Assignment

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Roll No: 86 Subject: Numerical methods and algebra

1] Area of circular plate $= \pi r^2$, i.e. $A(r) = \pi r^2$

$A'(r) = 2\pi r$

Change in Area $= A'(12.5)(0.15) = 3.75\pi\,cm^2$

Exact calculation of the change in Area

$= A(12.65) - A(12.5)$

$= 160.0225\pi - 156.25\pi$

$= 3.7725\pi\,cm^2$

i) Absolute error = Actual value - Approximate value

$= 3.7725\pi - 3.75\pi$

$= 0.0225\pi\,cm^2$

ii) Relative error $= \dfrac{\text{Actual value} - \text{Approximate value}}{\text{Actual value}}$

$= \dfrac{3.7725\pi - 3.75\pi}{3.7725\pi}$

$= \dfrac{0.0225\pi}{3.7725\pi}$

$= 0.006\,cm^2$

iii) Percentage error = Relative error $\times 100$

$= 0.006 \times 100$

$= 0.6\%$

2] a)

$$f(5) = f(4) + \frac{f(6) - f(4)}{6-4}(6-4)$$

$$= 0.60206 + \frac{0.7781513 - 0.60206}{6-4}(5-4)$$

$$= 0.690106$$

$$\text{Error} = \frac{0.69897 - 0.690106}{0.69897} \times 100\% = 1.27\%$$

b)

$$f(5) = f(4.5) + \frac{f(5.5) - f(4.5)}{5.5 - 4.5}(5 - 4.5)$$

$$= 0.6532125 + \frac{0.7403627 - 0.6532125}{5.5 - 4.5}(5 - 4.5)$$

$$= 0.696788$$

$$\text{Error} = \frac{0.69897 - 0.696788}{0.69897} \times 100\% = 0.3\%$$

3]

a	b	c	f(a) * f(c)
1.5	2	1.75	15.264
1.75	2	1.875	2.419
1.75	1.875	1.812	2.419
1.812	1.875	1.844	-0.303
1.844	1.875	1.86	-0.027

Hence the root of $x^4 - x - 10 = 0$ is approx 1.86

4]

$x^3 - x - 1 = 0$

$f(x) = x^3 - x - 1$ Here $f(1) = -1 < 0$ and

$f'(x) = 3x^2 - 1$ $f(2) = 5 > 0$

x	$f(x)$
0	-1
1	-1
2	5

$\therefore$ Root lies between 1 and 2

$$x_0 = \frac{1+2}{2} = 1.5$$

1st iteration: $f(x_0) = f(1.5) = 0.875$

$f'(x_0) = f'(1.5) = 5.75$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 - \frac{0.875}{5.75}$$

$x_1 = 1.34783$

2nd iteration: $f(x_1) = f(1.34783) = 0.10068$

$f'(x_1) = f'(1.34783) = 4.44991$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.34783 - \frac{0.10068}{4.44991}$$

$x_2 = 1.3252$

3rd iteration: $f(x_2) = f(1.3252) = 0.00206$

$f'(x_2) = f'(1.3252) = 4.26847$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3252 - \frac{0.00206}{4.26847}$$

$x_3 = 1.32472$

4th iteration: $f(x_3) = f(1.32472) = 0$

$f'(x_3) = f'(1.32472) = 4.26463$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.32472 - \frac{0}{4.26463}$$

$x_4 = 1.32472$

The root of the equation $x^3 - x - 1 = 0$ is approximatelly 1.32472.

5]

$$7A-(I+A)^3 = 7A-(I^3+A^3+3A^2I+3AI^2)$$
$$= 7A-(I+A\cdot A^2+3A^2+3A)$$
$$= 7A-(I+A\cdot A+3A+3A) \quad (\because A^2=A$$
$$= 7A-(I+A^2+6A)$$
$$= 7A-(I+A+6A) \quad (\because A^2=A$$
$$= 7A-I-7A$$
$$= -I$$

Hence, the value of $7A-(I+A)^3$ is $-I$.

c] $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

First we find cofactors:

$\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} = -1-(6)$
$= \cancel{-6}\,-7$

$\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} = -4-(0)$
$= -4$

$\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = 4$

$\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1-2$
$= -1$

$\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1-2$
$= -3$

$\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1-0$
$= 1$

$\begin{vmatrix} -1 & 2 \\ 0 & 6 \end{vmatrix} = -6-0$
$= -6$

$\begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 6-8$
$= -2$

$\begin{vmatrix} 1 & -1 \\ 4 & 0 \end{vmatrix} = 0-(-4)$
$= 4$

$\begin{vmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1(\cancel{0}-6) + 1(-4-0) + 2(4-0)$

$= -6 + (-4) + 8$

$= -6 - 4 + 8$

$= \cancel{-10}\; -2$

6)

$$A = \begin{bmatrix} -7 & -4 & 4 \\ -1 & -3 & 1 \\ -6 & -2 & 4 \end{bmatrix}$$

To get adjoint matrix,

$$A = \begin{bmatrix} -7 & -4 & 4 \\ -1 & -3 & 1 \\ -6 & -2 & 4 \end{bmatrix} \times \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 4 & 4 \\ 1 & -3 & -1 \\ -6 & 2 & 4 \end{bmatrix}$$

Now we will take transpose ~~to~~

$$Adj(A) = \begin{bmatrix} -7 & 1 & -6 \\ 4 & -3 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{det(A)} \times adj(A)$$

$$= \frac{1}{-\cancel{18}2} \times adj(A)$$

$$\cancel{\therefore A^{-1} = \begin{bmatrix} -7/-18 & 1/-18 & -6/-18 \\ 4/-18 & -3/-18 & 2/-18 \\ 4/-18 & -1/-18 & 4/-18 \end{bmatrix}}$$

$$\therefore A^{-1} = \begin{bmatrix} 7/2 & -1/2 & 3 \\ -2 & 3/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

8] $A = 75$, $B = 25$, $\theta = 30^\circ$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30^\circ}$

$= 97.46$ meter

1) $\vec{A} = (8\hat{i} - 9\hat{j})$ and $\vec{B} = (7\hat{i} - 9\hat{j})$

$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right]$

$= \cos^{-1}\left[\frac{(8\hat{i} - 9\hat{j})\cdot(7\hat{i} - 9\hat{j})}{|(8\hat{i} - 9\hat{j})||(7\hat{i} - 9\hat{j})|}\right]$

$= \cos^{-1}\left[\frac{(8)(7) + (-9)(-9)}{\sqrt{(8)^2 + (-9)^2}\sqrt{(7)^2 + (-9)^2}}\right]$

$= 3.76^\circ$

$\therefore$ The smallest non-negative angle is 3.76°

9] The lowest 3 digit number is 101 and the highest number is 998, which would give remainder of 2 when divided by 3.

$\therefore AP = 101, 104, 107, \ldots, 998$

where $a = 101$, $d = 104 - 101 = 3$, $a_n = 998$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$(n-1) = \frac{998 - 101}{3}$

$n - 1 = 299$

$n = 299 + 1$

$n = 300$

$S = \frac{n}{2}(2a + (n-1)d)$

$= \frac{300}{2}(2(101) + (300-1)3)$

$= 150((202) + (299)3)$

$= 150(202 + 897)$

$= 150(1099)$

$= 164850$

Hence, the sum of all 3 digit numbers that leave a remainder of 2 when divided by 3 is 164,850.

11] $(4+3x)^5$

$$\cancel{(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots}$$

$$\cancel{(4+3x)^5 = 4^5 + 5(4)^4(3x) + 5(4)(4)^3(3x)^2 + (5)(4)(3)(4)}$$

$$= \frac{5!}{0!(5-0)!} \cdot 4^5(3x)^0 + \frac{5!}{1!(5-1)!} \cdot 4^4(3x)^1 + \frac{5!}{2!(5-2)!} \cdot 4^3(3x)^2$$

$$+ \frac{5!}{3!(5-3)!} \cdot 4^2(3x)^3 + \frac{5!}{4!(5-4)!} \cdot 4^1(3x)^4 + \frac{5!}{5!(5-5)!} \cdot 4^0(3x)^5$$

$$= \frac{120}{1(120)} \cdot 1024(1) + \frac{120}{1(24)} \cdot 256(3x) + \frac{120}{2(6)} \cdot 64(9x^2)$$

$$+ \frac{120}{6(2)} \cdot 16(27x^3) + \frac{120}{24(1)} \cdot 4(81x^4) + \frac{120}{120(1)} \cdot 1(243x^5)$$

$$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$$

13]

$f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

First iteration $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{-13}{13} =$

Second iteration $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{f(6)}{f'(6)} = 6 - \frac{9}{32} = 5.7$

$$\boxed{x_2 = 5.71875}$$

[14] $(1-x+x^2)^4$

$= (1-x+x^2)(1-x+x^2)(1-x+x^2)(1-x+x^2)$

$= (x^4-2x^3+3x^2-2x+1)(1-x+x^2)(1-x+x^2)$

$= (x^6-3x^5+6x^4-7x^3+6x^2-3x+1)(1-x+x^2)$

$= x^6\cdot 1 + x^6(-x) + x^6x^2 - 3x^5\cdot 1 - 3x^5(-x) - 3x^5x^2 + 6x^4\cdot 1 + 6x^4(-x) + 6x^4x^2 - 7x^3\cdot 1 - 7x^3(-x) - 7x^3x^2 + 6x^2\cdot 1 + 6x^2(-x) + 6x^2x^2 - 3x\cdot 1 - 3x(-x) - 3xx^2 + 1\cdot 1 + 1\cdot(-x) + 1\cdot x^2$

$= x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

[5]

a	b	c	f(a) * f(b)
0.5	1.5	1	0.555
1	1.5	1.25	-1.555×10^{-3}
1	1.25	1.125	0.063
1.125	1.25	1.187	0.014

The value of x approximatelly is 1.187

[12] $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

To find eigen value Consider $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda)\begin{vmatrix} -(5+\lambda) & 0 \\ 0 & (3-\lambda) \end{vmatrix} + 3\begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$\therefore (2-\lambda)[-(5+\lambda)(3-\lambda)] + 3(6-2\lambda) = 0$

$\therefore -(2-\lambda)(5+\lambda)(3-\lambda) + 6(3-\lambda) = 0$

$\therefore [3-\lambda]\{(\lambda-2)(\lambda+5)+6\} = 0$

$\therefore (3-\lambda)(\lambda^2+3\lambda-4) = 0$

$\therefore (3-\lambda)(\lambda-1)(\lambda+4) = 0$

$\therefore \lambda = 3, 1, -4$

Eigen vectors

for $\lambda = 3$

$(A-\lambda I)x = 0$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 + 2R_1$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 = R_2/-14$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_1 + R_1 + 3R_2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 =$ Indeterminate $= \alpha$

for $\lambda = 3$, eigen vector $= \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

for $\lambda = 1$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = 0$, $x_1 - 3x_2 = 0$, $x_1 = 3x_2$

$$\text{eigen vector} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} x_2$$

For $\lambda = -4$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 = R_1 - 3R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 \\ 2x_1 - x_2 \\ 7x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$7x_3 = 0$ $\quad x_3 = 0$

$2x_1 - x_2 = 0$

$x_2 = 2x_1$, $x_1 = \frac{1}{2}x_2$

$$\text{eigen vector} = \begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} x_1 \text{ or } \begin{bmatrix} 1/2\, x_2 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} x_2$$

10] Let a be the first term of GP with common ratio r.

The nth term, $a_n = ar^{n-1}$

Given 6^{th} term $= 32$

$ar^5 = 32$ — (i)

Given 8^{th} term $= 128$

$ar^7 = 128$ — (ii)

Divide (ii) by (i)

$ar^7 / ar^5 = 128/32$

$r^2 = 4$

$r = 2$

∴ Common ratio is 2.