$

Now

$$x_1 = 2.468766$$

$$x_2 = 2.5$$

$$y_1 = -0.015819$$

$$y_2 = 0.625$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.469537$$

$$y'_n = -0.000176$$

The root of given equation is 2.469537

Ques 4 $3x - \cos x - 1 = 0$, b/w 0 and 1

Function is, $y = 3x - \cos x - 1$

x	y
0.3	-1.099986
0.4	-0.799976
0.5	-0.499962
0.6	-0.199945
0.7	0.100075

$$x_1 = 0.6$$

$$x_2 = 0.7$$

$$y_1 = -0.199945$$

$$y_2 = 0.100075$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.666644$$

$$y_n = -0.00006313$$

Root of $3x - \cos x - 1$ is 0.666644

Ques 5 - 1 $x^3 - 3x - 5$ b/w 2 and 2.5

x	y
2.0	-3
2.1	-2.039
2.2	-0.952
2.3	0.267

$$x_1 = 2.2 \quad x_2 = 2.3$$

$$y_1 = -0.952 \quad y_2 = 0.267$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$x_n = 2.278097$$

$$y_n = -0.011592$$

$$\text{Now, } x_1 = 2.278097, \quad x_2 = 2.3$$

$$y_1 = -0.011592, \quad y_2 = 0.267$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.279008$$

$$y'_n = -0.000136$$

Thus, the root of $x^3 - 3x - 5$ is 2.279008.

Ques 6 $\Rightarrow x \log_{10} x - 1.2 = 0$ b/w 2.3

$$f(x) = x \log_{10} x - 1.2$$

x	y
2.4	-0.287493
2.5	-0.205149
2.6	-0.121069
2.7	-0.035318
2.8	0.052042

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.740428$$

$$y_n = -0.0001902$$

Thus, root of function 2.740428

Ques 7 $\Rightarrow$

$$x^3 - 4x + 1 = 0$$

$$f(x) = x^3 - 4x + 1$$

x	y
0	1
0.1	0.601
0.2	0.208
0.3	-0.173

$$x_1 = 0.2$$

$$x_2 = 0.3$$

$$y_1 = 0.208$$

$$y_2 = -0.173$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.254593$$

$$y_n = -0.001869$$

Now,

$$x_1 = 0.2$$

$$x_2 = 0.254593$$

$$y_1 = 0.208$$

$$y_2 = -0.001869$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.254107$$

$$y'_n = -0.00002022$$

Thus, the root of given equation is 0.254107 .

Ques 8 $\Rightarrow x^3 + 2x^2 - 8 = 0$

Function is $y = x^3 + 2x^2 - 8$

x	y
0	-8
1	-5
2	8

$x_1 = 1$, $x_2 = 2$

$y_1 = -5$, $y_2 = 8$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.384615$$

$$y_n = -1.511156$$

Now $x_1 = 1.384615$, $x_2 = 2$

$y_1 = -1.511156$, $y_2 = 8$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.482389$$

$$y'_n = -0.347529$$

Now,

$x_1 = 1.482389$, $x_2 = 2$

$y_1 = -0.347529$, $y_2 = 8$

$$x''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = 1.509938$$

$$y''_n = -0.074689$$

Now,

$$u_1 = 1.503938, \quad u_2 = 2$$

$$y_1 = -0.074689, \quad y_2 = 8$$

$$u'''_n = \frac{u_1 y_2 - u_2 y_1}{y_2 - y_1}$$

$$= 1.508526$$

$$y'''_n = -0.0158204$$

Now, $u_1 = 1.508526, \quad u_2 = 2$

$$y_1 = -0.0158204, \quad y_2 = 8$$

$$u''''_n = \frac{u_1 y_2 - u_2 y_1}{y_2 - y_1}$$

$$= 1.509496$$

$$y''''_n = -0.003339$$

Now,

$$u_1 = 1.509496, \quad u_2 = 2$$

$$y_1 = -0.003339, \quad y_2 = 8$$

$$u_n = \frac{u_1 y_2 - u_2 y_1}{y_2 - y_1}$$

$$= 1.50970064$$

$$y_n = -0.000704$$

So, the root of the given equation is 1.50970064

Ques 9 $\Rightarrow$

$$x^3 - 9x + 1 = 0$$

function : $y = x^3 - 9x + 1$

x	y
0	1
0.1	0.101
0.2	-0.792

$$x_1 = 0.1$$

$$x_2 = 0.2$$

$$y_1 = 0.101$$

$$y_2 = -0.792$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0.1 \times (-0.792) - 0.2 \times 0.101}{-0.792 - 0.101}$$

$$= 0.1113102$$

$$y_n = -0.000413$$

Thus, the root of given equation is 0.1113102

Ques 10 =)

$$x^3 - x - 4 = 0$$

Function is

$$y = x^3 - x - 4$$

x	y
0	-4
1	-4
2	2

$$x_1 = 1, x_2 = 2$$

$$y_1 = -4, y_2 = 2$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.333334$$

$$y_n = -2.96296$$

Now,

$$x_1 = 1.333334, x_2 = 2$$

$$y_1 = -2.96296, y_2 = 2$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = 1.7313434$$

$$y'_n = -0.541555$$

$$\text{Now, } x_1 = 1.7313434, x_2 = 2$$

$$y_1 = -0.541555, y_2 = 2$$

$$x''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.788589$$

$$y''_n = -0.066802$$

Now,

$$x_1 = 1.788589, \quad x_2 = 2$$

$$y_1 = -0.066802, \quad y_2 = 2$$

$$x'''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = 1.795422$$

$$y'''_n = -0.007807$$

$$\text{Now, } x_1 = 1.795422, \quad x_2 = 2$$

$$y_1 = -0.007807, \quad y_2 = 2$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.796217$$

$$y_n = -0.0009105$$

Thus a root of $x^3 - x - 4$ is 1.796217.