Calculus Assignment - 1

① $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$\lim_{h \to 0} \frac{2(9-6h+h^2)-18}{h}$

$\lim_{h \to 0} \frac{18-12h+2h^2-18}{h}$

$\lim_{h \to 0} \frac{-12h+2h^2}{h}$

$\lim_{h \to 0} \frac{2\cancel{h}(-6+h)}{\cancel{h}}$

$\lim_{h \to 0} -12+2h$

Ans) -12

② $g(x) = \begin{cases} 2x, & x<6 \\ x-1, & x \geq 6. \end{cases}$

at $x=4$, $x=6$.

$\underset{x \to 4}{g(x)} = 2x$

$= 8$

$\lim_{x \to 6} = x-1$

$= 5$ $\therefore$ $g(x)$ is discontinuous

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(3) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$\lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{-(\sqrt{x}-3)}$

$\lim_{x \to 9} -(\sqrt{x}+3)$

$\lim_{x \to 9} -(\sqrt{9}+3)$

$\lim_{x \to 9} -(3+3) = -6$.

(4) $f(x) = \frac{4x+5}{9-3x}$ at $x = -1, 0, 3$.

i) $x = -1$

$\lim_{x \to -1} \frac{4(-1)+5}{9-3(-1)}$

$\lim_{x \to -1} \frac{1}{12} = f(-1)$

∴ continuous at $x = -1$.

$\lim_{x \to -1} f(x) = f(-1)$

ii) $x = 0$

$\lim_{x \to 0} \frac{4(0)+5}{9-3(0)}$

$\lim_{x \to 0} \frac{5}{9}$.

continuous at $x = 0$.

$\lim_{x \to 0} f(x) = f(0)$

$x = 3$

$\lim_{x \to 3} \frac{4x+5}{9-3x}$

$\lim_{h \to 0} \frac{4(3-h)+5}{9-3(3-h)}$

$\lim_{h \to 0} \frac{4(3-h)+5}{9-9+3h}$

$\lim_{h \to 0} \frac{17-4h}{3h} = \infty$

$\lim_{x \to 3^+} \frac{4x+5}{9-3x}$

$\lim_{h \to 0} \frac{12+5+4h}{-h}$

$x \in R - \{3\}$

LHL $\neq$ RHL

$f(x)$ is discontinuous at $x = 3$

(5) $\lim\limits_{x\to\infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\lim\limits_{x\to\infty} \dfrac{6e^{6x} - 1}{8e^{6x} - e^{4x} + 3e^{-x}}$

$\lim\limits_{x\to\infty} e^{6x}\left(\dfrac{6x - 1/e^{6x}}{8 - \dfrac{1}{e^{2x}} + \dfrac{3}{e^{5x}}}\right)$

$\dfrac{1}{e^{2x}} = \dfrac{1}{e^{6x}} = \dfrac{1}{e^{5x}} = 0$, when $x \to \infty$.

$\lim\limits_{x\to\infty} \dfrac{6}{8} = \dfrac{3}{4}$

(6) $g(y) = \begin{cases} y^2 + 5, & y \leq -2 \\ 1 - 3y, & y \geq -2 \end{cases}$

(a) $\lim\limits_{y\to 6} g(y) = 1 - 3y$

$1 - 3(6)$

$= -17.$

(b) $\lim\limits_{y\to -2} g(y) = y^2 + 5$

$= (-2)^2 + 5$

Ans $= 9$

⑦ $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$\frac{d}{dx}\{(4t^2 - t)(t^3 - 8t^2 + 12)\}$.

$\left((4t^2 - t)\frac{d}{dx}(t^3 - 8t^2 + 12)\right) + (t^3 - 8t^2 + 12)\frac{d}{dx}(4t^2 - t)$

$(4t^2 - t)(3t^2 - \cancel{8}^{16}t) + (t^3 - 8t^2 + 12)\, d(8t - 1)$

$12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2$
$+ 96t - 12$

$\Rightarrow 20t^4 - 132t^3 + 24t^2 + 96t - 12$

⑧ $R(\omega) = \frac{3\omega + \omega^4}{2\omega^2 + 1}$.

$= \frac{(2\omega^2 + 1)\frac{d}{dx}(3\omega + \omega^4) - (3\omega + \omega^4)\frac{d}{dx}(2\omega^2 + 1)}{(2\omega^2 + 1)^2}$

$= \frac{8\omega^5 + 6\omega^2 + 4\omega^3 + 3 - [12\omega^2 + 4\omega^5]}{(2\omega^2 + 1)^2}$

$= \frac{4\omega^5 - 6\omega^2 + 4\omega^3 + 3}{(2\omega^2 + 1)^2}$.

(9) $f(x) = 2e^{x} - 8^{x}$

$= 2\frac{d}{dx} e^{x} - \frac{d}{dx} 8^{x}$

$= 2e^{x} - 3\ln(2)$

(10) $y = z^{5} - e^{z} \ln(z)$

$\log y = 5\log z - z\log e - \log(\log z)$

$\frac{1}{y}\frac{dy}{dx} = \frac{5}{z} - 1 - \frac{1}{\log z} \times \frac{1}{z}$

$\frac{dy}{dx} = z^{5} - e \ln z \left[\frac{5\log z - z\log z}{\log z}\right]$

(11) a) $f(x) = \cancel{6x^{2}+7}\,(6x^{2}+7x)^{4}$

$f'(x) = 4(6x^{2}+7x)^{3} \times (12x+7)$

b) $f(t) = 5 + e^{4t+t^{7}}$

$f'(t) = 0 + e^{4t+t^{7}}$

$(e^{4t+t^{7}}) \times (4 + 7t^{6})$

Ans $= (e^{4t+t^{7}})(4+7t^{6})$

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(12) $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \frac{d}{dx} \log(7-x^3)$$

$$\frac{d'}{dx} = \frac{1}{(7-x^3)} \times (-3x^2).$$

$$\frac{d''}{dx} = \frac{-3x^2}{(7-x^3)}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4.}{(7-x^3)^2}.$$

$$= \frac{-42x - 3x^4.}{(7-x^3)^2}$$

(b) $g'(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$\frac{-2[4(6+8v-v^2)^3(2-2v)]}{(6-2v-v^2)}$$

$g(v) \Rightarrow [-16+16v][6+2v-v^2]^{-5}$

$g''v \Rightarrow [6+2v-v^2]^{-5}[16] + [16v-16][-5(6+2v-v^2) \times (2-2v)]$

Ans) $16(6+2v-v^2)^{-5} + (2-2v)(16v-16) - 5(6+2v$

(c) $f(t) = \ln(1+t^2)$

$f'(t) = \dfrac{1}{(1+t^2)} \times 2t$

$$f''(t) = \frac{(1+t^2)\frac{d}{dx}2t - 2t\frac{d}{dx}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)2 - [2t \times 2t]}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

Q.13

(13) $f(x) = x^3 - 3x^2 - 9x + 12$.

$f'(x) = 3x^2 - 6x - 9$.
$= 3(x^2 - 2x - 3)$
$= 3(x-3)(x+1)$.

$f''(x) = 0$ and $x = 3$ and -1

at 3

$f''(x) = 6x - 6$.
$= 12 > 0$.

at -1.

$f''(x) = 6x - 6$.
$= -12 < 0$

$x = 3$ pt of minimum

$x = -1$ pt of maximum

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$.

$f(x) = 4x^3 - 18x^2 + 24x - 7$.

$f'(x) = 12(x^2 - 3x + 2)$
$= 12(x-2)(x-1)$

$f''(x) = 0$ and $x = 2$ and 1

at 2

$f''(x) = 24x - 36$.
$= 12 > 0$

$x = 2$ pt of minimum.

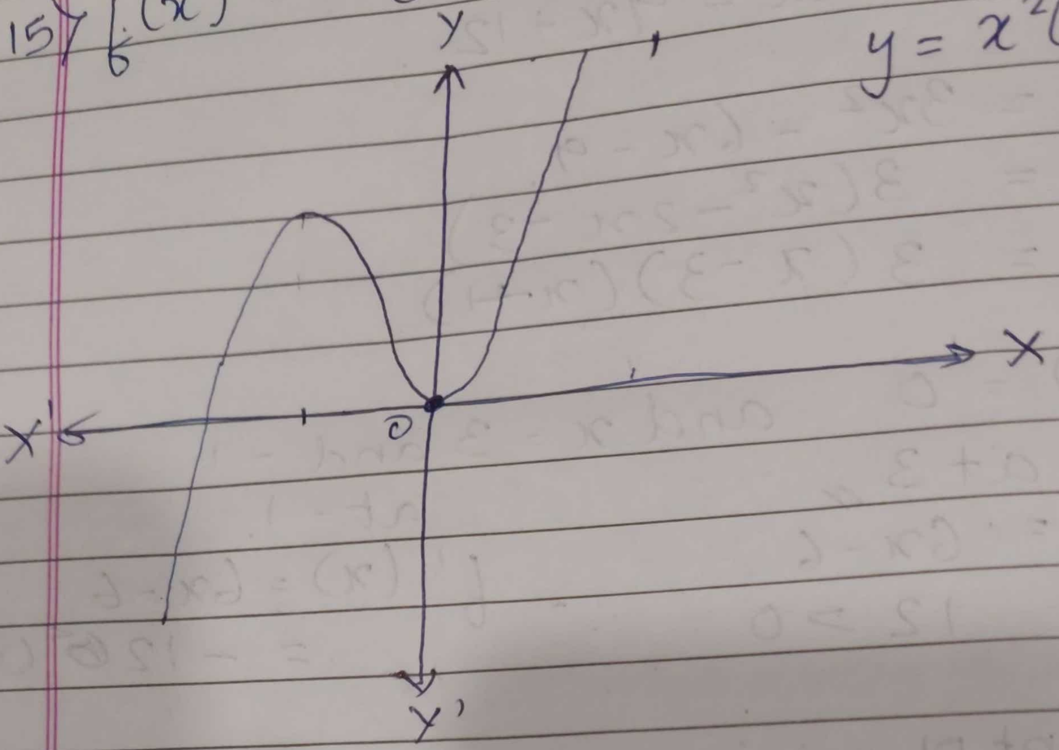
at 1

$f''(x) = 24x - 36$.
$= -12 < 0$

$x = 1$ pt of maximum.

15) $f(x) = x^2(x+3)$

$y = x^2(x+3)$



(16) $f(x) = 3x^2 - 6x$

$y = 3x^2 - 6$