ASSIGNMENT - 1

1. $$\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$$

$$\lim_{h\to 0} \frac{2(9-6h+h^2)-18}{h}$$

$$\lim_{h\to 0} \frac{18-12h+2h^2-18}{h}$$

$$\lim_{h\to 0} \frac{h(2h-12)}{h}$$

$$\lim_{h\to 0} 2h-12$$

$$\lim_{h\to 0} 2\times 0 - 12$$

$$= -12$$

2. $$g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$$

a) $x = 4$

$$g(4) = 2\times 4 = 8$$

LHL

$$g(4-h) = \lim_{h\to 0^-} = 2(4-h) = 8$$

RHL

$$g(4+h) = \lim_{h\to 0^+} 2(4+h) = 2(4+0) = 8$$

Continous

b) $x = 6$

$$g(6) = 6-1 = 5$$

LHL

$$g(6-h) = \lim_{h\to 0^-} \cancel{2(6-h)} \cancel{= 12} \; 2(6-h) = 12$$

RHL

$$g(6+h) = \lim_{h\to 0^+} (6+h)-1 = 5$$

Discontinous

3. $$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$\lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$$

$$\lim_{x\to 9} \frac{-(9-x)(3+\sqrt{x})}{(9-x)} \Rightarrow \lim_{x-9} -(3+\sqrt{x})$$

$$\Rightarrow -3-\sqrt{9} = -6$$

4. $f(x) = \dfrac{4x + 5}{9 - 3x}$

a) $x = -1$

$f(-1) = \dfrac{4(-1) + 5}{9 - 3(-1)} = \dfrac{-4+5}{9+3} = \dfrac{1}{12}$ Continuous

b) $x = 0$

$f(0) = \dfrac{4\times 0 + 5}{9 - 3\times 0} = \dfrac{5}{9}$ Continuous

c) $x = 3$

$f(3) = \dfrac{12 + 5}{9 - 3\times 3} = \dfrac{17}{0}$ Discontinuous

5. $\lim\limits_{x\to\infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\lim\limits_{x\to\infty} \dfrac{e^{4x}(6 - e^{-6x})}{e^{4x}(8 - e^{-2x} + 3e^{-6x})}$

$\lim\limits_{x\to\infty} \dfrac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-6x}}$

$\dfrac{6 - 1/e^{600}}{8 - 1/e^{200} + 3(1/e^{600})}$

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6. $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} 1-3(6) = -17$

b) $\lim_{y \to -2} g(y) = \lim_{y \to -2} 1-3(-2) = 7$

7. $f(t) = (4t^2-t)(t^3-8t^2+12)$

$f'(t) = (4t^2-t)\frac{d}{dt}(t^3-8t^2+12) + (t^3-8t^2+12)\frac{d}{dt}(4t^2-t)$

$f'(t) = (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$f'(t) = 12t^4 - 64t^3 - 3t^3 - 16t^2 + 8t^4 - t^3 - 64t^3 + 8t + 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

$= 4(5t^4 - 33t^3 + 6t^2 + 24t - 3)$

8. $R(w) = \frac{3w+w^4}{2w^2+1}$

$R'(w) = \frac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$

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$$R'(\omega) = \frac{(6\omega^2 + 8\omega^5 + 3 + 4\omega^3) - (12\omega^2 + 4\omega^5)}{(2\omega^2+1)^2}$$

$$R'(\omega) = \frac{6\omega^2 + 8\omega^5 + 4\omega^3 + 3 - 12\omega^2 - 4\omega^5}{(2\omega^2+1)^2}$$

$$= \frac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2+1)^2}$$

9. $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log 8$

10. $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dx} = 5z^4 - \left[\frac{e^z}{z} + \ln(z)e^z\right]$$

11. a) $f(x) = (6x^2 + 7x)^4$

$$f'(x) = 4(6x^2+7x)^3 \frac{d}{dx}(6x^2 + 7x)$$

$$= 4(6x^2+7x)^3 (12x+7)$$

b) $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = e^{4t+t^7} \frac{d}{dt} 4t + t^7$$

$$= (e^{4t+t^7}) \cdot (4 + 7t^6)$$

12. a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7-x^3} \cdot \frac{d}{dx}(7 - x^3)$$

$$= \frac{1}{7-x^3}(-3x^2)$$

$$= \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3) \cdot \frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

b) $Q(v) = \frac{2}{(6+2v-v^2)^4}$

$$Q'(v) = 2(6+2v-v^2)^{-4}$$

$$= -8(6+2v-v^2)^{-5}(2-2v)$$

$$G''(v) = -8\left[(6+2v-v^2)^{-5}\frac{d}{dv}(2-2v) + (2-2v)\frac{d}{dv}(6+2v-v^2)\right]$$

$$= -8\left[(6+2v-v^2)^{-5}(-2) + (2-2v)(2-2v)\right]$$

$$= 16(6+2v-v^2)^{-5} - 8(4+4v^2-8v)$$

$$= 16(6+2v-v^2)^{-5} - 32(1+v^2-2v)$$

$$= 16\left[(6+2v-v^2)^{-5} - 2(1+v^2-2v)\right]$$

c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{1}{(1+t^2)}\frac{d}{dt}(1+t^2)$$

$$= \frac{2t}{1+t^2}$$

$$f''(t) = \frac{(1+t^2)(2) - (2t)^2}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

13. $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

$f''(x) = 6x - 6$

• $x = 3$ $\therefore$ Minimum at $x = 3$

$f''(3) = 18 - 6 = 12$

• $x = -1$

$f''(-1) = 6(-1) - 6 = -12$ $\therefore$ Maximum at $x = -1$

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

Put $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

$f''(x) = 24x - 36$

$x = 1$ $\therefore$ Maximum at $x = 1$

$f''(1) = 24 - 36$

$= -12$

$x = 2$ $\therefore$ Minimum at $x = 2$

$f''(2) = 48 - 36$

$= 12$

15. $f(x) = x^2(x+3)$

~~$f'(x)$~~ $f(x) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

Put $f'(x) = 0$

$3x^2 + 6x = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x = -2$

$f''(x) = 6x + 6$

• $x = 0$

$f''(0) = 0 + 6$
$= 6$

$\therefore$ ~~Maximum~~ Minimum at $x = 0$

• $x = -2$

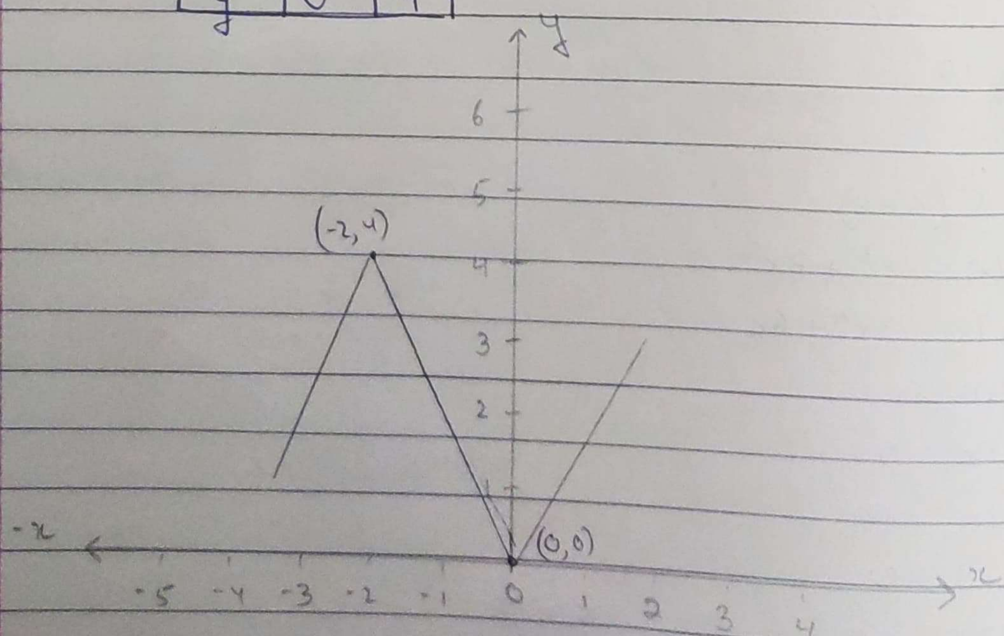
$\therefore$ ~~Minimum~~ Maximum at $x = -2$

$f''(-2) = -12 + 6$
$= -6$

Put $x = 0$ in $f(x)$
Minimum value $= f(0) = 0$

Put $x = -2$ in $f(x)$
Maximum value $= f(-2) = 4(-2+3) = \cancel{0}\ 4$

x	0	-2
y	0	4



16. $f(x) = 3x^2 - 6x$

i.e. $y = 3x^2 - 6x$

Put $x = -3$

$y = 3\times(-3)^2 - 6(-3)$
$= 27 + 18$
$= 45$

Put $x = -2$

$y = 3\times(-2)^2 - 6(-2)$
$= 12 + 12$
$= 24$

Put $x = -1$

$y = 3\times(-1)^2 - 6(-1)$
$= 3 + 6$
$= 9$

Put $x = 0$

$y = 3\times(0)^2 - 6\times(0)$
$= 0$

Put $x = 1$

$y = 3\times(1)^2 - 6\times(1)$
$= -3$

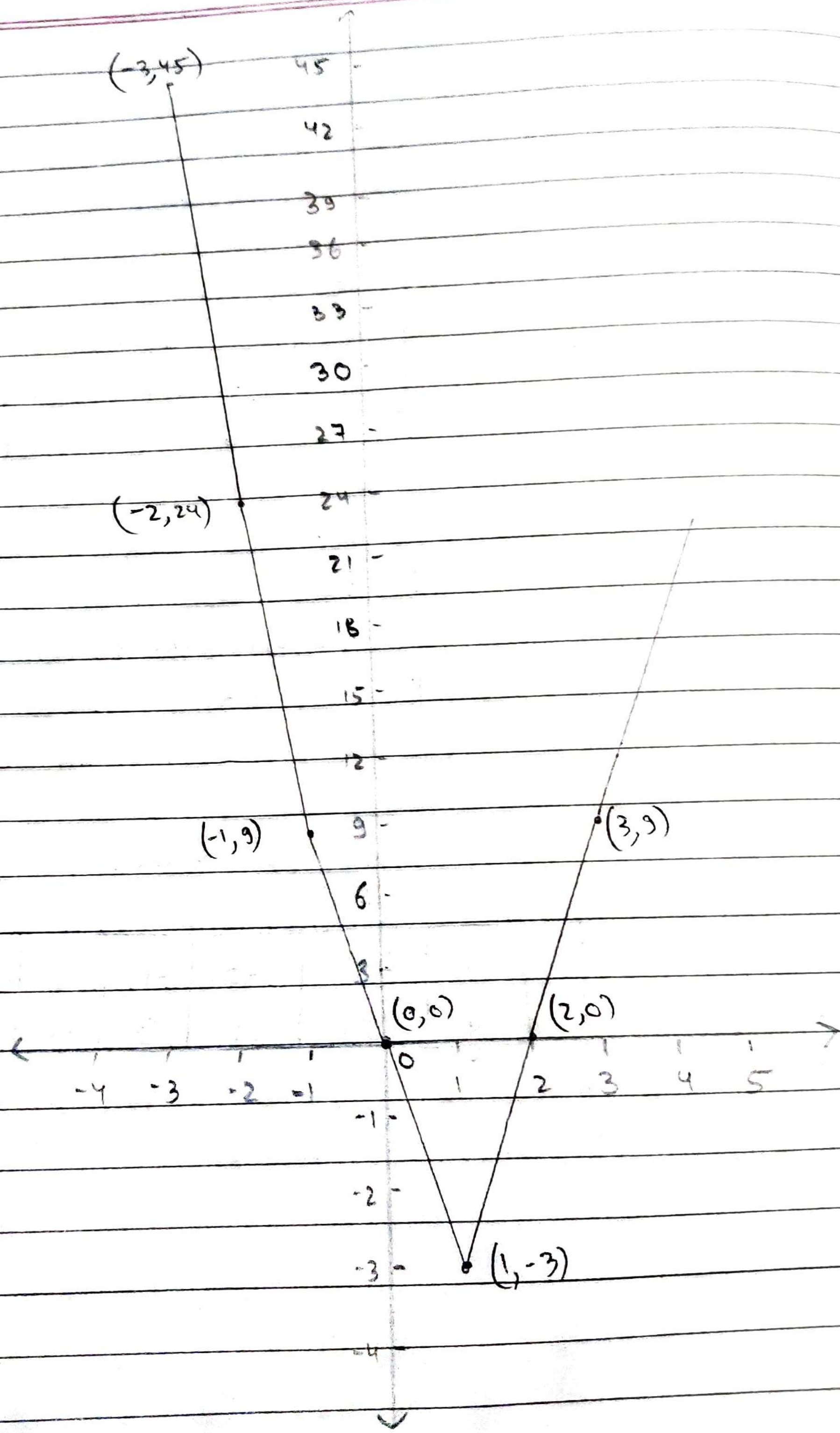
Put $x = 2$

$y = 3\times4 - 12$
$= 12 - 12$
$= 0$

Put $x = 3$

$y = 3(9) - 18$
$= 27 - 18$
$= 9$

x	-3	-2	-1	0	1	2	3
y	45	24	9	0	-3	0	9


(-3,45)
(-2,24)
(-1,9)
(0,0)
(2,0)
(3,9)
(1,-3)
45
42
39
36
33
30
27
24
21
18
15
12
9
6
3
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4
5