ASSIGNMENT - 1

1) Actual length $= \cancel{2}r_A = 12.\cancel{8}5$ cm
Approximate radius $r_B = 12.65$ cm.

→ Actual area $= \pi r_A^2$
$= \pi (12.5)^2$
$= 490.87385 \text{ cm}^2$

→ Approximate area $= \pi r_B^2 = \pi (12.65)^2$
$= 502.72551 \text{ cm}^2$

→ Absolute error = Approximated area − Actual area
$= 502.72551 - 490.87385$
$= 11.58166 \text{ cm}^2$

→ Relative error $= \dfrac{\text{Absolute error}}{\text{Actual area}}$
$= \dfrac{11.58166}{490.87385} = 0.024144$

→ Percentage error = Relative error × 100 = 0.024144 × 100
$= 2.4144\%$

2) $x_1 = 4$ $\quad y_1 = 0.60206$
$x_2 = 6$ $\quad y_2 = 0.7781513$

→ $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1}$

⇒ $\dfrac{x - 4}{6 - 4} = \dfrac{y - 0.60206}{0.7781513 - 0.60206}$

⇒ $(x - 4)\, 0.1760913 = 2(y - 0.60206)$

Date

→ For $x = 5$

$$y = \frac{5 - 4(0.1760913)}{2} + 0.60206$$

$$= 0.690105650$$

2) $x_1 = 4.5$ $\quad$ $y_1 = 0.6532125$

$x_2 = 5.5$ $\quad$ $y_2 = 0.7403627$

For $x = 5$

$$\rightarrow \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\Rightarrow \frac{5 - 4.5}{5.5 - 4.5} = \frac{y - 0.6532125}{0.7403627 - 0.6532125}$$

$$\Rightarrow \frac{0.5}{1} = \frac{y - 0.632125}{0.0871502}$$

$$\Rightarrow y - 0.632125 = 0.0435751$$

$$\Rightarrow \boxed{y = 0.6967876}$$

~~3) $x^4 - x - 10 = 0$~~

~~a = 2~~

~~b = 2~~

4. $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	y
0	1
1	-1
2	5

$x_0 = 1$
$f(x_0) = -1$
$f'(x_0) = 2$

$\rightarrow x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1 - \dfrac{(-1)}{2} = \boxed{1.5}$

$\rightarrow f(x_1) = 0.875$
$f'(x_1) = 5.75$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 1.5 - \dfrac{0.875}{5.75} = 1.34783$

$\rightarrow f(x_2) = 0.100699$
$f'(x_2) = 4.44994$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 1.34783 - \dfrac{0.100699}{4.44994} = 1.3252$

$\rightarrow f(x_3) = 0.00205$
$f'(x_3) = 4.26846$

$x_4 = 1.32472$

$\rightarrow f(x_4) = 0.000008712$

5. $A^2 = A$

$$7A - (I + A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$$
$$= 7A - (I + A^2(A) + 3A^2 I + 3A)$$
$$= 7A - (I + A^2 + 6A)$$
$$= 7A - (I + A + 6A)$$
$$= 7A - 7A - I$$
$$= \boxed{-I}$$

6. $$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$|A| = 1(2-6) + 1(-4-0) + 2(4-0)$$
$$= -6 + (-4) + 8$$
$$= -2$$

$$\text{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$= \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

7. $\vec{a} = 8\hat{i} - 9\hat{j}$
$\vec{b} = 7\hat{i} - 9\hat{j}$

$\rightarrow$ $$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{a \cdot b}$$

$\rightarrow \vec{a}\cdot\vec{b} = 56 + 81$

$= 137$

$\rightarrow a = \sqrt{64 + 81} = \sqrt{145}$

$\rightarrow b = \sqrt{49+81} = \sqrt{130}$

$\rightarrow \cos\theta = \dfrac{137}{(12.0415)(11.40175)}$

$= 0.99784916$

$\Rightarrow \theta = \cos^{-1}(0.99784916)$

$\theta = \boxed{3.75856^\circ}$

8. $R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30^\circ$

$\Rightarrow R^2 = 75^2 + 25^2 + 2(75)(25)\cos 30^\circ$

$= 9497.595264$

$\Rightarrow \boxed{R = 97.4556 \text{ m}}$

9. $d = 3$

$a = 101$

$n = ?$

$S_n = ?$

$\rightarrow l = a+(n-1)d$

$\rightarrow 998 = 101 + 3n - 3$

$\rightarrow 3n = 900$

$\rightarrow \boxed{n = 300}$

$\rightarrow S_n = \dfrac{n}{2}(a+l)$

$= \dfrac{300}{2}(101+998)$

$= 150(1099)$

$= \boxed{1,64,850}$

10. $a_6 = 32$ $\rightarrow a_8 = 128$

$ar^5 = 32 \rightarrow (1)$ $\Rightarrow ar^7 = 128 \rightarrow (2)$

From $(2) \div (1)$ $r^2 = 4$

$\Rightarrow \boxed{r = 2}$

11. Expanding $(4+3x)^5$

$\rightarrow (3x+4)^5 = \binom{5}{0}(3x)^5 + \binom{5}{1}(3x)^4(4)^1 + \binom{5}{2}(3x)^3 4^2 + \binom{5}{3}(3x)^2 4^3$

$+ \binom{5}{4}(3x)(4)^4 + \binom{5}{5} 4^5$

$= 3x^5 + 5(4)((3x)^4) + 10(16)(3x)^3 + 640(3x)^2 + 1280(3)$

$+ 1024$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

Expanding the matrix from row 3:-

for $|A - \lambda I| = (3-\lambda)[-10 - 2\lambda + 5\lambda + \lambda^2 + 6]$

$= (3-\lambda)[\lambda^2 + 3\lambda - 4]$

$= 3\lambda^2 + 9\lambda - 12 - \lambda^3 - 3\lambda^2 + 4\lambda$

$= -\lambda^3 + 13\lambda - 12$

$\Rightarrow -\lambda^3 + 13\lambda - 12 = 0$

$\Rightarrow \lambda^3 - 13\lambda + 12 = 0$

$\therefore$ 1 is a root of the equation.

1	1	0	−13	12
	1	1	1	−12
	1	1	−12	0

$\lambda^2 + \lambda - 12 = 0$

$\Rightarrow (\lambda + 4)(\lambda - 3) = 0$

$\Rightarrow \lambda = 3$ or -4

$\therefore$ The eigen values will be $-4, 1$ & 3.

14. $(x^2 - x + 1) = [(x^2+1) - x]^4$

let $(x^2+1) = y$

$\Rightarrow (y-x)^4 = \binom{4}{0} y^4 x^0 - \binom{4}{1} y^3 x + \binom{4}{2} y^2 x^2 - \binom{4}{3} y x^3$

$+ \binom{4}{4} y^0 x^4$

Substituting the value gives

$= (x^2+1)^4 - 4(x^2+1)^3 x + 6x^2(x^2+1)^2$

$- 4(x^2+1)x^3 + x^4$

15. $e^{-x} = 3\log(x)$ (Bisection Method)

$\Rightarrow e^{-x} - 3\log(x) = 0$

$\rightarrow f(x) = e^{-x} - 3\log x$

x	$f(x)$
0.3	-0.82782
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	0.03188

$\therefore x_0 = 0.6$
$x_1 = 0.7$

$\rightarrow x_2 = \dfrac{x_0 + x_1}{2} = 0.65$

$\rightarrow f(x_2) = 0.03921$

$\rightarrow x_3 = \dfrac{x_1 + x_2}{2} = 0.6875$

$\rightarrow f(x_3) = 0.01465$

$\rightarrow x_4 = \dfrac{x_2 + x_3}{2} = 0.68125$

$f(x_4) = 0.0059$

13. $f(x) = x^3 - 7x^2 + 8x - 3 \rightarrow f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$x_2 = ?$ (Newton's Method)

$\rightarrow f(x_0) = 5^3 - 7(5)^2 + 8(5) - 3$

$= -13$

$\rightarrow f'(x) = 13.$

$\rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{(-13)}{13} = 6$

$\rightarrow f(x_1) = 9$

$\rightarrow f'(x_1) = 32$

$\rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{9}{32} = \boxed{5.71875}$

3. $x^4 - x - 10 = 0$

$x_0 = 1.5$

$x_1 = 2$

$\rightarrow x_2 = \frac{x_0 + x_1}{2} = 1.75$

$\rightarrow f(x_2) = -2.37109$

$\rightarrow x_3 = \frac{x_1 + x_2}{2} = 1.375$

$\rightarrow f(x_3) = 0.48462$

$\rightarrow x_4 = \frac{x_2 + x_3}{2} = 1.8125$

$\rightarrow f(x_4) = -1.02025$

$\rightarrow \quad x_5 = \frac{x_4 + x_3}{2} = 1.84375$

$\therefore \quad f(x_5) = -0.28773$

$\rightarrow \quad x_6 = \frac{x_3 + x_5}{2} = 1.859375$

$\therefore \quad f(x_6) = 0.93378$