1) $\lim\limits_{h\to 0} \dfrac{2(-3+h)^2-18}{h}$

$= \lim\limits_{h\to 0} \dfrac{2(h-3)^2-18}{h}$

$= \lim\limits_{h\to 0} \dfrac{2(h^2+9-6h)-18}{h}$

$= \lim\limits_{h\to 0} \dfrac{2h^2+18-12h-18}{h}$

$= \lim\limits_{h\to 0} \dfrac{2h^2-12h}{h}$

$= \cancel{2h} \lim\limits_{h\to 0} 2h-12$

$= 2\times 0 - 12$

$= \underline{\underline{-12}}$

2) (a) $\lim\limits_{x\to 4} g(x) = \lim\limits_{x\to 4} 2x = 2\times 4 = 8.$

$\therefore$ the function is continuous at $x=4$

(b) $\lim\limits_{x\to 6^-} g(x) = \lim\limits_{x\to 6^-} 2x = 2\times 6 = 12$

$\lim\limits_{x\to 6^+} g(x) = \lim\limits_{x\to 6^+} (x-1) = 6-1 = 5$

$\lim\limits_{x\to 6^-} g(x) \neq \lim\limits_{x\to 6^+} g(x)$

$\therefore$ the function is discontinuous at $x=6$.

3) $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$= \lim_{x \to 9} \dfrac{(\sqrt{x})^2 - 3^2}{3-\sqrt{x}}$

$= \lim_{x \to 9} \dfrac{(\sqrt{x}+3)(\sqrt{x}-3)}{3-\sqrt{x}}$

$= \lim_{x \to 9} -(\sqrt{x}+3)$

$= -(3+3)$

$= -6$

4) (a) $f(-1) = \dfrac{(4x-1)+5}{9-(3x-1)} = \dfrac{-4+5}{9+3} = \dfrac{1}{12}$

~~$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-}$~~

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \dfrac{4x+5}{9-3x} = \dfrac{(4x-1)+5}{9-(3x-1)} = \dfrac{1}{12}$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \dfrac{4x+5}{9-3x} = \dfrac{(4x-1)+5}{9-(3x-1)} = \dfrac{1}{12}$

$\therefore \lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(x)$

$\therefore$ the function is continuous at $x=-1$.

(b) $f(0) = \frac{(4\times 0)+5}{9-(3\times 0)} = \frac{5}{9}$

$\lim_{x\to 0^-} f(x) = \frac{(4\times 0)+5}{9-(3\times 0)} = \frac{5}{9}$

$\lim_{x\to 0^+} f(x) = \frac{(4\times 0)+5}{9-(3\times 0)} = \frac{5}{9}$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$.

$\therefore$ the function is continuous at $x=0$.

(c) $f(3) = \frac{(4\times 3)+5}{9-(3\times 3)} = \frac{17}{0}$, which is not defined.

$\therefore f(3)$ does not exist.

$\therefore$ the function is discontinuous at $x=3$

5) $\lim_{x\to\infty} \frac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+3e^{-x}}$

$= \lim_{x\to\infty} \frac{e^{4x}(6-e^{-6x})}{e^{4x}(8-e^{-2x}+3e^{-5x})}$

$= \lim_{x\to\infty} \frac{6-e^{-6x}}{8-e^{-2x}+3e^{-5x}}$

$= \lim_{x\to\infty} \frac{6-0}{8-0+0} \quad (\because e^{-\infty}=0)$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

6) (a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} (1 - 3y) = 1 - (3 \times 6) = 1 - 18 = -17$

(b) $\lim_{y \to -2^-} g(y) = \lim_{y \to -2^-} (y^2 + 5) = (-2)^2 + 5 = 4 + 5 = 9$

$\lim_{y \to -2^+} g(y) = \lim_{y \to -2^+} (1 - 3y) = 1 - (3 \times -2) = 1 + 6 = 7$

$\therefore \lim_{y \to -2^-} g(y) \neq \lim_{y \to -2^+} g(y)$

$\therefore$ limit does not exist

7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12) \times (8t - 1)$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t$
$- 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8) $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$$R'(w) = \frac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w+0)}{(2w^2+1)^2}$$

$$= \frac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$$

$$= \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

9) $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log 8$

10) $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left(e^z \times \frac{1}{z} + \ln(z) \times e^z\right)$$

$$= 5z^4 - \frac{e^z}{z} - e^z \ln(z)$$

11) (a) $f(x) = (6x^2 + 7x)^4$

$f'(x) = 4(6x^2+7x)^3 \times (12x + 7)$

$= 4(6x^2+7x)^3(12x+7)$

(b) $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = 0 + e^{4t+t^7} \times (4 + 7t^6)$$

$$= \underline{\underline{(4 + 7t^6)\, e^{4t+t^7}}}$$

12) (a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7-x^3} \times -3x^2 = \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3) \times -6x - [-3x^2 \times -3x^2]}{(7-x^3)^2}$$

$$= \frac{-6x(7-x^3) - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \underline{\underline{\frac{-3x^4 - 42x}{(7-x^3)^2}}}$$

(b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$Q'(v) = \frac{(6+2v-v^2)^4 \times 0 - 2 \times 4\,(6+2v-v^2)^3\,(2-2v)}{(6+2v-v^2)^8}$$

$$= \frac{-16\,(6+2v-v^2)^3(1-v)}{(6+2v-v^2)^8}$$

$$= \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$= \frac{16v-16}{(6+2v-v^2)^5}$$

$$Q''(v) = \frac{(6+2v-v^2)^5 \times 16 - (16v-16) \times 5\,(6+2v-v^2)^4\,(2-2v)}{(6+2v-v^2)^{10}}$$

$$= \frac{16\,(6+2v-v^2)^5 - 16 \times 5 \times 2\,(v-1)(1-v)(6+2v-v^2)^4}{(6+2v-v^2)^{10}}$$

$$= \frac{16\,(6+2v-v^2)^5 + 16 \times 10\,(v-1)^2(6+2v-v^2)^4}{(6+2v-v^2)^{10}}$$

$$= \frac{16(6+2v-v^2)^4\left[6+2v-v^2+10(v-1)^2\right]}{(6+2v-v^2)^{10}}$$

$$= \frac{16\,(6+2v-v^2+10v^2+10-20v)}{(6+2v-v^2)^6}$$

$$= \frac{16\,(9v^2-18v+16)}{(6+2v-v^2)^6}$$

(c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{1}{1+t^2} \times 2t$$

$$= \frac{2t}{1+t^2}$$

$$f''(t) = \frac{(1+t^2)\times 2 - 2t \times 2t}{(1+t^2)^2}$$

$$= \frac{2(1+t^2) - 4t^2}{(1+t^2)^2}$$

$$= \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}$$

$$= \frac{2 - 2t^2}{(1+t^2)^2}$$

$$= \frac{2(1-t^2)}{(1+t^2)^2}$$

13) $f(x) = x^3 - 3x^2 - 9x + 12$

$$f'(x) = 3x^2 - 6x - 9$$

$$= 3(x^2 - 2x - 3)$$

$$= 3(x+1)(x-3)$$

At maxima and minima, slope of the function will be 0.

$\therefore$ if $f'(x) = 0$,

$3(x+1)(x-3) = 0$

$\Rightarrow x = -1$, $x = 3$ will be the points of either maxima or minima.

$f''(x) = 6x - 6$

$f''(-1) = (6 \times -1) - 6 = \underline{\underline{-12}}$

$f''(3) = (6 \times 3) - 6 = \underline{\underline{12}}$

$\therefore$ Maxima is at $x = -1$
Minima is at $x = 3$

$f(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times -1 + 12$

$= -1 - 3 + 9 + 12$

$= \underline{\underline{17}}$

$f(3) = 3^3 - 3 \times 3^2 - 9 \times 3 + 12$

$= \underline{\underline{-15}}$

$\therefore$ maximum value of the function is $f(-1) = 17$.
minimum value of the function is $f(3) = {}^-15$.

14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$= 12(x-2)(x-1)$

At maxima and minima, slope of the function is 0.

$\therefore$ if $f'(x) = 0$,

$(x-2)(x-1) = 0$

$\Rightarrow x = 2,\ x = 1$ are either maxima or minima.

$f''(x) = 24x - 36$

$f''(2) = (24 \times 2) - 36 = \underline{\underline{12}}$

$f''(1) = (24 \times 1) - 36 = \underline{\underline{-12}}$

$\therefore$ Maxima is at $x = 1$
Minima is at $x = 2$

$f(1) = 4 \times 1^3 - 18 \times 1^2 + 24 \times 1 - 7$

$= 4 - 18 + 24 - 7$

$= \underline{\underline{3}}$

$f(2) = 4 \times 2^3 - 18 \times 2^2 + 24 \times 2 - 7$

$= 32 - 72 + 48 - 7$

$= \underline{\underline{1}}$

$\therefore$ Maximum value of the function is $f(1) = 3$.
Minimum value of the function is $f(2) = 1$.

15) $f(x) = x^2(x+3) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

$= 3x(x+2)$

At maxima and minima, the slope of the function will be zero.

$\therefore$ if $f'(x) = 0$,

$3x(x+2) = 0$

i.e., $x = 0$ or $x = -2$ are either maxima or minima.

$f''(x) = 6x + 6 = 6(x+1)$

$f''(0) = 6(0+1) = 6$

$f''(-2) = 6(-2+1) = -6$

$\therefore$ Maxima is at $x = -2$

Minima is at $x = 0$

$f(-2) = (-2)^2(-2+3) = 4$

$f(0) = 0$

$\therefore$ Maxima of the curve will be at $(-2, 4)$ and minima of the curve will be at $(0, 0)$.

Also,

for $x < 2$,

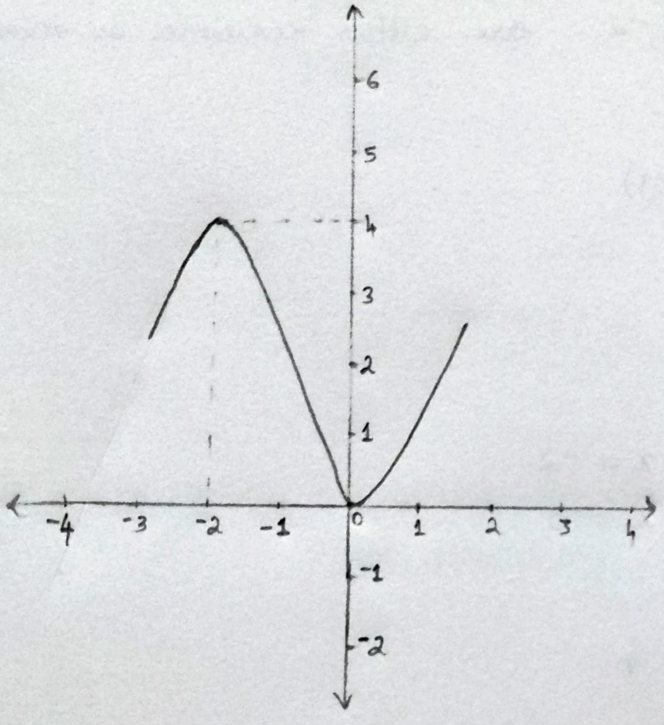
$f'(-3) = (3x-3)(-3+2) = 9 \Rightarrow$ +ve slope, i.e., slope is increasing.

for $-2 < x < 0$,

$f'(-1) = (3x-1)(-1+2) = -3 \Rightarrow$ -ve slope, i.e., slope is decreasing.

for $x > 0$,

$f'(1) = (3\times1)(1+2) = 9 \Rightarrow$ +ve slope, i.e., the slope is increasing.



16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

At maxima and minima, slope of the function is zero.

$\therefore$ if $f'(x) = 0$

$6(x-1) = 0$

$x = 1$ is either maxima or ~~ma~~ minima.

$f''(x) = 6$

$f''(1) = 6$

$\therefore$ the minima of the curve is at $x = 1$.

$f(1) = 3 \times 1^2 - 6 \times 1 = -3$

$\therefore$ the minima of the curve will be at $(1, -3)$

Also,

for $x < 1$,

$f'(0) = 6(0-1) = -6 \Rightarrow$ -ve slope, i.e., the slope is decreasing

for $x > 1$

$f'(2) = 6(2-1) = 6 \Rightarrow$ +ve slope, i.e., the slope is increasing.