Assignment-1

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Subject : Probability & Statistics-2
Section : FY-8.

Soln ① $X \sim Pois(\theta)$.

$$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$$

$P[X_1 < \theta < X_2] = 0.95$.

$X \sim Pois(200)$.

$$95\% \ CI = \mu \pm 1.96\sqrt{\sigma}$$
$$= 200 \pm 1.96\sqrt{200}$$

Hence the interval is (172.28, 227.7186)

Soln ② $\bar{x} = \sum_{i=1}^{n} x_i$ = sample mean

$S^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})$: sample Variance

i) $E(\bar{x}) = E\left[\frac{x_i}{n}\right] = \frac{1}{n}(\Sigma x_i)E = \frac{1}{n}\left[\Sigma E(x_i)\right] = \frac{n\mu}{n} = \boxed{\mu}$

$V(\bar{x}) = V\left[\frac{\Sigma x_i}{n}\right] = \frac{1}{n^2} V[E(x)] = \frac{n\sigma^2}{n^2} = \sigma^2/n.$

(ii) $E(S^2) = E\left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]$

$= \frac{1}{n-1} E\left[\Sigma x_i^2 - \bar{x}^2\right)$

$= \frac{1}{n-1}\left[E(\Sigma x_i^2) - E(n\bar{x}^2)\right] = \frac{1}{n-1}\left[E\left[\sigma^2 + \mu^2\right) - n\left[\frac{\sigma^2}{n} + \mu^2\right)\right]$

$$\frac{1}{n-1}\left[n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}\right]$$

$$= \frac{1}{n-1}\left[(n-1)\sigma^2\right] = \boxed{\sigma^2}$$

Now here $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$$Var\left[\frac{(n-1)s^2}{\sigma^2}\right] = \chi^2_{n-1}$$

$$\frac{(n-1)^{\cancel{2}}\, Var[s^2]}{\sigma^4} = 2\cancel{(n-1)}$$

$$\boxed{Var[s^2] = \frac{2\sigma^4}{(n-1)}}$$

(iv) $n = 10$

$\sigma^2 = 100$.

$$\frac{(n-1)s^2}{\sigma^2} = 0.5$$

$$\frac{9 \times s^2}{100} = 0.5$$

$$s^2 = \frac{50}{9}$$

$$\boxed{s^2 = 5.56 \cdot 1}$$

③

No. of Claim	0	1	2	3	4
No. of Policies.	86	75	16	2	1

(i) MLE

$X \sim Bin(4, p)$.

$$L = P(X=0)^{86} \times P(X=1)^{75} \times P(X=2)^{16} \times P(X=3)^{2} \times P(X=4)^{1}$$

$$= \left[{}^4C_0 \cdot p^0 \cdot (1-p)^4\right]^{86} \times \left[{}^4C_1 \cdot p^1 (1-p)^3\right]^{75} \times \left[{}^4C_2\, p^2 (1-p)^2\right]^{16} \times \left[{}^4C_3\, p^3 (1-p)\right]^{2} \times \left[{}^4C_4\, p^4 (1-p)^0\right]^{1}$$

$$= \left[Cons. \times (1-p)^4\right]^{86} \times \left[Cons \times p \cdot (1-p)^3\right]^{75} \times \left[Con\; p^2 (1-p)^2\right]^{16} \times \left[Con\; p^3 (1-p)\right]^{2} \cdot \left[{}^4C_4\, p^4\right]^{1}$$

$$\ln(L) = 344\ln(1-p) + 75\ln(p) + 255\ln(1-p) + 32\ln p + 32\ln(1-p) + 6\ln p + 2\ln(1-p) + 4\ln p.$$

$$\ln(L) = 633\ln(1-p) + 117\ln p$$

$$\frac{d[\ln(L)]}{dp} = \frac{117}{p} - \frac{633}{1-p}$$

let $\frac{d[\ln(L)]}{dp} = 0$

$$\frac{117}{p} = \frac{633}{1-p}$$

$$117 - 117p = 633p$$

$$117 = 750p$$

$$\boxed{\hat{p} = 0.156.}$$

(ii) $P(X=0) = {}^4C_0\, p^0 (1-p)^4$
$= 91.34$

$P(X=1) : {}^4C_1 (p)(1-p)^3$
$: 67.53$

$P(X=2) - {}^4C_2 (p)^2 (1-p)^2$
$: 18.72$

$P(X=3) = {}^4C_3 (p)^3 (1-p)^1$
$-$ ~~[illegible]~~ 2.31

$P(X=4) = {}^4C_4 (p)^4 (1-p)^0$
$= 0.11$

$f(x)$	O_i	E_i	$(O_i-E_i)^2/E_i$
$P(X=0)$	86	91.34	0.3123
$P(X=1)$	75	67.53	0.8263
$P(X=2)$	19	~~21.79~~ 2	0.3982
~~$P(X=3)$~~	~~2~~	~~2.31~~	~~0.0416~~
~~$P(X=4)$~~	1	~~0.11~~	~~7.2009~~
			39.8563

~~$X_{cal} = 8.7793$ at dof = 3.~~ $\chi^2_{cal} = 39.856$
~~$X_{tab} = 7.815$.~~ $\chi^2_{tab} = 3.841$

$X_{cal} > X_{tab}$

Thus we will reject H_0 at 5%.

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(4) $f(x) = \dfrac{cB^2}{(x+B)^4}$: $x>0, B>0$

(i) $\int_0^\infty f(x)\,dx = 1$

$cB^3 \int_0^\infty \dfrac{1}{(x+B)^4} = 1$

$cB^3 \left[\dfrac{(x+B)^{-3}}{-3}\right]_0^\infty = 1$

$\dfrac{c\cancel{B^3}}{3\cancel{B^3}} = 1$

$\boxed{c=3}$

$E(x) = \int_0^\infty x \cdot f(x) \cdot dx$

$= 3B^3 \int \dfrac{x}{(x+B)^4} dx$

$= 3B^3 \left[\dfrac{(x+B)^{-2}}{-2} - \dfrac{B(x+B)^{-3}}{-3}\right]_0^\infty$

$= 3B^3 \left[0 - 0 - \left(\dfrac{B^{-2}}{-2} + \dfrac{B^{-2}}{3}\right)\right]$

$= 3B^2 \left[\dfrac{1}{2B^2} + \dfrac{1}{3B^2}\right] = B/2$

$E(x^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx = 3B^3 \int_0^\infty \dfrac{x^2}{(x+B)^4} dx$

(ii) $B = x_1, x_2, x_3 \ldots x_n$

$E(x) = \bar{x}$ $\sim$ $\boxed{\hat{B} = 2\bar{x}_n}$

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$E(\hat{B}) = E(2\bar{X}_n) = \frac{2n}{n} \times \frac{B}{2} = B$ $\therefore$ unbiased//

(iii) $MSE = Var(b\bar{X}_n) + (Bias[b\bar{X}_n])^2$

$$= b^2 \times \frac{3}{4} \times \frac{B^2}{n} + \left(E(b\bar{X}_n) - B\right)^2$$

$$= \frac{B^2}{n}\left[\frac{3b^2}{4} + n\left(\frac{b}{2} - 1\right)^2\right]$$

(iv) It seen is that $\hat{B} = 2\bar{X}_n$ is an unbiased estimator. The estimt $b\bar{X}_n$ in (iii) wh $b = \frac{2}{1+3/n}$ is only biased estimator.

It is also consitant.

(5) (i) $X \sim U(a+b)$

$$\bar{x} = \frac{1}{2}(a+b)$$

$$s = \frac{1}{\sqrt{12}}(b-a)$$

$$\sqrt{12} + a = b$$

$$\bar{x} = \frac{1}{2}(a + \sqrt{12}s + a)$$

$$\bar{x} = \frac{1}{4}[2a + 2\sqrt{3}s]$$

$$\boxed{\hat{a} = \bar{x} - \sqrt{3}s}$$

(ii) $\bar{x} = \frac{1}{2}(a+b)$

$s = \frac{1}{\sqrt{12}}(b-a)$

$a = b - \sqrt{12}s$

$\therefore \bar{x} = \frac{1}{2}[b - \sqrt{12}s + b]$

$= \frac{1}{2}[2b - 2\sqrt{3}s$

$\boxed{\hat{b} = \bar{x} + \sqrt{3}s}$

(iii) Sample : 1, 2, 3, 4, 10.

$\bar{x} = 20/5 = 4$

$\$var = \frac{1}{12} \times 81 = 27/4$

$\hat{a} = \bar{x} - \sqrt{3 \times 27/4}$

$= 4 - \sqrt{3 \times 27/4} = -7.6913$

$\hat{b} = 4 + \sqrt{3}(27/4) = 15.69134$, Hence the result is false.

(iv) $f(x) = \frac{1}{b-a} = 1/b$

$\ln L = -n \log b$

$\frac{dl}{db} = -n/b = 0 \rightarrow \quad b \rightarrow \infty$

$\frac{d^2l}{db^2} = n/b^2 > 0$, minimum

$\boxed{\hat{\theta_b} = \max . x_i}$

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(6) $H_0: p = 0.1$

$H_1: p > 0.1$

(i) $P(\text{type } 1) = P[\text{Reject } H_0 | H_0 \text{ is } T]$

$= P[X \geq 3 | p = 0.1] + P[X = 0 | p = 0.1] \cdot P[Y \geq 5 | p = 0.1]$
$+ P[X = 1 | p = 0.1] \cdot P[Y \geq 4 | p = 0.1] + P[X = 2 | p = 0.1] \cdot P[Y \geq 3 | p = 0.1]$

$= 0.14727 \boxed{\simeq 15\%}$

(ii) $P(X \geq 3) | p = 0.3) + P[X = 0 | p = 0.3) = 0.91253 = \boxed{91\%}$

(iii) $P[\text{type II error}] = 1 - 0.9253 = 0.08747 \boxed{9\%}$

(iv) $X \sim Bin(120, 1/3)$, $\hat{p} = 40/120 = 1/3$

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

$$P\left[-1.2816 < \frac{\hat{p} - p}{\sqrt{pq/n}} < 1.2816\right]$$

99 CI $= (0.333, 0.2782)$

(v) $\frac{X - np_0}{\sqrt{np_0q_0}} \sim N(0,1) \ldots \boxed{1.2511}$

(vii) $P.Q = \frac{(\bar{x}_1 - \bar{x}) - (u_1 - u_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ $S_p^2 = 402700$

$\boxed{\leftarrow 0.2034}$

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(7) (i) Public =

$\bar{x}_1 = 30$

$s = 8.0195$. $s^2 = 64.3124$

Private:

$\bar{x}_2 = 35.06$

$s = 8.2099$ → $s^2 = 67.4024$.

(ii) $H_0: \bar{x}_1 = \bar{x}_2$ $H_1: \bar{x}_1 \neq \bar{x}_2$.

$$\frac{(30-35.06)}{s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(30-35.06)}{\sqrt{8.1153}\sqrt{\frac{2}{9}}} = -1.3226.$$

$$s_p^2 = \frac{8(64.3124)+8(67.4024)}{16} = 65.8575$$

$Z_{tab} = (-2.624, 2.624)$.

~~accept~~ H_0. not reject H_0.

(iii) Private hospital don't result in higher claim.

(8)

(i) $E(\hat{\theta}) = \theta$, it will be unbiased.

(ii) $E(\hat{\theta}) \neq \theta$, then will be biasness of parameter

(iii) MSE = var + bias^2.

(iv) $\hat{\theta}$ is more effecient than $\tilde{\theta}$, since MSE of $\hat{\theta}$ is lower than $\tilde{\theta}$

(v) when MSE move toward 0, as the n increase, the estimate would be more consistent.

(vi) Method of Moments.
Maximum liklihood estimator.

(a) $t_K \sim \frac{Z}{\sqrt{\chi_K / K}}$

$Z = N(0,1)$
χ_K^2 = Chi-square dist with degree of freedom K
K = degree of freedom

(ii) $\mu(t_K) = 0$ $\quad Var(t_K) = \frac{K}{K-2}$

(iii) $n = 10$
$\mu = 50$
$s^2 = 48.667$

$t_{0.005, 9} = 3.25$

(a) $CI = 50 \pm 3.25 \sqrt{\frac{48.667}{10}} = (42.803, 57.1699)$

(b) $CI = 50 \pm 2.5758 \sqrt{\frac{48.667}{10}} = (44.3176, 55.6823)$

(10) $n = 1000$, $X \sim Pois(3)$ $X \sim N(3000, 3000)$

(i) Type 1 error = $P[H_0 \text{ reject} / H_0 \text{ is true})$

$= P[X > 3100 / \lambda = 3]$

$= P\left[Z > \frac{3100 - 3000}{\sqrt{3000}}\right]$

$= P[Z > 1.8257)$

$= 1 - P[Z < 1.8257]$

$= 3.39\%$

(ii) Type 2 error = $P[\text{do not reject } H_0 / H_0 \text{ is false}]$

$= P[X < 3100 / \lambda = n\lambda)$

$= P\left[X < \frac{3100 - n\lambda}{\sqrt{n\lambda}}\right]$

(iii) Power = $P[\text{ reject } H_0 / H_0 \text{ is false}]$

$= P[X > 3100 / \lambda = n\lambda]$

$= 1 - P\left[\frac{3100 - n\lambda}{\sqrt{n\lambda}}\right]$

(iv) $n = 2900$.

$CI = 3 \pm 2.5758\sqrt{\frac{3}{2900}}$

$(2.9172, 3.0828)$.

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(11) $\Sigma x = 213,200$

$\Sigma x^2 = 4,919,860,000$

$\bar{x} = 17,767$

$s.d = 10,144.$

$n = 12$

$\Sigma x' = 213200 - 11000 - 48000 = 154200.$

$\Sigma x'' = 154200 + 27500 + 31000 = 213200$

$\boxed{\bar{x}' = 17767}$

$\Sigma x^{2'} = 4919860000 - 11000^2 - 48000^2 = 2494860000$

$\Sigma x^{2''} = 24948600 + 27500^2 + 31500^2 = 4243360000$

$$s = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$$

$\boxed{s = 6434.0326}$

(12) $X_1, X_2, X_3 \ldots X_n \sim U(0,\theta).,\quad a = 0,\ b = \theta$

(i) $E(Y) = \theta/2,\quad V(Y) = \theta^2/12$

$L = \prod_{i=1}^{n} \left(\frac{1}{\theta}\right) = 1/\theta^n$

$\ln(L) = -\cancel{\theta}n/\theta$

$dl/d\theta = -n/\theta$ here θ = infinite.

hence CRLB would not be applicable.

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(ii) $P[\text{Max } X < Y] = \prod_{i=1}^{n} \left[\int_0^y \frac{dx}{\theta} \right] = \left(\frac{y}{\theta}\right)^n$

$$E[Y^k] = \int y^k \cdot \frac{n}{\theta^n} \cdot y^{n-1} \, dy$$

$$= \frac{n}{n+k} \int_0^{\theta} \frac{n+k}{\theta^n} \cdot y^{n+k-1} \cdot dy$$

$$= \frac{n}{n+k} \cdot \left(\frac{\theta^{n+k} - 0}{\theta^n} \right)$$

$$\boxed{= \frac{n\theta^k}{n+k}}$$

(iii) $$\text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}(c) - \theta]$$

$$= E[cy - \theta]$$

$$= cE[y] - \theta.$$

$$= c \cdot \frac{n\theta}{n+1} - \theta$$

$$= \theta\left(\frac{cn}{n+1} - 1\right).$$

(iv) $$\frac{d}{dc} MSE[\theta(c)] = \frac{d}{dc} \left[\frac{c^2 n\theta^2}{n+2} - \frac{2n\theta^2}{n+1} c + \theta^2 \right]$$

$$c_m = \frac{2n\theta^2}{n+1} \cdot \frac{n+2}{2n\theta} = \frac{n+2}{n+1}$$

$$\frac{d^2}{dc^2}[MSE(\hat{\theta}(c))] = \frac{2n\theta^2}{n+2} > 0, \text{ min}^m \text{ value}$$

(vi) $\frac{n+2}{n+1} Y \to Y$, two estimator become one same $n \to \infty$.

$\times$ $\times$