

Rohit jain A34 numerical assignment

Numerical Methods & Algebra

HW-1

i)

x	7	8	9	10
f(x)	3	1	1	9

$f(9.5) = ?$

$$f(9.5) = (3) \times \frac{(7-1.5)(8-0.5)(9-0.5)}{(-1)(-2)(-3)} + (1) \times \frac{(7-2.5)(8-0.5)(9-0.5)}{(1)(-1)(-2)} \\ + (1) \times \frac{(7-3.5)(8-1.5)(9-0.5)}{(2)(1)(-1)} + (9) \times \frac{(7-8.5)(8-1.5)(9-0.5)}{(3)(2)(1)} \\ = 0.1875 + (-0.8125) + (0.9375) + 2.8125 \\ \underline{f(9.5) = 8.625}$$

2.

x	3	7	9	10
f(x)	168	120	72	63

when $x=6$, $f(x) = ?$

$$f(6) = (168) \times \frac{(6-7)(-3)(-4)}{(48) \times (96) \times (105)} + (120) \times \frac{(3)(-3)(-4)}{(-48)(48)(57)} \\ + (72) \times \frac{(3)(-1)(-4)}{(-96) \times (-48) \times (9)} + (63) \times \frac{(3)(-1)(-3)}{(-105) \times (-57)(-9)} \\ f(6) = (-0.0041667) + (-0.0000947) \\ + (0.000333) + (-0.0002631)$$

2.

x	3	7	9	10
f(x)	168	120	72	63

when $x=6$, $f(x) = ?$

$$f(6) = 168 \times \frac{(-1)(-3)(-4)}{(-4)(-6)(-7)} + (120) \times \frac{3 \times (-3)(-4)}{(4)(-2)(-3)} \\ + 72 \times \frac{(3)(-1)(-4)}{(6)(2)(-1)} + 63 \times \frac{(3)(-1)(-3)}{(7)(3)(1)} \\ = 12 + 180 + (-12) + 27 \\ \underline{f(6) = 147}$$

3.

x	1	2	7	8
f(x)	4	5	5	4

find $f(6)$

$$f(6) = (4) \times \frac{(4)(-1)(-2)}{(-1)(-6)(-7)} + (5) \times \frac{(5)(-1)(-2)}{(1)(-5)(-6)} \\ + (5) \times \frac{(3)(4)(-2)}{(6)(3)(-1)} + (4) \times \frac{(5)(4)(-1)}{(7)(6)(1)} \\ = (-0.761904) + (1.6667) + (6.6667) + (-1.904761) \\ \underline{f(6) = 5.666736}$$

4. x	1	2	3	4	7
$f(x)$	2	4	8	16	128

find $f(5)$

$$\begin{aligned}
 f(5) &= \frac{(2) \times (3) \times (2) \times (1) \times (-2)}{(-1) \times (-2) \times (-3) \times (-6)} + \frac{(4) \times (4) \times (2) \times (1) \times (-2)}{(1) \times (-1) \times (-2) \times (-5)} \\
 &+ \frac{(8) \times (4) \times (3) \times (1) \times (-2)}{(2) \times (1) \times (-1) \times (-4)} + \frac{(16) \times (4) \times (3) \times (2) \times (-2)}{(3) \times (2) \times (1) \times (-3)} \\
 &+ \frac{(128) \times (4) \times (3) \times (2) \times (1)}{(6) \times (5) \times (4) \times (3)} \\
 &\rightarrow (-0.6667) + (6.4) + \frac{(-48)}{(-24)} + (42.6667) + \frac{(-10.6667)}{(8.6333)} \\
 \boxed{f(5) = 32.9333}
 \end{aligned}$$

5. x	10	20	14	16
$f(x)$	0.25	0.2	0.15	0.10

 $f(15) = ?$

$$\begin{aligned}
 f(15) &= \frac{(0.25) \times (-5) \times (1) \times (-1)}{(-10) \times (-4) \times (-6)} + \frac{(0.2) \times (5) \times (1) \times (-1)}{(10) \times (6) \times (4)} \\
 &+ \frac{(0.15) \times (5) \times (-5) \times (-1)}{(4) \times (-6) \times (-2)} + \frac{(0.10) \times (5) \times (-5) \times (-1)}{(6) \times (-4) \times (2)} \\
 &= (-0.00520833) + (-0.00416667) + 0.018125 + 0.0620833 \\
 \boxed{f(15) = 0.1208333}
 \end{aligned}$$

HW 2

1. $x^3 + x - 1 = 0 \quad \therefore f(x) = x^3 + x - 1$

x	y	$\left. \begin{array}{l} x_1 = 0.5 - (-0.375 \times 1) \\ \quad \quad \quad + 0.125 \end{array} \right\} \begin{array}{l} = 0.625 \\ y_1 = -0.12376 \end{array}$
0.5	-0.375	
1	1	
x	y	$\left. \begin{array}{l} x_2 = 0.625(1) - (-0.12376(1)) \\ \quad \quad \quad + (-0.12376) \end{array} \right\} \begin{array}{l} x_2 = 0.673091 \\ y_2 = -0.02194 \end{array}$
0.625	-0.12376	
1	1	
x	y	$\left. \begin{array}{l} x_3 = 0.68011521 \\ \quad \quad \quad y_3 = -0.0052921 \end{array} \right\}$
0.673091	-0.02194	
1	1	
x	y	$\left. \begin{array}{l} x_4 = 0.6817994 \\ \quad \quad \quad y_4 = -0.0012658 \end{array} \right\}$
0.68011521	-0.00529	
1	1	
x	y	$\left. \begin{array}{l} x_5 = 0.68220141 \\ \quad \quad \quad y_5 = -0.0003028 \end{array} \right\}$
0.6817994	-0.001265	
1	1	

Root = 0.6817994

2. $x^3 - 2x + 0.5 = 0$

x	y	$\left. \begin{array}{l} x_1 = 0.5(1) - 0(-0.5) \\ \quad \quad \quad 0.5 + 0.5 \end{array} \right\} \begin{array}{l} 0.5 = x_1 \\ y_1 = -0.375 \end{array}$
0.1	-0.5	
0	-0.5	
x	y	$\left. \begin{array}{l} x_2 = 0.285714 \\ \quad \quad \quad y_2 = -0.048104 \end{array} \right\}$
0.5	-0.375	
0	0.5	
x	y	$\left. \begin{array}{l} x_3 = 0.260638 \\ \quad \quad \quad y_3 = -0.00357029 \end{array} \right\}$
0.285714	-0.048104	
0	0.5	
x	y	$\left. \begin{array}{l} x_4 = 0.2587219 \\ \quad \quad \quad y_4 = -0.000125126 \end{array} \right\}$
0.260638	-0.00357029	
0	0.5	

Root = 0.260638

$$3. x^3 + 2x - 20 = 0$$

$$\begin{array}{l|l} x & y \\ 2 & -8 \\ 2.5 & 0.625 \end{array} \quad \left. \begin{array}{l} x_1 = 2(0.625) - 2.5(-8) \\ 3.0.625 \end{array} \right\} \quad \begin{array}{l} x_1 = 2.463768 \\ y_1 = -0.117015 \end{array}$$

$$\begin{array}{l|l} x & y \\ 2.463768 & -0.117015 \\ 2.5 & 0.625 \end{array} \quad \left. \begin{array}{l} x_2 = 1.537865 + 0.2725375 \\ 0.742015 \end{array} \right\} \quad \begin{array}{l} x_2 = 2.467481 \\ y_2 = -0.001312105 \end{array}$$

$$\begin{array}{l|l} x & y \\ 2.467481 & -0.001312105 \\ 2.5 & 0.625 \end{array} \quad \left. \begin{array}{l} x_3 = 1.5434256 + 0.002230 \\ 0.6263121 \end{array} \right\} \quad \begin{array}{l} x_3 = 2.467544 \\ y_3 = -0.00003349 \end{array}$$

$$\rightarrow \therefore \text{Root} = 2.467544$$

$$4. 3x - \cos x - 1 = 0$$

→ Trigonometry

$$5. x^3 - 3x - 5 = 0$$

$$\begin{array}{l|l} x & y \\ 2 & -3 \\ 2.5 & 3.125 \end{array} \quad \left. \begin{array}{l} x_1 = 6.250 + 7.5 \\ 6.125 \end{array} \right\} \quad \begin{array}{l} x_1 = 2.24489 \\ y_1 = -0.421366 \end{array}$$

$$\begin{array}{l|l} x & y \\ 2.24489 & -0.42133 \\ 2.5 & 3.125 \end{array} \quad \left. \begin{array}{l} x_2 = 7.0152812 + 1.06337 \\ 3.54625 \end{array} \right\} \quad \begin{array}{l} x_2 = 2.2752001 \\ y_2 = -0.047944 \end{array}$$

$$\begin{array}{l|l} x & y \\ 2.27520 & -0.047944 \\ 2.5 & 3.125 \end{array} \quad \left. \begin{array}{l} x_3 = 7.11 + 0.11986 \\ 3.172944 \end{array} \right\} \quad \begin{array}{l} x_3 = 2.278596 \\ y_3 = -0.006365 \end{array}$$

$$\begin{array}{l|l} x & y \\ 2.278596 & -0.006365 \\ 2.5 & 3.125 \end{array} \quad \left. \begin{array}{l} x_4 = 7.1206125 + 0.01326 \\ 3.130365 \end{array} \right\} \quad \begin{array}{l} x_4 = 2.27897 \\ y_4 = -0.0006 \end{array}$$

$$\text{Root} = 2.27897$$

$$5. x \log_{10} x - 1.2 = 0$$

$$\begin{array}{rcl} x & y & \\ 2 & -0.597940 & \\ 2.5 & 0.281363 & \end{array} \left. \vphantom{\begin{array}{rcl} x & y & \\ 2 & -0.597940 & \\ 2.5 & 0.281363 & \end{array}} \right\} \begin{array}{l} x_1 = \frac{0.4627275 + 1.49485}{0.829303} = 2.36050938 \\ y_1 = -0.319516 \end{array}$$

$$\begin{array}{rcl} x & y & \\ 2.360509 & -0.319516 & \\ 2.5 & 0.281363 & \end{array} \left. \vphantom{\begin{array}{rcl} x & y & \\ 2.360509 & -0.319516 & \\ 2.5 & 0.281363 & \end{array}} \right\} \begin{array}{l} x_2 = \frac{0.5461344 + 0.79879}{0.550879} = 2.44141526 \\ y_2 = -0.2536057 \end{array}$$

$$\begin{array}{rcl} x & y & \\ 2.441415 & -0.25360 & \\ 2.5 & 0.281363 & \end{array} \left. \vphantom{\begin{array}{rcl} x & y & \\ 2.441415 & -0.25360 & \\ 2.5 & 0.281363 & \end{array}} \right\} \begin{array}{l} x_3 = \frac{0.6540143 + 0.5665049}{0.43523} \rightarrow 2.472643 \\ y_3 = -0.22821 \end{array}$$

$$7. x^3 - 4x + 1 = 0$$

$$\begin{array}{cc|cc} x & y & x & y \\ 0 & 1 & 1 & -2 \\ 1 & -2 & 0 & 1 \end{array} \left\{ \begin{array}{l} x_1 = 1 \\ y_1 = -0.833, y_1 = -0.2962139 \end{array} \right.$$

$$\begin{array}{cc|cc} x & y & x & y \\ 0.833 & -0.296213 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array} \left\{ \begin{array}{l} x_2 = 0.25411449 \\ y_2 = -0.0115162 \end{array} \right.$$

$$\begin{array}{cc|cc} x & y & x & y \\ 0.254114 & -0.011516 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array} \left\{ \begin{array}{l} x_3 = 0.2541977 \\ y_3 = -0.00036549 \end{array} \right.$$

$$\text{Root} = 0.2541977$$

$$8. x^3 + 2x^2 - 8 = 0$$

$$\begin{array}{cc|cc} x & y & x & y \\ 1.5 & -0.125 & 12 & 8 \\ 2 & 8 & 12.11462 & 8.02653 \end{array} \left\{ \begin{array}{l} x_1 = 12 + 0.250 \\ y_1 = 1.50769 \\ y_1 = -0.026519 \end{array} \right.$$

$$\begin{array}{cc|cc} x & y & x & y \\ 1.50769 & -0.02653 & 12.11462 & 8.02653 \\ 2 & 8 & 12.11462 & 8.02653 \end{array} \left\{ \begin{array}{l} x_2 = 12.11462 \\ y_2 = 1.509318 \\ y_2 = -0.00562725 \end{array} \right.$$

$$\begin{array}{cc|cc} x & y & x & y \\ 1.509318 & -0.00562725 & 12.0112545 & 8.00562725 \\ 2 & 8 & 12.0112545 & 8.00562725 \end{array} \left\{ \begin{array}{l} x_3 = 0.0112545 + 12.0112545 = 1.509629 \\ y_3 = -0.001901 \end{array} \right.$$

$$\begin{array}{cc|cc} x & y & x & y \\ 1.509629 & -0.001901 & 12.00233032 & 8.001901 \\ 2 & 8 & 12.00233032 & 8.001901 \end{array} \left\{ \begin{array}{l} x_4 = 0.00233032 + 12.00233032 = 1.509735 \\ y_4 = -0.000256 \end{array} \right.$$

$$\text{Root} = 1.509735$$

9. $x^3 - 9x + 1 = 0$

$$\begin{array}{cc|l} x & y & \\ 0 & -1 & \} x_1 = \frac{1}{8} = 0.125 \quad y_1 = -0.125046 \\ 8 & 1 & \end{array}$$

$$\begin{array}{cc|l} x & y & \\ 0.125 & -0.423046 & \} x_2 = 0.111804 \quad y_2 = -0.000851085 \\ 0 & 1 & \end{array}$$

Root = 0.111804

10. $x^3 - x - 4 = 0$

$$\begin{array}{cc|l} x & y & \\ 1 & -4 & \} x_1 = \frac{10}{6} = 1.6667 \quad y_1 = -1.0367925 \\ 2 & 2 & \end{array}$$

$$\begin{array}{cc|l} x & y & \\ 1.667 & -1.0367925 & \} x_2 = \frac{3.22234 + 2.073585}{3.0367925} = 1.7439806 \quad y_2 = -0.4402062 \\ 2 & 2 & \end{array}$$

$$\begin{array}{cc|l} x & y & \\ 1.7439206 & -0.4402062 & \} x_3 = \frac{0.8801125 + 3.487111}{2.4402062} = 1.790116 \quad y_3 = -0.0586554 \\ 2 & 2 & \end{array}$$

$$\begin{array}{cc|l} x & y & \\ 1.790116 & -0.0586554 & \} x_4 = \frac{0.1043116 + 3.98282}{2.0586554} = 1.79594 \quad y_4 = -0.006268 \\ 2 & 2 & \end{array}$$

$$\begin{array}{cc|l} x & y & \\ 1.79594 & -0.006268 & \} x_5 = \frac{0.2225368 + 3.891188}{2.006268} = 1.7962327 \quad y_5 = -0.000771 \\ 2 & 2 & \end{array}$$

Root = 1.7962327