Financial Mathematics
Assignment - 2
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Q.1. Loan Amount = INR 20,000
Monthly Repayments = INR 427.9
Yearly Repayments = 12×427.9
$= 5134.8$

term of loan = 5
Total Repayment = 5×5134.8
$= 25674.00$

Total Interest = Total Repayment - Original loan

$= 25674 - 20000$
$= 5674.$

~~Flat Rate~~ = Annual Interest = $\frac{5674}{5} = 1134.8$

Flat Rate = $\frac{1134.8}{20000} = 0.05674$
$= 5.674\%$

(i) APR = $2 \times$ flat Rate = 11.348%.

(ii) After one year capital left to pay = $12 \times a_{\overline{4}|}^{(12)}$ @ 11.348% $\times 427.9$

$= 12 \times \frac{1 - (1.11348)^{-4}}{0.17954926} \times 427.9$

iii) Original Loan had 4 years of payment remaining
Total Interest $= 21 \times 12 \times 427.90 - 16775.68$
$= 3763.22$

New loan has a term of 7.25 years
Total Interest $= 7.25 \times 12 \times 274.9 - 16775.68$
$= 7104.65$
$7104.65 - 3763.23 = 3341.23$
Hence They will have to pay 3341.23 more interest.

Q2. Discounted Pay back Period.
The DPP is the ~~the~~ time 't' for which the accumulated or the present value of the of the return upto time t exceeds the accumulate or present value of the cost upto time t

Pay back Period: The payback period is the same as the dpp, except the present value calculation is carried out using an interest rate of 0.
It is the time for which the monetary value of the return exceeds the monetary value of the costs.

(ii) Unlike the NPV, neither the DPP nor the payback period gives an indication how profitable a project is as they ignore cash flows after the accumulated value zero is reached.
There may ~~be~~ not be one unique time when the balance in the investors' account changes from negative to positive. However

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NPV can always be calculated. The payback Period can be misleading as it does not give misleading results and not take into account the time value of money. Hence NPV is a superior measure compared to DPP or payback period

(iii) IRR. The IRR for Project A. is the rate of interest that satisfies the equation value.

$-170000 - 200000 - 10000v^2 + 20000v + 20000v^2 + 20000v^3 = 0$

$10v^2 + 200v^3 = 170$

$10v^2 + 200v^3 - 1700 = 0$

$0.9 \rightarrow -16.1$

$0.94 \rightarrow 4.9538$

$0.9309 \longrightarrow 0.0046$

$\therefore v = 0.9309$

$$\frac{1}{1+i} = 0.9309$$

$\Rightarrow i = 0.074229224032$

$i = 7.422924\%$

So IRR of the project is around 7%

$149 a_{\overline{6}|} \; 20 + 200V^6 = 200$

(b) NPV – The net present value are found by discounting the payment at 6%.

Project A NPV $= -170000 + 1000\,v^2 + 20000\,v^3 = 6823.82$

Project B NPV $= -200000 + 14000\,a_{\overline{6}|} + 200000\,v^6$
$= 9834.645$

Maximum Interest Rate ~~by~~ Beyond which projects are unprofitable. Each project will be profitable if the rate of interest at which funds can be borrowed is less than the IRR.

So to be profitable Project A requires a borrowing rate less than 7.42292% and Project B require a rate lower that 7%.

Other factor to be considered include

- Project A does not provide any net income during the 1st year
- ~~Pro~~ The lower IRR for Project B applies for a longer period.
- Rate of interest available in year (4–6) will affect the comparison between the accumulated profits at the end of year 6.

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Q.3. Value of 1st November 1985 5]

Annuity 1 = $2000 \times v^{11/4} \times \ddot{a}_{\overline{27}|}$ at 8%.

$= 200 \times 0.9809444 \times 10.2007 \times \frac{0.08}{0.0740711}$

$= 2161.366301$

Annuity 2 $= 320\,(1-e^{i})^{1/12} \times a^{(4)}_{\overline{16}|}$ at 8%.

$= 320 \times 1.006434 \times \frac{1-(0.925426)}{0.087708}$

$= 2957.870071$

Annuity 3 : 180 $a^{(12)}_{\overline{18.75}|}$

$= 180 \times \frac{1-(0.925926)^{18.75}}{0.07782208}$

$= 1780.664166$

Let the revised annuity be A

Value of revised annuity is $x(1+i)^{1/4} \times a^{(4)}_{\overline{20}|}(1+i)^{1/4}$

$A = \frac{6899.90}{1.019427 \times 10.30886}$

$= 656.56$

Q. 4)

1 2 3 4 5 . . . . . . n

0 1 2 3 4 . . . . . . (n-1) n

$$(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + \dots + nv^{(n-1)}$$

$$(I\ddot{a})_{\overline{n}|} = (1+i)(Ia)_{\overline{n}|}$$

$$(I\ddot{a})_{\overline{n}|} = (1+i)\frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{\frac{i}{(1+i)}} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

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Q.5.)

(i) TWRR of Property fund

$$(1+i)^{1} = \cancel{F} \frac{16.4}{12.4}$$

$$(1+i) = 1.322580645$$

$$\therefore i = 0.322580645$$

$$= 32.2580645\ \%$$

TWRR of Equity Fund

$$(1+i) = \cancel{\frac{12.1}{15}} \frac{15.5}{12.1}$$

$$1+i = 1.280991736$$

$$\therefore i = 28.0991736\%$$

(ii) a) Assuming that the investor bought 100 units per quater in property fund. The equation value is given by

$$1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1580(1+i)^{1/4} = 4 \times 1640$$

If $(1+i) = x$

$$1240x + 1310(x)^{3/4} + 1480(x)^{1/2} + 1580\, x^{1/4} - 6560 = \cancel{501}$$

$\cancel{1.245}$

$1.2 \to -29.31$

$1.2 \to 2.145$

$1.29 \to -0.991$

$1.293 \to -0.047$

$1.204 \to 0.2637$

$1.293 \to -0.017$

$1.2912 = 0.0120$

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$x \approx 1.2932$

$1+i \approx 1.2932$

$i \approx 29.32\%$

(5) Assuming that the investor purchases INR 100 worth of units each quater $400 \times \ddot{s}_{\overline{1}|}^{(4)} = 100 \times 16.4 \left(\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.7} + \frac{1}{15.0} \right)$

$\ddot{s}_{\overline{1}|}^{(4)} = 1.1801$

$\frac{(1+i)^1 - 1}{i^{(4)}} = 1.807$

$\frac{i}{i^{(4)}} = 1.807$ ... $\therefore$ $i^{(4)} = \left[4(1+i)^{1/4} - 1\right]$

(iii) $1210(1+i) + 920(1+i)^{3/4} + 1030(1+i)^{1/2} + 1310(1+i)^{1/4} = 4 \times 1330$

Let $(1+i) = x$

$1210x + 920x^{3/4} + 1030x^{1/2} + 1310x^{1/4} = 62000$

$x \to 1.6 \to -14.89$

$x \to 1.7 \to 6.5487$

$x \to 1.67 \to -0.752$

$x \to 1.68 \to 1.6841$

$x \to 1.673 \to -0.021$

$x \to 1.674 \to 0.2224$

$x \to 1.6731 \to 0.000342481$

$x = 1.6731$

$\therefore i = 0.6731$

$\therefore i = 67.31\%$

(b) $400 \times \ddot{s}^{(4)}_{\overline{1|}} = 100 \times 15.5 \left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right)$

$\ddot{s}^{(4)}_{\overline{1|}} = \frac{15.5}{4}\left(0.0826462 + 0.1086956 5 + 0.09 708 73786 + 0.07633587786\right)$

$\ddot{s}^{(4)}_{\overline{1|}} = 1.413456705$

$\frac{x - 1}{\left[1 - \left(\frac{1}{x}\right)^{1/4}\right] \times 4} = 1.41345 8705$

$1.7 \to -0.00496$

$1.8 \to 0.0493$

$1.71 \to 0.00005$

$1.708 \to -0.00005$

$1.0709 \to 0.000005$

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$\therefore 1+i = 1.7089$

$\therefore i = 70.89\%$

Q.6.

(i) Let x be the initial annual repayment

$160000 = x\, a_{\overline{7}|} @ 8\%$

$x = \dfrac{160000}{a_{\overline{10}|}} = \dfrac{160000}{6.7101} = 23844.65$

(ii) The loan outstanding immediately after the 4th payment

$23844.65\, a_{\overline{6}|} @ 8\% = 23844.65 \times 4.6229 = 110,231.442$

Let y be the revised annual installment The equation of value $y\, a_{\overline{6}|} = 110231.44 @ 10\%$

$\therefore y = 25309.782$

(iii) The loan outstanding immediately after 7th installment

$25309.72\, a_{\overline{3}|} @ 10\%$

$= 25309.72 \times 2.4869 = 62942.74$

Let z be the revised annual installment

$z\, a_{\overline{3}|} = 62942.74 @ 9\%$

$z = \dfrac{62942.74}{a_{\overline{3}|}} = \dfrac{62942.74}{2.5313} = 24865.77$

~~The can~~ be

$160000 + 1465.07\, a_{\overline{4}|} - 443.95\, a_{\overline{7}|} - 24865.77\, a_{\overline{10}|}$

$160000 + 1465.07\left(\dfrac{1-v^4}{i}\right) - 442.95\left(\dfrac{1-v^7}{i}\right)$ ~~2t~~

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8.7) Investing in property may not be preferred ~~after~~ as alternative to investing in government securities. The return on property is lower than that in government securities. It is difficult to invest in property. Investing in property is less flexible because of the large unit size. The market ability for property is poor. Large buying & selling expense.

Tender usually refers to process to hereby government & financial institutions invite bids for large projects that must be submitted within a finite deadline. Government may raise money by floating a loan on stock exchange. The term of the issue are set out by tender. The investors are invited to nominate the price and it is issued

For the Government:

Advantages:- They will know the price that the investor will pay at the outset so it will know the cost of borrowing money. It will be administratively less difficult by tender.

Disadvantages: The investor will be willing to pay more for the bond than the set price and so the money could be borrowed more cheaply.

The securities with uncertain returns are as follows.

Equity:- The dividend depends on the companies ~~[illegible]~~ performance.

Property: Rent payments are subject to review.

Index linked bond: Coupon payment & final redemption price may increase in relation to the index of inflation.

Q.3) (i) The Present value of ~~outlay~~ ~~[illegible]~~ outflow:

$2.7 + a_{\overline{3}|} @ i$

The expected PV = $1.03v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) @ i$

PV of outflow to the PV of inflow

$2.7 + 0.2\,\bar{a}_{\overline{3}|} = 0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) @ i$

(ii) PV of inflow = $0.1v^3 + 0.35\,\bar{a}_{\overline{10}|}\,v^3$

$0.1v^2 + 0.35\,\bar{a}_{\overline{10}|} - 2.7 - 0.2\,a_{\overline{3}|} @ i = 10\%$

$= 0.1 \times 0.7513 + 0.35 \times \cancel{[illegible]}\ 0.7513 \times 0.4487 -$

$2.70 - 0.2 \times 2.04866$

$= 4.19 - 3.19$

$= 1.0099$.

NPV = +ve hence project is worth investing.

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Q.9.

TWRR

$$(1+i)^3 = \frac{33}{30.50} \times \frac{41.05-4.5}{33+6} \times \frac{45.6}{41.05} \times \frac{47}{466-250}$$

$$(1+i)^3 = 1.08196713 \times 0.93717948372 \times 1.110840439 \times 1.090437$$

$(1+i)^3 = 1.2281327$

$(1+i) = 1.070951231$

$i = 7.0951231\%$.

MWRR

$30.5(1+i)^2 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} - 47$

$(1+i) = x$

x	$\to$ RHS	x	$\to$ RHS
1	$\to -9$	1.07	$\to -0.367$
1.1	$\to 2.9346$	1.08	$\to 0.8702$

$1.072 \to -0.00240710$ $\quad 1.0721 \to 0.008495827$

$1.073 \to 0.121695165$ $\quad 1.07204 \to 0.00095563$

$x = 1.07204$

$i = 0.07204$

$= 7.204\%$

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(ii) Linked rate of return.

Year 2011 → ~~of~~ Equation of value

$30.5(1+i) + 6(1+i)^{0.25} = 38.50$

of $(1+i) = x$

$30.5x + x^{0.25} - 38.50 = 0$

$x \rightarrow LHS$

$1 \rightarrow -2$

$1.1 \rightarrow 1.194682135$

$1.06 \rightarrow -0.087$

$1.07 \rightarrow 0.2373$

$1.062 \rightarrow -0.98$

$1.063 \rightarrow 0.0138$

$1.0625 \rightarrow -0.00212044569$

$1.0626 \rightarrow 0.0010712882058$

$1+i = 1.0626$

$i = 6.26\%$

Year 2012

Let $(1+i) = x$

$38.5x + 4.850x^{0.5} - 45 = 0$

$x \rightarrow LHS$

$1 \rightarrow -2$

$1.1 \rightarrow 2.0696$

$1.04 \rightarrow -0.37$

$1.05 \rightarrow 0.0361$

$1.049 \rightarrow -0.0045884$

$1.0492 \rightarrow 0.0035208$

$1+i = 1.0492$

$\therefore i = 4.92\%$

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Year 2013

$(1+i) = x$

$46x - 2.5\,x^{0.5} - 47.20$

$x \to RHS$

$1.1 \to -0.122$

$1.\cancel{1} \to 4.2013$

$1.12 \to 0.376$

$1.102 \to -0.034404$

$1.10279 \to 0.00020473$

$1.1028 \to 0.0000428814$

$1.103 \to 0.0094043$

$1.1027 \to -0.0037...$

$1+i = 1.10279$

$i = 10.279\%$

Linked rate of return

$(1+i)^3 = (1.0626) \times (1.0492) \times 1.010279$

$(1+i)^3 = 1.229478427$

$(1+i) = 1.071289807.$

$i = 7.1289037\%.$

(iii) TWRR eliminates the effect of the cash flow amount and timing and hence gives a fairer bases for assessing the investment performance for the fund. MWRR is sensitive to the amount of net cashflows.

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Q.10.] (i)

$180000\ a_{\overline{15}|} + 2000\ (Ia)_{\overline{15}|}$ @ $i = 12\%$

$= 180000 \times 6.5109 + 20000 \times \left\{ \frac{\ddot{a}_{\overline{15}|} - 15v^{15}}{i} \right\}$

$= 1225962 + 20000 \times 40.43123739$

$= 2040588$

Total cost of house $= \frac{2040588}{0.8} = 25507.55$

(ii) The loan outstanding at the beginning of the 9th year.

$= (180000v + 180000v^2 \cdots 180000v^7) + [(9 \times 2000v) + 10 \times 2000v^2 - \cdots 15(2000v^7)]$

$= 340000\ a_{\overline{7}|} + 20000\ (Ia)_{\overline{7}|}$

$= 1557672 + 324158.123 = 1875350.23$.

Loan Schedule.

Year	Loan O/s	Loan Installment	Interest	Cap. Repayment
9	18,75850.35	3,60000	225102.04	134897.96
10	1740952.37	3,6000	208914.28	171085.72
11	15 69,866.65			

(iii) Let the new installment be y

$1569866.65 = y\, a_{\overline{5}|} @ 10\%$

$1569866.65 = y \times 3.790786$

$\therefore y = 414126.367$

Extra payment by Mr. Sam if the loan continued with the first bank.

Total Installment - Loan o/s at the end of 10th year

$= (400000 + 420000 + 440000 + 460000 + 480000) - 1569866.65$

$= 630133.35$

Extra payment made by Mr Sam if the loan transferred to 2nd banker

Total installment + 1% processing fees - loan o/s at the end of 10th year

$= 5 \times 414126.90 + 0.01 \times 1569866.65 - 1569866.65$

$= 57466.35$

Since the interest payment to the 1st banker is higher than the payment to 2nd banker. It is advisable to Mr. Sam to arrange his loan with the 2nd bank.

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11) (i) TWRR.

$$(1+i)^2 = \left(\frac{550+50}{500+40}\right) \times \frac{X}{530} \times \frac{\cancel{500}}{460}$$

$(1+i)^2 = 1.44$

$\therefore 1.44 = \frac{600\,X}{4.6 \times 540 \times 11}$

$\therefore X = 655.776$

(ii) MWRR

$460(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.776$

$(1+i) = X$

$460X^2 + 40X^{7/4} - 50X - 655.776 = 0$

$1.1 \rightarrow -106.91682$ $1.198 \rightarrow -0.607$

$1.2 \rightarrow 1.6575$ $1.199 \rightarrow 0.5257$

$1.19 \rightarrow -9.06\,816$ $1.1985 \rightarrow -0.042$

$1.19854 \rightarrow 0.0025233$

$(1+i) = 1.19854$

$i = 0.19754$

$= 19.754\%$

(iii) The effective rate of return for year 2015 is given by value i -

$460(1+i) + 40(1+i)^{3/4} = 600$

$460X + 40X^{3/4} - 600 = 0$

$1.2 \rightarrow -2.137$ $1.204 \rightarrow -0.134$

$1.3 \rightarrow 46.698$ $1.205 \rightarrow 0.3044$

$1.21 \rightarrow 2.7446$ $1.2043 \rightarrow -0.037$

$1.20439 = 0.00134301$

$i = 20.437\%$

Effective rate of return for the year 2016

$$(1+i) = \frac{655.78}{550} = 1.92327$$

So linked Rate of return

$$(1+i)^2 = 1.92327 \times 1.202133$$

$$(1+i)^2 = 1.43001521$$

$$(1+i) = 1.19833843$$

$$i = 19.8338483\%$$

$$LIRR = 19.83384833\%$$

(iv) When the sub interval chosen for calculating LIRR coincide with the interval between net cash flow being received then the LIRR will be identical to the TWRR.

(v) (i) TWRR is a better measure of fund manager's performance

(ii) MWRR is sensitive to the amount and time of net ~~cha~~ cashflows but fund manager does not have any control over them

(iii) TWRR is calculated using growth reflecting factor which shows the change between time of consecutive cash flows. TWRR is hence independent of the amount & the ~~amo~~ timing of the cashflows.

(iv) Hence TWRR is a better measure of the fund manager's performance

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Q.12. (i) The DPP of an investment project is the first time when the ~~net~~ NPV of the cashflow from the project is +ve.

(ii)(a) Let's work in units of 1000

$$-800(1+i)^t - 50(1+i)^{t-1} + 100\,\bar{s}_{\overline{t-2}|} \ @7\% = 0$$

$$-800(1.07)^t - 50(1.07)^{t-1} + 100 \times \frac{(1.07)^{t-2} - 1}{\ln(1.07)} = 0$$

$t \to 17 \to -74.79$ $\quad 17.76 \to -0.757$
$18 \to 23.4626$ $\quad 17.77 \to 0.2426$
$17.7 \to -6.741$ $\quad 17.767 \to -0.057$
$17.8 \to 3.24082$ $\quad 17.768 \to 0.0426.$

(b) Accumulated Profit after 22 years is found by accumulating the income after DPP has elapsed at 6% p.a.

$$100\,\bar{s}_{\overline{4.232}|} \text{ at } 6\% = 100 \times \frac{(1.06)^{4.232} - 1}{\ln(1.06)}$$

$$= 480.0743066$$

Accumulated amount = ₹ 480074.3066.

(iii) The businessman may accumulate his rental income for each year @ 6%.

$$100\,\bar{s}_{\overline{17}|} = 100 \times \frac{(1.06)^{17} - 1}{\ln(1.06)}$$

$$= 102.9708672$$

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The DPP is the smallest integer:

$$-800(1+i)^t - 50(1+i)^{t-1} + 102.97085\, s_{\overline{t-2}|} @7\%$$

$$-800(1.04)^t - 50(1.04)^{t-1} + 102.9708\, \frac{(1.04)^{t-2}-1}{\ln(0.04)}$$

$t \rightarrow 17 \rightarrow -87.1$

$t \rightarrow 18 \rightarrow 9.7866$

$t = 18$ years

The balance is the businessman account after translation 18 years. is.

$$\cancel{800} \quad -800(1+i)^{18} - 50(1+i)^{17} + 102.9716\, s_{\overline{17}|}$$

$$= 2703.945827 - 157.9407605 + 287.16655545.$$

$$= 9.77896$$

Av after 22 year.

$$9.77896(1+i)^4 + 100\, s_{\overline{4}|} @6\%.$$

$$+100 \times s_{\overline{4}|} \times 1$$

$$= \cancel{12.34571167+460}$$

$$= 12.34571167 + 460.205454.$$

$$= 462.8015571$$

Accumulated amount after 22 years is 462801.3571

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Q. 13. (i) Let $x/12$ be the monthly repayment

$x\, a_{\overline{25}|}^{(12)} = 9330$

$x\,(11.6536) \times \frac{0.07}{0.067850} = 9330$

$x = 821.7669096$

$\frac{x}{12} = 68.4805758$

The monthly repayment = 68.4805757

(ii) Loan o/s immediately after repayment on 10th March 2026

$x\, a_{\overline{34/3}|}^{(12)}$ @ 7% $\cancel{821.7669096}$ $821.7669096 \times \frac{1-(1.07)^{-32/2}}{0.067850}$

$= 6485.745315$

(iii) The loan o/s just after the repayment on 10th September is made $x\, a_{\overline{83/6}|}^{(12)}$ @ 7%.
The loan o/s just after the repayment on 10th Oct 2020 is made
$\cancel{x\, a_{\overline{83/4}|}^{(12)}}$ @ 7%.

$x\, a_{13.75}^{(12)}$

Capital repaid on 10th October 2026 is thus

$x\left[a_{\overline{83/6}|}^{(12)} - a_{\overline{55/4}|}^{(12)}\right]$ @ 7%.

$x\left[\frac{(1.07)^{-13.75} - (1.07)^{-83/6}}{0.067850}\right]$

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$$821.7669096\,(0.03268447611) = 26.85902093$$

(iv) The capital repayment contained in these 12 installments

$$\left[a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|}\right] @ 7\%.$$

$$821.7669096\left[\frac{(1.07)^{-19/3} - (1.07)^{-22/3}}{0.06785}\right]$$

$$= 516.1973861$$

(ii) Total interest in these 12 installments is

$$821.7669096 - 516.1973861 = 305.5695235$$