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Homework - 1 Lagranges' Interpolation

①

x	y	Δy	Δy^2	Δy^3
7	3			
		-2		
8	1		2	
		0		6
				$= \Delta^3 f(x_n)$
9	1		8	
		8	$= \Delta^2 f(x_n)$	
10	9	$= \Delta f(x_n)$		
	$= f(x_n)$			

$h = 1$, $u = \frac{x - x_n}{h} = \frac{9.5 - 10}{1} = -0.5 = -\frac{1}{2}$

$f(9.5) = f(x_n) + \frac{u \Delta f(x_n)}{1!} + \frac{u(u-1)}{2!} \Delta^2 f(x_n) + \frac{u(u-1)(u-2)}{3!} \Delta^3 f(x_n)$

$= 9 + (-0.5)(8) + \frac{(-0.5)(-1.5)}{2} (8)^4 + \frac{(-0.5)(-1.5)(-2.5)}{6} (6)$

$= 9 - 4 + 30 - 1.875$

$= \underline{6.125}$

②

P.T.O

x	3	7	9	10
y	168	120	72	63

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)}(168) + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)}(120)$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)}(72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)}(63)$$

$$= \frac{(-1)(-3)\cancel{(-4)}}{\cancel{(-4)}(-6)(-7)}(168) + \frac{(3)\cancel{(-3)}(-4)}{(4)(-2)\cancel{(-3)}}(120)$$

$$\cancel{= \frac{3}{42}(168)} \quad \frac{3(-1)(-4)}{6(+2)(-1)}(72) + \frac{3(-1)(-3)}{(7)(3)(1)}(63)$$

$$= \frac{3}{42}(168) + \frac{3}{2}(120) + (-72) + (+(27))$$

$$= 12 + 180 - 72 + 27$$

$$= \underline{147}$$

(3)

x	1	2	7	8
y	4	8	16	128

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$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)}(4) + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)}(8)$$

$$+ \frac{(6-1)(6-7)(6-2)}{(8-1)(8-7)(8-2)}(128) + \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)}(128)$$

$$= \frac{(4)(-3)(-2)(4)}{(-1)(-6)(-7)}(1) + \frac{(5)(-1)(-2)(8)}{(1)(-5)(-6)}$$

$$+ \frac{(5)(-1)(4)(128)}{(7)(+1)(6)} + \frac{(5)(4)(-2)(128)}{(6)(5)(-1)}$$

$$= -0.762 + 2.66667 + 3.6667 + (-1.904761)$$

$$= \underline{3.6666}$$

(4)

x	1	2	3	4	7
y	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)}(2) + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)}(4)$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)}(8) + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)}(16)$$

$$+ \frac{(5-1)(5-2)(5-4)(5-3)}{(7-1)(7-2)(7-4)(7-3)}$$

$$= -0.6667 + 6.4 - 24 + 42.6667 + 8.5333 = 32.9333$$

⑤ x	10	14	16	20
y	0.25	0.15	0.10	0.2

$$f(15) = \frac{(15-14)(15-16)(15-20)}{(10-14)(10-16)(10-20)}(0.25) + \frac{(15-10)(15-16)(15-20)}{(14-10)(14-16)(14-20)}(0.15)$$

$$+ \frac{(15-10)(15-14)(15-20)}{(16-10)(16-14)(16-20)}(0.1) + \frac{(15-10)\cancel{(15-14)}\cancel{(15-16)}}{(20-10)(20-14)(20-16)}(0.2)$$

$$= -0.00521 + 0.078125 + \cancel{0.078}\ 0.052083$$

$$-0.00417 \ldots$$

$$= \cancel{0.12083833}\ \ 0.120828$$

Homework - 2

① $f(x) = x^3 + x - 1$

→

x	$f(x)$
0.5	-0.375
1	1

$$\bar{x} = \frac{0.5(1) - (-0.375)(1)}{1 - (-0.375)}$$

$$\bar{x} = 0.63636$$

$$\bar{y} = (0.63636)^3 + 0.63636 - 1$$

$$\bar{y} = -0.10594$$

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→

x	$f(x)$
0.63636	-0.10594
1	1

$\bar{x} = \dfrac{0.63636(1) - (-0.10594)(1)}{1 - (-0.10594)}$

$\bar{x} = 0.6711937$

$\bar{y} = (0.6711937)^3 + 0.6711937 - 1$

$\bar{y} = -0.0264$

→

x	$f(x)$
0.6711937	-0.0264
1	1

$\bar{x} = \dfrac{0.6711937(1) - (-0.0264)(1)}{1 - (-0.0264)}$

$\bar{x} = 0.6796$

$\bar{y} = (0.6796)^3 + 0.6796 - 1$

$\bar{y} = -0.006522$

$\therefore$ The approximate value of the root is 0.6796.

(2) $f(x) = x^3 - 2x + 0.5$

→

x	$f(x)$
0	0.5
1	-0.5

$\bar{x} = \dfrac{0(-0.5) - (0.5)(1)}{-0.5 - 0.5}$

$\bar{x} = 0.5$

$\bar{y} = (0.5)^3 - 2(0.5) + 0.5 = -0.375$

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→

x	$f(x)$
0	0.5
0.5	-0.375

$$\bar{x} = \frac{0(-0.375) - (0.5)(0.5)}{-0.375 - 0.5}$$

$$\bar{x} = 0.2857$$

$$\bar{y} = (0.2857)^3 - 2(0.2857) + 0.5$$

$$\bar{y} = -0.0481$$

→

x	$f(x)$
0	0.5
0.2857	-0.0481

$$\bar{x} = \frac{0(-0.0481) - (0.5)(0.2857)}{-0.0481 - 0.5}$$

$$\bar{x} = 0.26063$$

$$\bar{y} = (0.26063)^3 - 2(0.26063) + 0.5$$

$$\bar{y} = -0.003556$$

∴ The approximate value of the root is 0.26063.

③ $f(x) = x^3 + 2x - 20$

x	$f(x)$
2	-8
2.5	0.625

$$\bar{x} = \frac{2(0.625) - (-8)(2.5)}{0.625 - (-8)}$$

$$\bar{x} = 2.46377$$

$$\bar{y} = (2.46377)^3 + 2(2.46377) - 20 = \cancel{4.9721}\ -0.11697$$

→ x $f(x)$

x	$f(x)$
2.46377	-0.11697
2.5	0.625

$\bar{x} = \dfrac{2.46377(0.625) - (-0.11697)(2.5)}{0.625 - (-0.11697)}$

$\bar{x} = \cancel{2.30}\ 2.46948$

$\bar{y} = (2.46948)^3 + 2(2.46948) - 20$

$\bar{y} = -0.0013034$

∴ The approximate value of root is $\underline{2.46948}$.

(4) $f(x) = 3x - \cos x - 1$

→

x	$f(x)$
0	-2
1	2

$\bar{x} = \dfrac{0(2) - (-2)(1)}{2 - (-2)}$

$\bar{x} = 0.5$

$\bar{y} = 3(0.5) - \cos(0.5) - 1$

$\bar{y} = -0.49996$

→

x	$f(x)$
0.5	-0.49996
1	2

$\bar{x} = \dfrac{(0.5)(2) - (-0.49996)(1)}{2 - (-0.49996)}$

$\bar{x} = 0.59999$

$\bar{y} = 3(0.59999) - \cos(0.59999) - 1$

$\bar{y} = -0.199975$

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→ x $f(x)$

x	$f(x)$
0.59999	-0.199975
1	2

$$\bar{x} = \frac{(0.59999)(2) - (-0.199975)(1)}{2 - (-0.199975)}$$

$\bar{x} = 0.63635$

$\bar{y} = -0.09088$

→ x $f(x)$

x	$f(x)$
0.63635	-0.09088
1	2

$$\bar{x} = \frac{(2)(0.63635) - (-0.09088)(1)}{2 - (-0.09088)}$$

$\bar{x} = 0.652156$

$\bar{y} = -0.043467$

→ x $f(x)$

x	$f(x)$
0.652156	-0.043467
1	2

$$\bar{x} = \frac{2(0.652156) - (-0.043467)(1)}{2 - (-0.043467)}$$

$\bar{x} = 0.659555$

$\bar{y} = 3(0.659555) - \cos(0.659555) - 1$

$\bar{y} = -0.02127$

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→ x $f(x)$ $\bar{x} = \dfrac{2(0.659555) - (-0.02127)(1)}{2 - (-0.02127)}$

x	$f(x)$
0.659555	-0.02127
1	2

$\bar{x} = 0.66314$

$\bar{y} = (0.66314)3 - \cos(0.66314) - 1$

$\bar{y} = -0.0105$

→ x $f(x)$ $\bar{x} = \dfrac{2(0.66314) - (-0.0105)(1)}{2 - (-0.0105)}$

x	$f(x)$
0.66314	-0.0105
1	2

$\bar{x} = 0.6649$

$\bar{y} = (0.6649)3 - \cos(0.6649) - 1$

$\bar{y} = -0.00523$

∴ The approximate value of the root is $\underline{\cancel{-0.00523}}$ 0.6649.

(5) $f(x) = x^3 - 3x - 5$

x	$f(x)$
2	-3
2.5	3.125

$\bar{x} = \dfrac{2(3.125) - (-3)(2.5)}{3.125 - (-3)}$

$\bar{x} = 2.244897$

$\bar{y} = -0.4213777$

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→ x $f(x)$

~~244~~

2.244897 -0.4213777

2.5 3.125

$$\bar{x} = \frac{(2.244897)(3.125) - (-0.4213777)(2.5)}{3.125 - (-0.4213777)}$$

$\bar{x} = 2.2752$

$\bar{y} = -0.04795$

→ x $f(x)$

2.2752 -0.04795

2.5 3.125

$$\bar{x} = \frac{2.2752(3.125) - (-0.04795)(2.5)}{3.125 - (-0.04795)}$$

$\bar{x} = 2.2786$

$\bar{y} = -0.00526$

∴ The approximate value of root is 2.2786.

(6) $x \log_{10} x - 1.2 = f(x)$

→ x $f(x)$

2 -0.59794

3 0.23136

$$\bar{x} = \frac{2(0.23136) - (-0.59794)(3)}{0.23136 - (-0.59794)}$$

$\bar{x} = 2.72102$

$\bar{y} = -0.017086$

x	$f(x)$
2.72102	-0.017086
3	0.23136

$$\bar{x} = \frac{(2.72102)(0.23136) - (-0.017086)(3)}{0.23136 - (-0.017086)}$$

$\bar{x} = 2.7394$

$\bar{y} = -0.001086$

$\therefore$ The approximate value of the root is 2.7394.

(7) $f(x) = x^3 - 4x + 1$

x	$f(x)$
0	1
1	-2

$$\bar{x} = \frac{0(-2) - 1(1)}{-2-1}$$

$\bar{x} = 0.3333$

$\bar{y} = -0.296174$

x	$f(x)$
0	1
0.3333	-0.296174

$$\bar{x} = \frac{0(-0.296174) - 1(0.3333)}{-0.296174 - 1}$$

$\bar{x} = 0.257141$

$\bar{y} = -0.01156$

x	$f(x)$
0	1
0.257141	-0.01156

$$\bar{x} = \frac{0(-0.01156) - (1)(0.257141)}{-0.01156 - 1} = 0.254202$$

$\bar{y} = -0.0003818$

$\therefore$ The approximate value of root is 0.254202.

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(7) $f(x) = x^3 + 2x^2 - 8$

→

x	$f(x)$
0	-8
1	-5
2	8

$$\bar{x} = \frac{1(8) - (-5)(2)}{8 - (-5)}$$

$\bar{x} = 1.3846$

$\bar{y} = (1.3846)^3 + 2(1.3846)^2 - 8$

$\bar{y} = -2.57636$

→

x	$f(x)$
1.3846	-2.57636
2	8

$$\bar{x} = \frac{8(1.3846) - (-2.57636)(2)}{8 - (-2.57636)}$$

$\bar{x} = 1.53451$

$\bar{y} = 0.32278$

→

x	$f(x)$
1.3846	-2.57636
1.53451	0.32278

$$\bar{x} = \frac{(0.32278)(1.3846) - (-2.57636)(1.53451)}{0.32278 - (-2.57636)}$$

$\bar{x} = 1.51782$

$\bar{y} = 0.10427$

→

x	$f(x)$
1.3846	-2.57636
1.51782	0.10427

$$\bar{x} = \frac{(0.10427)(1.3846) - (-2.57636)(1.51782)}{0.10427 - (-2.57636)}$$

$\bar{x} = 1.51264$ $\bar{y} = 0.0372$

→ x $\quad f(x)$ $\quad \bar{x} = \dfrac{(1.3846)(0.0372) - (-2.57636)(1.51264)}{0.0372 - (-2.57636)}$

x	$f(x)$
1.3846	-2.57636
1.51264	0.0372

$\bar{x} = 1.51082$

$\bar{y} = 0.01372$

→ x $\quad f(x)$ $\quad \bar{x} = \dfrac{(0.01372)(1.3846) - (-2.57636)(1.51082)}{0.01372 - (-2.57636)}$

x	$f(x)$
1.3846	-2.57636
1.51082	0.01372

$\bar{x} = 1.51015$

$\bar{y} = 0.00508$

∴ The approximate value of root is 1.51015.

(9) $f(x) = x^3 - 9x + 1$

→ x $\quad f(x)$ $\quad \bar{x} = \dfrac{0(-7) - (1)(1)}{-7 - 1}$

x	$f(x)$
0	1
1	-7

$\bar{x} = \cancel{-0.125}\ 0.125$

$\bar{y} = -0.12305$

∴ The approximate value of root is 0.111304.

→ x $\quad f(x)$ $\quad \bar{x} = \dfrac{0(-0.12305) - (1)(0.125)}{-0.12305 - 1}$

x	$f(x)$
0	1
+0.125	-0.12305

$\bar{x} = 0.111304$ $\quad \bar{y} = -0.000357$

(10) $f(x) = x^3 - x - 4$

→

x	$f(x)$
0	-4
1	-4
2	2

$$\bar{x} = \frac{2(1) - (-4)(2)}{2-(-4)}$$

$\bar{x} = 1.6667$

$\bar{y} = -1.0368$

→

x	$f(x)$
1.6667	-1.0368
2	2

$$\bar{x} = \frac{2(1.6667) - (-1.0368)(2)}{2-(-1.0368)}$$

$\bar{x} = 1.7805$

$\bar{y} = -0.136$

→

x	$f(x)$
1.7805	-0.136
2	2

$$\bar{x} = \frac{2(1.7805) - 2(-0.136)}{2-(-0.136)}$$

$\bar{x} = 1.7945$ $\quad \bar{y} = -0.0158$

→

x	$f(x)$
1.7945	-0.0158
2	2

$$\bar{x} = \frac{2(1.7945) - (-0.0158)(2)}{2-(-0.0158)}$$

$\bar{x} = 1.79611$ $\quad \bar{y} = -0.00184$

∴ The approximate value of root is 1.79611.