Probability & Statistics Assignment

1. $x = 200$

$x \sim Poi(\theta)$

$$\frac{x-\theta}{\sqrt{\theta}} \sim N(0,1)$$

$P[x_1 < \theta < x_2] = 0.95 \Rightarrow P[-1.96 < \theta < 1.96] = 0.95$

95% $\Rightarrow \theta \pm 1.96\sqrt{\theta}$

$\Rightarrow 200 \pm 1.96\sqrt{200}$

$\Rightarrow \boxed{227.718585 \text{ and } 172.281414}$

2. mean $= \mu$, variance $= \sigma^2$

$$\bar{x} = \frac{\sum x_i}{\mu} \quad ; \quad s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

(i) Mean of $\bar{x} \Rightarrow E(\bar{x}) = E\left(\frac{\sum x_i}{n}\right) = \frac{1}{n}(E(\sum x_i)) \Rightarrow \frac{n(\mu)}{n} \Rightarrow \boxed{\mu}$

Variance of $\bar{x} \Rightarrow V(\bar{x}) =$

$$\Rightarrow V\left(\frac{\sum x_i}{n}\right) \Rightarrow \frac{1}{n^2} V(\sum x_i) \Rightarrow \frac{\sum V(x_i)}{n^2} \Rightarrow \frac{n\sigma^2}{n^2} \Rightarrow \boxed{\frac{\sigma^2}{n}}$$

(ii) Show that $E(s^2) = \sigma^2$

$$E(s^2) \Rightarrow E\left[\frac{1}{n-1}\left(\sum x_i^2 - n\bar{x}^2\right)\right]$$

$$\Rightarrow \frac{1}{n-1}\left[E(\sum x_i^2) - nE(\bar{x}^2)\right]$$

$$\Rightarrow \frac{1}{n-1} \sum \left[E\left(x_i^2 + \bar{x}^2 - 2x_i\bar{x}\right)\right]$$

$$\Rightarrow \frac{1}{n-1} \sum \left[E(x_i^2) + E(\bar{x})^2 - 2\bar{x}^2\right]$$

$$\frac{1}{n-1} \sum \left[E(x_i)^2 + E(\bar{x})^2\right]$$

$$\frac{1}{n-1} \Sigma\left[\sigma^2+\mu^2-\left(\frac{\sigma^2}{n}+\mu^2\right)\right]$$

$$\boxed{\frac{1}{n-1} \Sigma\left[\sigma^2\left(\frac{n-1}{n}\right)\right]}.$$

iii $S^2 \sim \frac{2\sigma^4}{n-1}$; $X_i \sim$ Normal (μ, σ^2)

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V\left(\frac{(n-1)S^2}{\sigma^2}\right) = \frac{(n-1)^2}{\sigma^4} V(S^2)$$

$$V(\chi^2_{n-1}) = \frac{(n-1)^2}{\sigma^4} V(S^2)$$

$$2(n-1) = \frac{(n-1)^2}{\sigma^4} V(S^2)$$

$$\frac{2\cancel{(n-1)}\sigma^4}{(n-1)^{\cancel{2}}} = V(S^2) \Rightarrow \boxed{\frac{2\sigma^4}{n-1}}$$

iv $\mu = 10$; $\sigma^2 = 100$

$$P[50 < S^2 < 100] = P[S^2 < 150] - P[S^2 < 50]$$

$$\Rightarrow P\left[\frac{(n-1)S^2}{\sigma^2} < \frac{9\times 3\cancel{0}}{2\cancel{0}}\right] - P\left[\frac{(n-1)S^2}{\sigma^2} < 9\times\frac{1}{2}\right]$$

$$\Rightarrow P[\chi^2_9 < 13.5] - P[\chi^2_9 < 4.5]$$

$$\Rightarrow 0.8587 - 0.1245 \Rightarrow \boxed{0.7342}.$$

$\left[\begin{array}{l} 100+50=150 \\ \& \ 100-50=50 \end{array}\right.$

3.

No. of claims (X)	0	1	2	3	4
No. of policies	86	75	16	2	1

i $L = \prod_{i=1}^{180} P(X=n) \Rightarrow [P(X=0)]^{86} \times [P(X=1)]^{75} \times [P(X=2)]^{16} \times [P(X=3)]^{2}$

$\times [P(X=4)]^{1}$

$L \Rightarrow [{}^4C_0\, p^0 (1-p)^4]^{86} \cdot [{}^4C_1\, p^1 (1-p)^3]^{75} \cdot [{}^4C_2\, p^2 (1-p)^2]^{16} \cdot [{}^4C_3\, p^3 (1-p)^1]^{2}$

$\cdot [{}^4C_4\, p^4 (1-p)^0]^{1}$

$L \Rightarrow \text{constant} \times p^{117} \times (1-p)^{603}$

$\log L = \log(\text{constant}) + 117 \log p + 603 \log(1-p)$ [Adding log on both the sides]

$\frac{dl}{dp} = 0 + 117 \times \frac{1}{p} + 600 \times \frac{1}{1-p}$

$= \frac{117}{p} - \frac{603}{1-p} \Rightarrow 0$ $\cancel{\frac{117(1-p) - 603p}{p(1-p)}}$

$= \frac{117}{p} = \frac{603}{1-p}$

$117(1-p) = 603p$

$117 - 117p = 603p \Rightarrow p \Rightarrow \boxed{\frac{117}{720}}$

ii Goodness of fit test

x_i	O_i	E_i
0	86	$180(P=0) \rightarrow 88.5547$
1	75	$180(P=1) \rightarrow 68.729$
2	16	$180(P=2) \rightarrow 20.0032$
3	2	$180(P=3) \rightarrow 2.5874$
4	1	$180(P=4) \rightarrow 0.1255$

2, 3 and 4 are combined together

hence.

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
86	88.5547	0.0737
75	68.729	0.5722
19	22.7159	0.6078

$X^2_{cal} \Rightarrow \sum \frac{(O_i - E_i)^2}{E_i} \Rightarrow \boxed{1.2537}$

$X^2_{tab} \Rightarrow X^2_1 \Rightarrow \boxed{3.841}$

X^2_{tab} is greater than X^2_{cal} hence it not rejected at 5% level of significance.

5.4. $X \sim U(a,b)$; $\bar{x} = \frac{\sum x_i}{n}$

(i) $E(x) = \frac{1}{2}(a+b) \Rightarrow \bar{x} \Rightarrow \frac{(a+b)}{2}$ =

$2\bar{x} \Rightarrow a+b$ — (i)

$V(x) = s^2$

$\frac{(b-a)^2}{12} = s^2 \Rightarrow \frac{b-a}{2\sqrt{3}} = s$ — (ii)

$\cancel{b} - a = s2\sqrt{3}$

$\cancel{b} + a = 2\bar{x}$

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$-2a = s2\sqrt{3} - 2\bar{x}$

$-\cancel{2}a = \cancel{2}(s\sqrt{3} - \bar{x})$

$\boxed{-a = s\sqrt{3} - \bar{x}}$

(ii) $\hat{b} \Rightarrow$

$b - a = s2\sqrt{3}$

$b + a = 2\bar{x}$

$\cancel{2}b = \cancel{2}(s\sqrt{3} + \bar{x})$

$\boxed{b \Rightarrow s\sqrt{3} + \bar{x}}$

ii let the sample be 5, 10, 15, 20, 25

$\bar{X} = 15$ $\qquad$ $\hat{a} \Rightarrow \bar{X} - 5\sqrt{3}$

$\cancel{\quad} \cdot S \Rightarrow$ $\qquad$ $\Rightarrow$

$\hat{b} \Rightarrow \bar{X} + 5\sqrt{3}$

$\Rightarrow$

conclusion $\Rightarrow$ $\hat{b}$ is closer to the sample as compared to $\hat{a}$.

iv $X \sim U(0, b)$ when $a = 0$

$$L = \prod_{i=4}^{n} f(x)$$

$$L = \left(\frac{1}{b-a}\right)^n \Rightarrow \left(\frac{1}{b}\right)^n = (b)^{-n}$$

$$\log L = \cancel{n\log(b)} - n \log b$$

$$\frac{d}{db}(\log L) = -n \times \frac{1}{b} \Rightarrow 0.$$

It gives 'b' = ∞ which is not possible.

$$\frac{d^2L}{db^2} = \frac{n}{b^2} \; ; \; \frac{n}{b^2} > 0.$$ it gives minimum likelihood estimate.

$$\therefore \hat{b} = \max(x_i)$$

6. $H_0 \Rightarrow p = 0.1$ and $H_1 \Rightarrow p > 0.1$

(i) let 'x' be no. of hit in 1^{st} step & 'Y' be in 2^{nd} step.

$$P(\text{type I error}) \Rightarrow P(X \geq 3 / p=0.1) + P(Y \geq 5 | P=0.1) \cdot P(X=0 | P=0.1) + P(Y \geq 4 | p=0.1) P(X=1 | p=0.1) + P(Y \geq 3 | p=0.1) \cdot P(X=2 | p$$

$$\Rightarrow P(X>2 | p=0.1) + P(Y=4 | p=0.1) \cdot P(X=0 | p=0.1) + P(Y>3 | p=0.1) P(X=1 | p=0.1) + P(Y>2 | p=0.1) \cdot P(X=2 | p=0.1)$$

$$\Rightarrow (1-0.8891) + (1-0.9953)(0.2824) + (1-0.9744)(0.6590-0.2824) + (1-0.8891)(0.8891-0.6590)$$

$$\Rightarrow \boxed{0.1433} \Rightarrow \boxed{14.33\%}$$

ii $P(\text{rejecting } H_0)$ when $p=0.3 \Rightarrow$

$$\Rightarrow P(x\geq 2 \mid p=0.3) + P(Y>4 \mid p=0.3)\cdot P(X=0 \mid p=0.3)$$
$$+ P(Y>3 \mid p=0.3)\cdot P(X=1 \mid p=0.3) + P(Y>2 \mid p=0.3)\cdot$$
$$P(X=2 \mid p=0.3)$$

$$\Rightarrow (1-0.2528) + (1-0.07237)(0.0138) + (1-0.4925)(0.0850-0.0138)$$
$$+ (1-0.2528)(0.2528-0.850)$$

$$\Rightarrow 0.9125 \Rightarrow \boxed{91.25\%}$$

iii $P(\text{type 2 error}) \Rightarrow 1 - P(\text{type 1 error})$

$$\Rightarrow 1 - 0.9125 \Rightarrow 0.08747 \Rightarrow \boxed{8.747\%}$$

iv $\hat{p} = \frac{40}{120} = \frac{1}{3} \Rightarrow 0.3333$

$$\bar{X} \sim N\left(p, \frac{p(1-p)}{n}\right) \Rightarrow \frac{\bar{X}-np}{\sqrt{\frac{p(1-p)}{n}}}$$

$$P\left[\frac{\bar{X}-p}{\sqrt{\frac{p(1-p)}{n}}} < X_1\right] = 0.1 \quad [\text{For } 90\% \text{ one sided}]$$

$$\bar{X} - Z_{0.1}\sqrt{\frac{p(1-p)}{n}} \Rightarrow 0.3333 - 1.2816\sqrt{\frac{0.2222}{120}}$$

$$\Rightarrow 0.2781514748 \Rightarrow \boxed{0.27815}$$

v. $H_0 \Rightarrow p = 0.278$ & $H_1 \Rightarrow p > 0.278$

$$TS \rightarrow \frac{\frac{x-\frac{1}{2}}{n} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

$$\Rightarrow \frac{\frac{40-0.5}{120} - 0.275}{\sqrt{\frac{0.278(0.722)}{120}}} \Rightarrow \frac{6.14}{\sqrt{\frac{(120)^2-(0.200716)}{120}}} \Rightarrow \boxed{1.251}$$

$Z_{tab} \Rightarrow Z_{10\%} \Rightarrow \boxed{1.2816}$

$\Rightarrow$ We will not reject H_0 because $Z_{cal} < Z_{tab}$.

vi. Comment $\longrightarrow$ The solution shows that ~~that~~ p could be less than or equal to 0.278 ~~change~~ chances in 90% of the times. and $p < 0.2782$ for 10% chances.

vii $n_1 \Rightarrow 12$; $n_2 = 15$
$\bar{X}_1 = 65$ & $\bar{X}_2 = 70$; $\sigma_1 = 504$ & $\sigma_2 = 70$.

$H_0 \Rightarrow \mu_1 = \mu_2$ & $H_1 \Rightarrow \mu_1 \neq \mu_2$

$$T.S. \Rightarrow \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$S_p = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} \Rightarrow \boxed{4027.04}$$

$$t_{cal} \Rightarrow \frac{(65-70)}{\sqrt{4027.04\left(\frac{1}{12}+\frac{1}{15}\right)}} \Rightarrow \frac{-5}{\sqrt{24.571}} \Rightarrow \boxed{-0.2034}$$

$$t_{tab} \Rightarrow \boxed{-2.0602060}$$

comment $\longrightarrow$ We ~~do~~ will not reject H_0 as t_{cal} lies in the 95% CI.

Assumption $\longrightarrow$ Population variance is equal in both the cases.

7 (i) Sample mean

Priv. hos. $(\bar{X}_{Pr.}) \Rightarrow \frac{\Sigma X_i}{n} \Rightarrow \boxed{35.056}$

Public hos. $(\bar{X}_{Pb}) \Rightarrow \frac{\Sigma X_i}{n} \Rightarrow \boxed{30}$

$S^2_{Private} \Rightarrow \frac{\Sigma X_i^2 - n\bar{X}}{n-1} \Rightarrow \boxed{67.403}$ $\quad S^2_{Public} \Rightarrow \frac{\Sigma X_i^2 - n\bar{X}}{n-1} \Rightarrow \boxed{64.312}$

(ii) Assumption — Population of variation of both is equal.

$\rightarrow$ 2-sided t-test can be used for testing equal mean.

Let $H_0 \Rightarrow \sigma_1^2 = \sigma_2^2$ and $H_1 \Rightarrow \sigma_1^2 \neq \sigma_2^2$

$$\frac{S^2_{Pvt} / S^2_{public}}{\sigma^2_{Pvt} / \sigma^2_{public}} \sim F_{n_{pr}-1,\, n_{pb}-1}$$

For 95% $\Rightarrow P\left[\frac{1}{x} < F_{9,6} < x\right] = 0.95$

$$\Rightarrow P\left[\frac{1}{4.433} < \frac{67.403}{64.3125}\frac{\sigma^2_{Public}}{\sigma^2_{Pvt}} < 4.433\right]$$

95% of C.I $\Rightarrow$ 0.2152 and 4.229.

$\rightarrow$ We do not reject H_0 because CI includes 1 at 5% LOS.

iii $H_0 \Rightarrow \mu_{Pvt} = \mu_{Public}$, $H_1 \Rightarrow \mu_{Pvt} > \mu_{Public}$

T-distribution will be used as population variance is unknown.

$$t_{cal} \Rightarrow \frac{(\bar{x}_{Pvt} - \bar{x}_{Public}) - (\mu_{Pvt} - \mu_{Public})}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\Rightarrow \frac{5.056}{8.6756\sqrt{\frac{2}{9}}} \Rightarrow \boxed{1.2363}$$

$$t_{tab} \Rightarrow t_{16,5\%} \Rightarrow \boxed{1.746}$$

H_0 cannot be rejected as of now as $t_{tab} > t_{cal}$.

Q i It is an estimator whose mean value is equal to the true parameter value i.e θ.

ii It is the measure of the difference between the expected value of the estimator and the true parameter value.

iii Parameter for mean square error is θ. It is the second moment estimator about the true parameter value.

$$MSE \Rightarrow E[X] + Bias^2$$

iv The estimator which is 'efficient' is $\tilde{\theta}$ i.e the biased estimator as it does not deviates much from the true value.

v When $n \to \infty$ and $MSE \to 0$ these would make an estimator consistent ~~when~~, MSE tends to be minimum.

vi Two methods are <u>methods of moments</u> and maximum likelihood estimate.

9 (i) t_k is defined as the (formula) $\Rightarrow \dfrac{Z}{\sqrt{C/k}} \sim t_k$,

when $Z \sim N(0,1)$ $C \sim \chi^2_{k=n-1}$ and $k \Rightarrow$ degree of freedom.

(ii) Mean of t_k for $k>2 \Rightarrow 0$.

Variance of t_k for $k>2 \Rightarrow \dfrac{k}{k-2}$.

(iii) $n=10$; $\bar{X}=50$; $S^2 = 48.667$.

a) $T.S \Rightarrow \dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

For 99% of CI $\Rightarrow \bar{X} \pm t_{n-1,\,0.5\%} \, S/\sqrt{n}$

$$\Rightarrow 50 \pm 3.250 \times \sqrt{\frac{48.667}{10}} \Rightarrow \boxed{(42.83,\ 57.1697)}$$

b) $\dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim N\left(0, \dfrac{k}{k-2}\right)$ where $N\left(0, \dfrac{9}{7}\right)$

$$\Rightarrow \frac{\bar{X}-\mu}{S/\sqrt{\frac{9}{7n}}} \sim N(0,1)$$

99% of CI $\Rightarrow$ CI $\Rightarrow \bar{X} \pm z_{0.5\%} \sqrt{\dfrac{9S^2}{7n}}$

$$\Rightarrow 50 \pm 2.5758 \sqrt{\frac{9(48.667)}{70}} \Rightarrow \boxed{(43.556,\ 56.443)}$$

10. $n = 1000$; $\lambda = 3$; $X \sim Poi(3)$

i Type 1 error is probability of rejecting H_0 when H_0 is true.

ii Type 2 error is probability of accepting H_0 when it is rejected.

iii Power of a test is the probability of rejecting H_0 when it false

Power of test $= P(X > 3100 \mid \tilde{X} \sim N(n\mu, n\mu))$

$\Rightarrow 1 - P(X < 3100 \mid \bar{X} \sim N(n\mu, n\mu))$

$\Rightarrow 1 - P\left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$

iv $\bar{X} \sim N\left(\mu, \frac{\mu}{n}\right)$

$\frac{\bar{X} - \mu}{\sqrt{\frac{\mu}{n}}} \sim N(0,1) \Rightarrow P\left[X_1 < \frac{\bar{X} - \mu}{\sqrt{\frac{\mu}{n}}} < X_2\right] < 0.99$

$\Rightarrow P\left[-2.5758 < Z < 2.5758\right] = 0.99$

99% CI $\Rightarrow \frac{2900}{1000} \pm 2.5758\sqrt{\frac{2.9}{1000}} \Rightarrow \boxed{(2.7613, 3.0387)}$

11. $\Sigma x = 213200$, $n = 12$; $\Sigma x^2 = 4919860000$, $\bar{x} = 17767$

$S = 10144$

$\Sigma X_{new} \Rightarrow \Sigma X$ + correct value − incorrect value

$\Rightarrow 213200 + (27500 + 31500) - (11000 + 48000) \Rightarrow \boxed{213200}$

$\bar{X}_{new} = \frac{\Sigma X_{new}}{n} \Rightarrow \boxed{177676}$

$\Sigma X_{i\,new}^2 \Rightarrow \Sigma X_i^2 - (11000^2 + 48000^2) + 27500^2 + 31500^2$

$\Rightarrow \boxed{424,336,000}$

$$S^2 \Rightarrow \frac{1}{n-1}\left(\Sigma X_{i\,new}^2 - n\bar{X}_{new}^2\right)$$

$$\Rightarrow \frac{1}{11}\left[4243360000 - 3787995468\right]$$

$$S^2 \Rightarrow \boxed{41396776} \quad \therefore S \Rightarrow \boxed{6434.0326}$$

12. $X \sim U(0,\theta)$

(i) Cramer-Rao lower bound of unbiased estimator does ~~not~~ not apply here because the parameter is already present in the range of distribution given.

(ii) $\theta(c) = cY$ where ~~y~~ Y = maximum X_i and C = constant.

$$F(Y) \Rightarrow F(X_{i\,max}) \Rightarrow F(max(x_1, x_2 \dots x_n))$$

$$\Rightarrow 1 - P(X \geq max(X_1, X_2 \dots X_n)$$

$$\Rightarrow 1 - P(X > x_1)\, P(X > X_2) \dots P(X > X_n)$$

$$\Rightarrow 1 - P(X \geq x)^n$$

$$P(X < x)^n \Rightarrow \boxed{F(x)^n}$$

$$F(Y) \Rightarrow \left(\frac{x-0}{\theta - 0}\right)^n \Rightarrow \left(\frac{x}{\theta}\right)^n$$

$$f(Y) \Rightarrow F'(Y) \Rightarrow \frac{d}{dY}\left(\frac{Y}{\theta}\right)^n \Rightarrow \boxed{\frac{1}{\theta}\left[ny^{n-1}\right]} \quad \text{when } 0 < y < \theta.$$

$$E(Y^k] \Rightarrow \int_0^\theta Y^k f(y)\, dy$$

$$\Rightarrow \int_0^\theta y^k \left(\frac{n}{\theta^n}\right) \times y^{n-1})\, dy$$

$$\frac{n}{\theta^n}\int_0^\theta y^{k+n-1}\, dy \Rightarrow \frac{n}{\theta^n}\left[\frac{y^{n+k}}{k+n}\right]_0^\theta$$

$$\Rightarrow \cancel{\frac{n\theta^{n+k}}{\theta^n(n+k)}} \Rightarrow \frac{n\theta^{n+k}}{\theta^n(n+k)} \Rightarrow \boxed{\frac{n\theta^k}{n+k}} \quad \text{--- (1)}$$

iii $\text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}(c)] - \theta$

$\Rightarrow E(cY) - \theta$

$c\,E(Y) - \theta \Rightarrow c\left[\frac{n\theta}{n+1}\right] - \theta$ [when $k=1$]

$\Rightarrow \boxed{\theta\left[\frac{cn}{n+1} - 1\right]}$

$\text{MSE}[\hat{\theta}(c)] = \text{Variance} + \text{Bias}^2$

$V(\hat{\theta}(c)) \Rightarrow V(cY) \Rightarrow c^2 V(Y)$

$\Rightarrow c^2[E(Y^2) - [E(Y)]^2]$

$\boxed{c^2\left[\frac{n\theta^2}{n+2} - \left(\frac{n\theta}{n+1}\right)^2\right]}$

$\text{MSE} \Rightarrow c^2\left[\frac{n\theta^2}{n+2} - \left(\frac{n\theta}{n+1}\right)^2\right] + \left[\theta\left(\frac{cn}{n+1} - 1\right)\right]^2$

$\Rightarrow c^2\left[\frac{n\theta^2}{n+2} - \frac{(n\theta)^2}{(n+1)^2}\right] + \theta^2\left[\frac{cn}{n+1} - 1\right]^2$

$\Rightarrow \boxed{\frac{c^2 n\theta^2}{n+2} - \frac{(cn\theta)^2}{(n+1)^2} + \theta^2\left[\frac{c^2n^2}{(n+1)^2} + 1 - \frac{2cn}{n+1}\right]}$

iv We need ~~Bias $(\hat{\theta}(c))$~~ Bias and $E(\hat{\theta}(c)) - \theta = 0$ for $\theta(c)$ to be unbiased estimator of θ

$E(\hat{\theta}(c)) = \theta$

$E(cY) = \theta \Rightarrow c\,E(Y) = \theta$

$c\left(\frac{n\theta}{n+1}\right) = \theta \Rightarrow \boxed{c = \frac{n+1}{n}}$

v. $\text{MSE} \Rightarrow \frac{c^2 n\theta^2}{n+2} - \frac{c(2n\theta^2)}{n+1} + \theta^2$

$$\frac{d}{dc}(\text{MSE}) = \frac{2cn\theta^2}{n+2} - \frac{2n\theta^2}{n+1} \Rightarrow 0$$

$$\Rightarrow \frac{\cancel{2}c\cancel{n}\cancel{\theta^2}}{n+2} = \frac{\cancel{2n\theta^2}}{n+1}$$

$$c(n+1) = n+2$$

$$c = \boxed{\frac{n+2}{n+1}}$$

$$\frac{d^2}{dc^2}(\text{MSE}) \Rightarrow \frac{2n\theta^2}{n+2} > 0$$ and minimum at $\boxed{c = \frac{n+2}{n-1}}$

vi $\hat{\theta}(c_m)$ is the one which I prefer for estimating θ because it does not deviates as compared to $\hat{\theta}(c_u)$ from the true value.
When 'n' becomes larger the estimator i.e. $\hat{\theta}c_u$ and $\hat{\theta}c_m$ become equal.

4.
(ii) Method of moment $\rightarrow E(x) = \frac{\sum X_i}{n}$

mean $\Rightarrow \frac{B}{C-1}$ and $c=3$

variance $\Rightarrow \frac{CB^2}{(c-1)^2(-2)} = \frac{CB^2}{4(\ldots)}$

$$\frac{\beta}{c-1} = \bar{X}_n$$

$$\boxed{\beta = \bar{X}_n(c-1)}$$

verification

$$\text{Bias} = E[g(x)] - \theta \Rightarrow E(\hat{B}) - B$$

$$= E(2\bar{X}_n) - B \Rightarrow 2E(\bar{X}_n) - B$$

$$\Rightarrow 2E\left(\frac{\sum X_{i,u}}{n}\right) - B$$

$$\frac{2\sum E(X_{i,u})}{n} - B$$

$$\frac{2}{n}\sum\left(\frac{B}{2}\right) - B \Rightarrow \frac{\cancel{2n}B - \overset{2n}{\cancel{2}}B}{2n} \Rightarrow \boxed{0}$$

Bias $= 0$.

∴ it is an unbiased estimator of B.