Uddhav Jadhav Calculus

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Q1 ans) $\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{h\to 0} \frac{4(-3+h) - 0}{1}$ .... {Use L'hospital rule $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$ }

$= \lim_{h\to 0} 4(-3+h)$

$= 4(-3+0)$

$= 4 \times -3$

$= \boxed{-12}$

Q2 ans) $g(x) = \begin{cases} 2x, & x < 6 \\ x-1, & x \geq 6 \end{cases}$

(a) $x = 4$

as $g(x) = 2x$ as $x = 4 < 6$

$g(x) = 2x$ is polynomial ~~also~~ — polynomial is continuous everywhere

$\therefore g(x)$ is continuous at $x = 4$

(b) $x = 6$

as at $x = 6$ definition of function changes as

$g(x) = 2x \quad x < 6$

$\quad\quad\; x - 1 \quad x \geq 6$

so we need to check it ~~$g(6^-) = g$~~ $g(6^-) = g(6^+) = g(6)$

$g(6^-) = 2 \times (6) = 12$ as $g(x) = 2x$ } for $x < 6$

$g(6^+) = 6 - 1 = 5$ $g(x) = x - 1$ } for $x \geq 6$

$g(6) = 6 - 1 = 5$ $g(x) = x - 1$ } for $x = 6$

So, $g(6^-) \neq g(6^+) = g(6)$ .... $6^- \Rightarrow x < 6$

~~so $g(x)$ is discontinuous~~ $6^+ \Rightarrow x > 6$

at $\therefore g(x)$ is discontinuous at $x = 6$

Q3 ans) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$= \frac{9-9}{3-\sqrt{9}} = \frac{0}{3-3} = \frac{0}{0}$

$\therefore$ Apply L'hospital rule

$= \lim_{x \to 9} \frac{1-0}{0-\frac{1}{2\sqrt{x}}}$

$= \lim_{x \to 9} \frac{1}{\frac{-1}{2\sqrt{x}}}$

$= \lim_{x \to 9} -2\sqrt{x}$

$= -2 \times \sqrt{9} = -2 \times 3$

$= -6$

$$\boxed{\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} = -6}$$

Q4 ans) $f(x) = \frac{4x+5}{9-3x}$

$\because$ the given function is defined at every serial number except the point where $9-3x = 0$

$\Rightarrow x = 3$

Hence, given function is continuous at every real number except $x=3$

$\therefore$ function is continuous at $x=1$ and $x=0$; and the function is discontinuous at $x=3$

Q5 ans) $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\lim_{x \to \infty} \dfrac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$

Apply limit

$= \dfrac{\cancel{6}3}{\cancel{8}4}$

$= \dfrac{3}{4}$

Q6ans) $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & y \geq -2 \end{cases}$

(a) at $g(y) = 6$

$\lim_{y \to 6} g(y)$

$= \lim_{y \to 6} (1 - 3y)$

$= 1 - 3(6)$

$= -17$

(b) at $y = -2$

$\lim_{y \to -2^-} g(y)$

$= \lim_{y \to -2^-} (y^2 + 5)$

$= (-2)^2 + 5$

$= 9$

$\lim_{y \to -2^+} g(y)$

$= \lim_{y \to -2^+} (1 - 3y) = \cancel{[1 - 3(-2)]}$

$= [1 - 3(-2)]$

$= 7$

$\therefore \lim_{y \to -2} g(y) =$ one .... $\{\because \lim_{y \to -2^-} g(y) \neq \lim_{y \to -2^+} g(y)\}$

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Q7ans) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$ .... {given}

Differenciate wrt t using product rule

$$f'(t) = \frac{d}{dt}(4t^2 - t)(t^3 - 8t^2 + 12)$$

$$f'(t) = (t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t) + (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12)$$

$$f'(t) = (t^3 - 8t^2 + 12)\left[\frac{4d(t^2)}{dt} - \frac{d(t)}{t}\right] + (4t^2 - t)\left[\frac{d(t^3)}{dt} - \frac{8d(t^2)}{dt} + \frac{d(12)}{dt}\right]$$

$$f'(t) = (t^3 - 8t^2 + 12)(4 \times 2t - 1) + (4t^2 - t)(3t^2 - 8 \times 2t + 0)$$

$$f'(t) = (t^3 - 8t^2 + 12)(8t - 1) + (4t^2 - t)(3t^2 - 16t)$$

$$\therefore f'(t) = (t^3 - 8t^2 + 12)(8t - 1) + (4t^2 - t)(3t^2 - 16t)$$

Q8ans) $R(w) = \frac{3w + w^4}{2w^2 + 1}$

$$\frac{dR}{dw} = \frac{(2w^2 + 1)\frac{d}{dw}(3w + w^4) - (3w + w^4)\frac{d}{dw}(2w^2 + 1)}{(2w^2 + 1)^2}$$

$$\frac{dR}{dw} = \frac{(3 + 4w^3)(2w^2 + 1) - 4w(3w + w^4)}{(2w^2 + 1)^2}$$

$$\boxed{\frac{dR}{dw} = \frac{4x^5 + 4x^3 - 6x^2 + 3}{(2x^2 + 1)^2}}$$

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Q9 ans) $f(x) = 2e^x - 8^x$

Differenciate $f(x)$ with respect to x

$$= \frac{d}{dx}(2e^x - 8^x)$$

$$= \frac{d}{dx}(2e^x) - \frac{d}{dx}(8^x)$$

$$= 2e^x - 8^x \ln(8)$$

We know that $\frac{d}{dx}(e^x) = e^x$ and $\cancel{\frac{d}{dx}(a^x) = a^x}$

$\cancel{\frac{d}{dx}(a^x) = a^x \ln(a)}$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$\therefore \cancel{f'(x) = 2}$

$$\therefore f'(x) = 2e^x - 8^x \ln(8)$$

Q10 ans) $y = z^5 - e^z \ln(z)$

~~Differenet~~ Differenciate wrt z

$$\frac{dy}{dz} = (z^5 - e^z \ln(z))'$$

$$\frac{dy}{dz} = (z^5)' - [(e^z \ln(z))]'$$

$$\frac{dy}{dz} = 5z^4 - [(e^z)'(\ln(z)) + (\ln(z))'(e^z)]$$

$$\frac{dy}{dz} = 5z^4 - \left(e^z \ln z + \frac{1}{z} e^z\right)$$

$$\frac{dy}{dz} = 5z^4 - e^z\left(\ln z + \frac{1}{z}\right)$$

Q11ans) $f(x) = (6x^2+7x)^4$

(a) Differenciate $f(x)$ with respect to x

$$\frac{d}{dx}(fx) = \frac{d}{dx}(6x^2+7x)^4$$

$$\frac{d}{dx} = 4(6x^2+7x)^3 \frac{d}{dx}(6x^2+7x)$$

$$\frac{d}{dx} = 4(6x^2+7x)^3 \times (12x+7)$$

(b) $f(t) = 5+e^{4t+t^7}$

Differenciate $f(t)$ with respect to t

$$\frac{df(t)}{dt} = \frac{d}{dt}(5+e^{4t+t^7})$$

$$\frac{d}{dt}f(t) = 0+e^{4t+t^7}\frac{d}{dt}(4t+t^7)$$

$$\frac{d}{dt}f(t) = e^{4t+t^7} \times (4+7t^6)$$

Q12 ans) (a) $z = \ln(7 - x^3)$

$$\frac{d}{dx}(z) = \left(\frac{1}{7-x^3}\right)(-3x^2) = \frac{-3x^2}{7-x^3} = \frac{3x^2}{x^3-7}$$

$$\frac{d''z}{dx} = \frac{(x^3-7)\frac{d}{dx}(3x^2) - 3x^2\frac{d}{dx}(x^3-7)}{(x^3-7)^2}$$

$$\frac{d''z}{dx} = \frac{6x(x^3-7) - 3x^2(3x^2)}{(x^3-7)^2}$$

$$\frac{d}{dx}\frac{d''z}{dx} = \frac{-3x^4 - 42x}{(x^3-7)^2}$$

$$\frac{d''z}{dx} = \frac{-3x(x^3-14)}{(x^3-7)^2}$$

(b)

$$Q(v) = \frac{2}{(6+2v-v^2)^4}$$

$$Q'(v) = \frac{-8\,(2-2v)}{(6+2v-v^2)^5}$$

$$Q'(v) = \frac{16\,(v-1)}{(6+2v-v^2)^5}$$

$$Q''(v) = 10\left[\frac{(6+2v-v^2)^5 - (v-1)\,5(6+2v-v^2)^4(2-2v)}{(6+2v-v^2)^{10}}\right]$$

$$Q''(v) = 16\left[\frac{(6+2v-v^2)+10(v-1)^2}{(6+2v-v^2)^{5}}\right]$$

$$Q''(v) = 16\left[\frac{9v^2-18v+16}{(6+2v-v^2)^5}\right]$$

(c) $f(t) = \ln(1+t^2)$

~~(c) f(t) = ln(~~ ~~f'(t)~~ $f'(t) = \dfrac{1}{1+t^2} \times 2t = \dfrac{2t}{(1+t^2)}$

$$f''(t) = 2\left[\frac{(1+t^2) - t(2t)}{(1+t^2)^2}\right]$$

$$f''(t) = 2\left[\frac{1-t^2}{(1+t^2)^2}\right]$$

Q13 ans) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

For maximum and minimum function

$f'(x) = 0$

$\therefore \quad 3x^2 - 6x - 9 = 0$

$\cancel{3}x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$\cancel{x = 3 \text{ or }} \; x = -1$ or $x = 3$

$f''(x) = 6x - 6$

~~at $x = 3$, $f''(3)$~~

~~$f''(3) = f(6(3) - 6 = 18 - 6 = 12$~~

at $x = -1$

$f''(-1) = 6 \times (-1) - 6 = -6 - 6 < 0$

$\therefore$ function ~~is~~ has maximum value at $x = -1$ and maximum value $=$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 \times 1 + 9 + 12$

$= -1 - 3 + 9 + 12$

$= 17$

$f''(x) = 6x - 6$ at $x = 3$

$f''(x) = 6 \times 3 - 6 = 18 - 6 > 0$

$\therefore$ function is minimum at $x = 3$ and minimum value

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= 0 - 15$

$= -15$

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Q14ans) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

For maximum and minimum ~~value~~ function $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$\cancel{x^2} - \cancel{3x}\ \cancel{2}\quad x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-2)(x-1) = 0$

$x = 1$ or $x = 2$

$f''(x) = 24x - 36$ at $x = 1$

$f''(1) = 24 \times 1 - 36 = 24 - 36 < 0$

$\therefore$ function has maximum value at $x = 1$ and maximum value

$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$f(1) = 4 \times 1 - 18 \times 1 + 24 \times 1 - 7$

$f(1) = 4 - 18 + 24 - 7$

$f(1) = 3$

$f''(x) = 24x - 36$ at $x = 2$

$f''(2) = 24 \times 2 - 36 = 48 - 36 > 0$

$\therefore$ function has minimum value at $x = 2$ and minimum value

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$f(2) = 4 \times 8 - 18 \times 4 + \cancel{48}\ 24 \times 2 - 7$

$f(2) = 32 - 72 + 48 - 7$

$f(2) = -40 + 48 - 7$

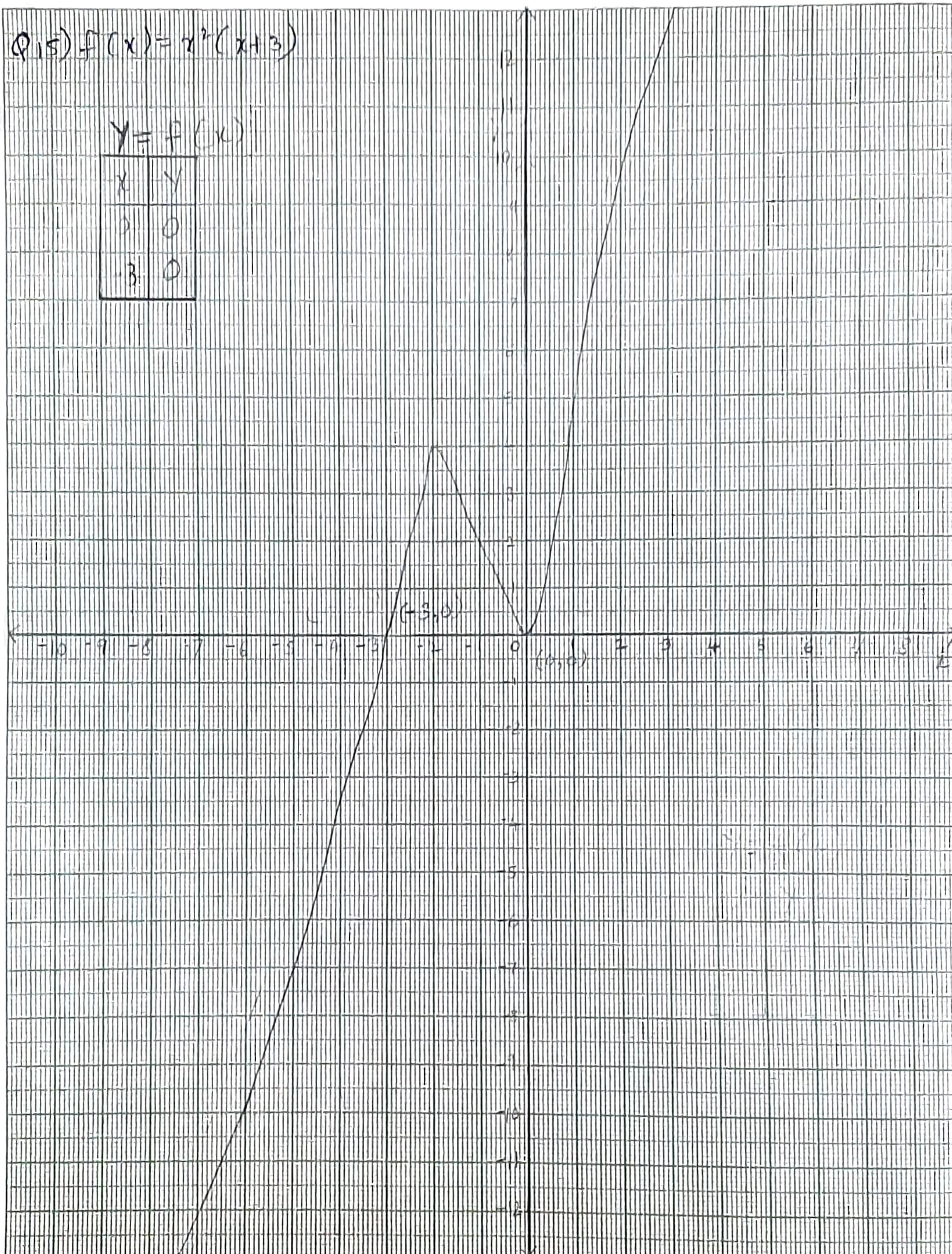
$f(2) = 8 - 7$

$f(2) = 1$

Q15) $f(x) = x^2(x+3)$

$y = f(x)$

x	y
0	0
-3	0



Q16) $f(x) = 3x^2 - 6x$

$y = f(x)$

x	y
0	0
2	0