classmate

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Calculus Assignment

1) $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$

$= \lim_{h\to 0} \frac{2(9-6h+h^2)-18}{h}$

$= \lim_{h\to 0} \frac{\cancel{18}-12h+2h^2\cancel{-18}}{h}$

$= \lim_{h\to 0} \frac{-12h+2h^2}{h}$

$= \lim_{h\to 0} \frac{2\cancel{h}(-6+h)}{\cancel{h}}$

$= \lim_{h\to 0} 2(-6+h)$

$= 2(-6+0)$

$= -12$

2) $g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$

a) $g(4) = 2(4)$
$= 8$ $\quad\therefore$ function is continuous at $x=4$

b) $g(6) = 6-1$
$= 5$ $\quad\therefore$ function is continuous at $x=6$

3) $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{(3-\sqrt{x})(3+\sqrt{x})}$

$= \lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim_{x\to 9} \frac{-1(9-x)(3+\sqrt{x})}{9-x}$

$= \lim_{x\to 9} -1(3+\sqrt{x})$

$= -1(3+\sqrt{9})$

$= -1(3+3)$

④ $f(x) = \dfrac{4x+5}{9-3x}$

a) $f(-1) = \dfrac{4(-1)+5}{9-3(-1)}$

$= \dfrac{-4+5}{9+3}$

$= \dfrac{1}{12}$

$\therefore$ function is continuous at $x = -1$.

b) $f(0) = \dfrac{4(0)+5}{9-3(0)}$

$= \dfrac{5}{9}$

$\therefore$ function is continuous at $x = 0$.

c) $f(3) = \dfrac{4(3)+5}{9-3(3)}$

$= \dfrac{17}{0}$

$\therefore$ function is discontinuous at $x = 3$.

⑤ $\lim\limits_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim\limits_{x \to \infty} \dfrac{\cancel{e^{4x}}(6 - e^{-6x})}{\cancel{e^{4x}}(8 - e^{-2x} + 3e^{-6x})}$

$= \dfrac{6 - \frac{1}{e}6(\infty)}{8 - \frac{1}{2}e(\infty) + 3\left(\frac{1}{e^{5\infty}}\right)}$

$= \dfrac{6 - 1/\infty}{8 - 1/\infty + 3/\infty}$

$= \dfrac{3}{4}$

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(6) $g(y) = \begin{cases} y^2+5 & y<-2 \\ 1-3y & y \geq -2 \end{cases}$

a) $\lim_{y \to 6} g(y) = 1-3(6)$
$= 1-18$
$= -17$

b) $\lim_{y \to -2} g(y) = 1-3(-2)$
$= 1+6$
$= 7$

(7) $f(t) = (4t^2-t)(t^3-8t^2+12)$

$f'(t) = (4t^2-t) \cdot \frac{d}{dt}(t^3-8t^2+12) + (t^3-8t^2+12) \cdot \frac{d}{dt}(4t^2-t)$

$= (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t$
$+ 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

$= 4(5t^4 - 33t^3 + 6t^2 + 24t - 3)$

(8) $R(w) = \dfrac{3w+w^4}{2w^2+1}$

$R'(w) = \dfrac{2w^2+1 \cdot \frac{d}{dw}(3w+w^4) - (3w+w^4) \cdot \frac{d}{dw}(2w^2+1)}{(2w^2+1)^2}$

$= \dfrac{2w^2+1\,(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$

$= \dfrac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$

$= \dfrac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$

(9) $f(x) = 2e^{x} - 8^{x}$

$f'(x) = 2e^{x}$ ~~[illegible]~~ $- 8^{x}\log 8$

(10) $y = z^{5} - e^{z}\ln(z)$

$$\frac{dy}{dx} = 5z^{4} - \left[e^{z}\cdot\frac{d}{dz}(\ln z) + \ln z\cdot\frac{d}{dz}e^{z}\right]$$

$$= 5z^{4} - \left[e^{z}\left(\frac{1}{z}\right) + \ln z\,(e^{z})\right]$$

$$= 5z^{4} - \frac{e^{z}}{z} - \ln z\,(e^{z})$$

(11) (a) $f(x) = (6x^{2} + 7x)^{4}$

$$f'(x) = 4(6x^{2} + 7x)^{3}\cdot\frac{d}{dx}(6x^{2} + 7x)$$

$$= 4(6x^{2} + 7x)^{3}(12x + 7)$$

(b) $f(t) = 5 + e^{4t + t^{7}}$

$$f'(t) = e^{4t + t^{7}}\cdot\frac{d}{dt}(4t + t^{7})$$

$$= e^{4t + t^{7}}\cdot(4 + 7t^{6})$$

(12) (a) $z = \ln(7 - x^{3})$

$$\frac{dz}{dx} = \frac{1}{7 - x^{3}}\cdot\frac{d}{dx}(7 - x^{3})$$

$$= \frac{1}{7 - x^{3}}(-3x^{2})$$

$$= \frac{-3x^{2}}{7 - x^{3}}$$

$$\frac{d^{2}z}{dx^{2}} = \frac{(7 - x^{3})\cdot\frac{d}{dx}(-3x^{2}) - (-3x^{2})\frac{d}{dx}(7 - x^{3})}{(7 - x^{3})^{2}}$$

$$= \frac{(7 - x^{3})(-6x) - (-3x^{2})(-3x^{2})}{(7 - x^{3})^{2}}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

(b) $g(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$= 2(6+2v-v^2)^{-4}$$

$$g'(v) = 2.(-4)(6+2v-v^2)^{-5} . \frac{d}{dv}(6+2v-v^2)$$

$$= -8(6+2v-v^2)^{-5}(2-2v)$$

$$g''(v) = -8\left[(6+2v-v^2)^{-5} . \frac{d}{dv}(2-2v) + (2-2v) . \frac{d}{dv}(6+2v-v^2)\right]$$

$$= -8\left[(6+2v-v^2)^{-5}(-2) + (2-2v)(2-2v)\right]$$

$$= 16(6+2v-v^2)^{-5} + 40(6+2v-v^2)^{-6}(2-2v)^2$$

(c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{1}{(1+t^2)}\frac{d}{dt}(1+t^2)$$

$$= \frac{2t}{1+t^2}$$

$$f''(t) = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}$$

$$= \frac{2 - 2t^2}{(1+t^2)^2}$$

$$= \frac{2(1-t^2)}{(1+t^2)^2}$$

(13) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0$ or $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

$f''(x) = 6x - 6$

Put $x = 3$ in $f''(x)$

$f''(3) = 6(3) - 6$

$= 18 - 6$

$= 12$

$\therefore f(x)$ is minimum at $x = 3$.

Put $x = -1$ in $f''(x)$

$f''(-1) = 6(-1) - 6$

$= -6 - 6$

$= -12$

$\therefore f(x)$ is maximum at $x = -1$.

Put $x = 3$ in $f(x)$

$\therefore$ Minimum value $= (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 3(9) - 27 + 12$

$= 27 - 27 - 27 + 2$

$= -15$

Put $x = 1$ in $f(x)$

$\therefore$ Maximum value $= (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3(1) + 9 + 12$

$= -1 - 3 + 9 + 12$

$= 17$

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

Put $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$12(x^2 - 3x + 2) = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-2)(x-1) = 0$

$x - 2 = 0$ or $x - 1 = 0$

$x = 2$ $\qquad$ $x = 1$

$f''(x) = 24x - 36$

Put $x = 2$ in $f''(x)$

$f''(2) = 24(2) - 36$

$= 48 - 36$

$= 12$

$\therefore f(x)$ is minimum at $x = 2$.

Put $x = 1$ in $f''(x)$.

$f''(1) = 24(1) - 36$

$= 24 - 36$

$= -12$.

$\therefore f(x)$ is maximum at $x = 1$

Put $x = 2$ in $f(x)$

$\therefore$ Minimum value $= 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 4(8) - 18(4) + 48 - 7$

$= 32 - 72 + 48 - 7$

$= 1$

Put $x = 1$ in $f(x)$

$\therefore$ Maximum value $= 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3$

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(15) $f(x) = x^2(x+3)$

$f'(x) = 2x(x+3) + (1)x^2$

$= 2x^2 + 6x + x^2$

$= 3x^2 + 6x$

Put $f'(x) = 0$

$3x^2 + 6x = 0$

$3(x^2 + 2x) = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x = -2$

$f''(x) = 6x^2 + 6$

Put $x = 0$ in $f''(x)$: $f''(0) = 6(0) + 6 = 6$ $f''(0) >$

Put $x = -2$ in $f''(x)$ $\therefore f''(-2) = 6(-2) + 6 = -6$ $f''(-2$

$\therefore$ $f(x)$ is maximum at $x = 2$ & minimum at $x = 0$

Put $x = -1$ in $f'(x)$

$\therefore f'(-1) = 3(-1)^2 + 6(-1)$

$= 3 - 6$

$= -3$

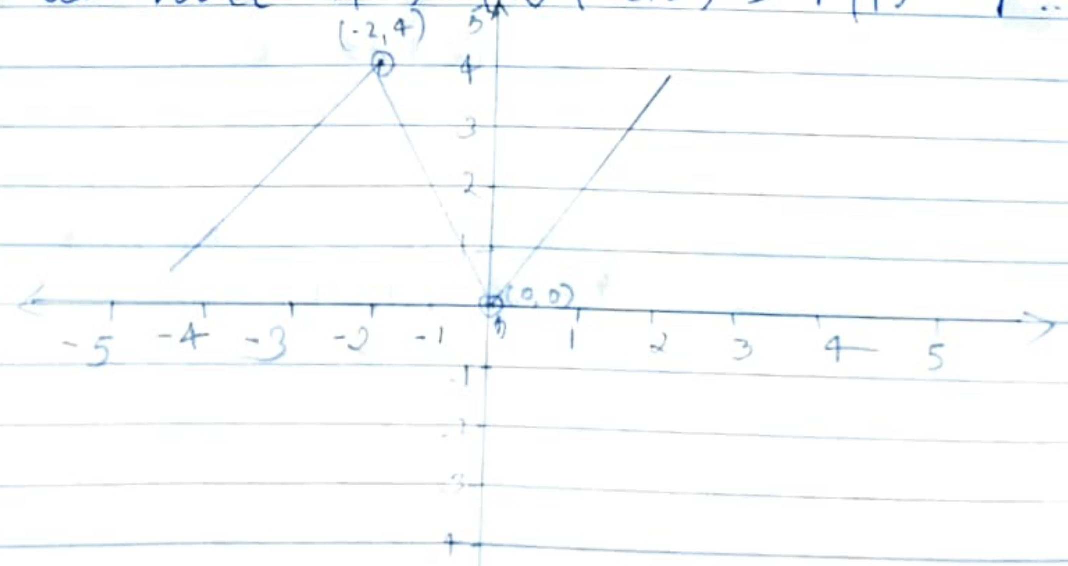
$\therefore$ The given function is deceasing between -2 & 0.

Put $x = 0$ in $f(x)$.

Minimum value $= f(0) = 0(0+3) = 0$ $\therefore x = 0, y = 0$

Put $x = -2$ in $f(x)$

Maximum value $= f(-2) = (-2)^2(-2+3) = 4(1) = 4$ $\therefore x = -2,$



(16) $f(x) = 3x^2 - 6x$

i.e. $y = 3x^2 - 6x$

Put $x = 0$ in $f(x)$

$y = 3(0)^2 - 6(0) = 0$

Put $x = -1$ in $f(x)$

$y = 3(-1)^2 - 6(-1)$

$= 3 + 6$

$= 9$

Put $x = -2$ in $f(x)$

$\therefore y = 3(2)^2 - 6(2)$

$= 12 - 12$

$= 0$

Put $x = -3$ in $f(x)$

$\therefore y = 3(-3)^2 - 6(-3)$

$= 3(9) + 18$

$= 45$

Put $x = 1$ in $f(x)$

$\therefore y = 3(1)^2 - 6(1) = 3$

Put $x = 2$ in $f(x)$

$\therefore y = 3(2)^2 - 6(2)$

$= 12 - 12$

$= 0$

Put $x = 3$ in $f(x)$

$y = 3(3)^2 - 6(3)$

$= 3(9) - 18$

$= 9$

x
0
1
-1
2
2
3
-3