Name : Dhanvini Shah

Subject : Pricing & Reserving for Life Insurance Products - 2

Assignment 1 : Unit 1 & Unit 2

Roll Number : 704

1] i) Prove that

$$(Ia)_{\overline{n}|} = \frac{(\ddot{a}_{\overline{n}|} - nv^n)}{i}$$

$$(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \dots + nv^n \rightarrow (1)$$

$$(1+i)(Ia)_{\overline{n}|} = 1 + 2v + 3v^2 + \dots + nv^{n-1} \rightarrow (2)$$

Subtracting (2) from (1)

$$(Ia)_{\overline{n}|} + i(Ia)_{\overline{n}|} - (Ia)_{\overline{n}|} = (1 + 2v + 3v^2 + \dots + nv^{n-1}) - (v + 2v^2 + 3v^3 + \dots + nv^n)$$

$$i(Ia)_{\overline{n}|} = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$$

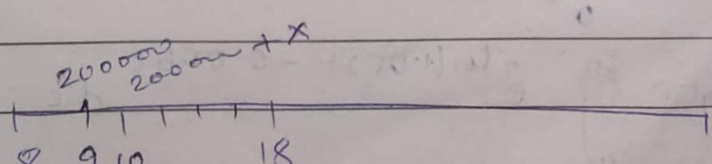
$$i(Ia)_{\overline{n}|} = \ddot{a}_{\overline{n}|} - nv^n$$

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

(ii) Inflation = 6% . Interest = 10% .

Cost of edu = 30,00,000

contri = 2,00,000



PV of contribution

$$= 200000\, \ddot{a}_{\overline{10|}}^{@10\%} + X\,(Ia)_{\overline{9|}}^{@10\%} = 30{,}00{,}000 \times (1.06)^{10} \times V^{10}_{@10\%}$$

$$200000 \times 6.759026 + X\,[25.18046] = 2071350$$

$$\boxed{X = 28{,}575}$$

2] 20 yr term $\quad i = 5\%$

$x = 40$

$\overline{SA} = 100000$

$$\begin{tikzcd} h \arrow[rr, "\sigma_x = 0.002"] \arrow[dr, "\mu_x = 0.004"'] & & i \arrow[dl, "\nu_x = 0.005"] \\ & d & \end{tikzcd}$$

$$EPV = 100000 \int_0^{20} v^t\, {}_tp_{40}^{h}\, (\sigma_{40+t} + \mu_{40+t})\, dt$$

$$= 100000 \int_0^{20} e^{-\ln(1.05)t}\, e^{-0.006t} \cdot 0.006\, dt$$

$$= 600 \int_0^{20} e^{-\ln(1.05)t - 0.006t}\, dt$$

$$\frac{600}{0.5497} \left[-e^{-0.5497t}\right]_0^{20}$$

$\boxed{EPV = 1290.3}$

3] 40 ———————————————— 60

Monthly P

SA = 50,00,000 $\quad$ i = 4%

ADB $\quad$ CI

50,00,000 $\quad$ 1500,000

select $\quad$ ultimate

i) EPV of Prem = $P\ddot{a}^{(12)}_{40:\overline{20}|}$

$$= P\left[\ddot{a}_{40:\overline{20}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]$$

$$= P\left[13.927 - \frac{11}{24}\left(1 - \frac{882.85}{2052.96}\right)\right]$$

$$= 13.6658\ P$$

EPV of claims = $75,00,000 \cdot A^{1}_{40:\overline{20}|}$ + $50,00,000\ A^{1}_{[40]:\overline{20}|}$

$$= 7500000\left[A_{40:\overline{20}|} - \frac{D_{60}}{D_{40}}\right] + 5000000$$

$$\left[A_{[40]:\overline{20}|} - \frac{D_{60}}{D_{[40]}}\right]$$

$= 427714.9658$

EPV of expenses $= 0.03 \times 42714.9658 +$
$0.35P\,\ddot{a}^{(12)}_{40:\overline{1}|} + 0.05P\left[\ddot{a}^{(12)}_{40:\overline{20}|} - 1\right]$
$+ 0.\cancel{0}P\,\ddot{a}^{(12)}_{40:\overline{20}|} \cdot 1.04^{0.5} + 500$

Solving for P
$P = 39009.7$.
Monthly Prem $= \dfrac{39009.7}{12}$

$= 3250.8$

ii) Policies with only accident.

${}_5V = 7500000\,A^{1}_{45:\overline{8}|} - 39009.7\,\ddot{a}^{(12)}_{45:\overline{8}|}$
${}_5V = 269387.1975 - 434750.63$
${}_5V_1 = 165363.4377$

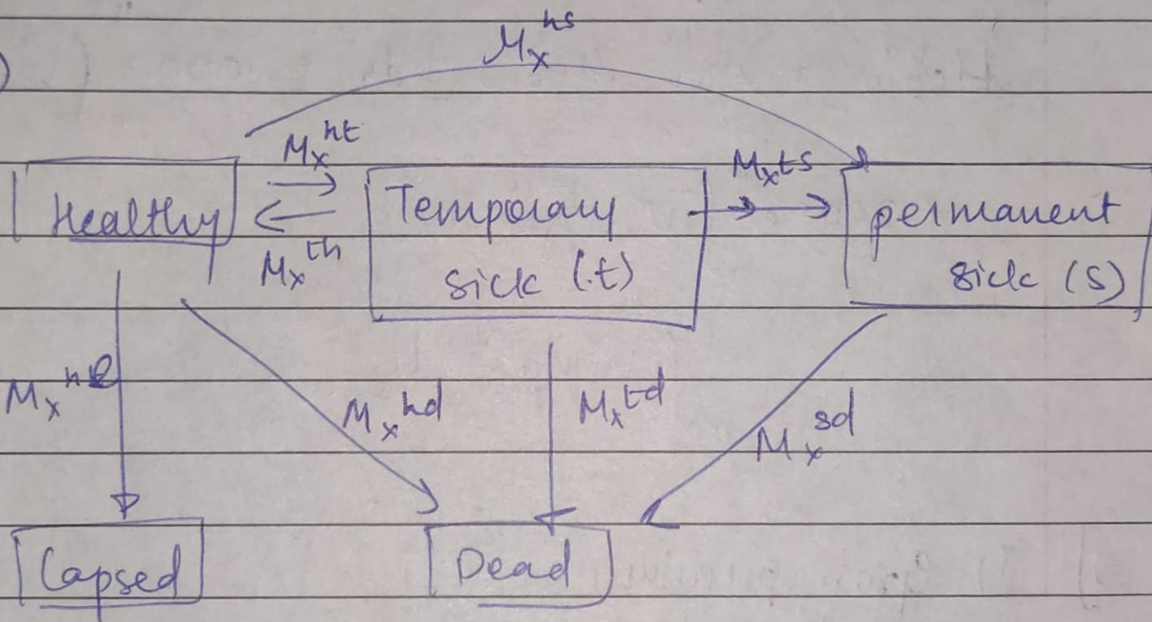
Policies with only illness claims
$= 500{,}000\,A^{1}_{45:\overline{15}|} - 39009.7\,\ddot{a}^{(12)}_{45:\overline{15}|}$
$= 50{,}00{,}000 \times 0.0359918.293 - 434750.6532$
${}_5V_2 = 255759.1702$

Policies with no claims

$$(7500000 + 5000000)\, A^{1}_{45:\overline{15}|} - 390097\, \ddot{a}^{(12)}_{45:\overline{15}|}$$

$${}_5V_3 = 14228.0273$$

4] i)



$n = 15 \quad x = 50$

ii) EPV (premium) = EPV (benefit)

$$\text{EPV of benefit} = 10000 \int_0^{15} v^u \, {}_up_{50}^{ht} \, du$$

$$+ 20000 \int_0^{15} v^u \, {}_up_{50}^{hs} \, du \, +$$

$$100000 \int_0^{15} v^u \left({}_up_{50}^{hh} \, \mu_{50+u}^{hd} + {}_up_{50}^{ht} \, \mu_{50+u}^{td} \right.$$

$$\left. + {}_up_{50}^{hs} \, \mu_{50+u}^{sd} \, du \right)$$

EPV of Premium $= P \times \int_0^{15} v^u \, {}_u p_{50}^{hh} \, du$

$$P = \frac{100000 \int_0^{15} v^u \left[{}_u p_{50}^{hh} \mu_{50+u}^{hd} + {}_u p_{50}^{ht} \mu_{50+u}^{td} + {}_u p_{50}^{hs} \mu_{50+u}^{sd} \right] du + 10000 \int_0^{15} v^u \, {}_u p_{50}^{ht} \, du + 20000 \int_0^{15} v^u \, {}_u p_{50}^{hs} \, du]}{\int_0^{15} v^u \, {}_u p_{50}^{hh} \, du}$$

5] i) Gross premium

EPV of Premium $= P \ddot{a}_{[40]:25}$

$= 15.887 P$

EPV of expenses $= 1200 + 500 \bar{A}_{[40]} + 2\%$ of $P \cdot (\ddot{a}_{[40]:25} - 1)$

$= 1200 + 500 (1.04)^{0.5} \times 0.23041 + 0.02 \times P \times (15.887 - 1)$

$= 1317.48 + 0.29774P$

EPV of benefits $= 200000 \left(v^{0.5} q_{[40]} + v^{1.5} (1.04) \; {}_{1|}q_{[40]} + v^{2.5} (1.04)^2 \; {}_{2|}q_{[40]} + \ldots \right)$

$$= \frac{200000}{1.04^{0.5}}\left(q_{[40]} + (1.04)\, {}_{1|}q_{[40]} + \cancel{q}\,(1.04)^2\right.$$

$$\left. {}_{2|}q_{[40]} + \ldots\right)$$

$$= 196116.14$$

$$15.887P = 196116.14 + 1317.487 + 0.29224P$$

$$P = 12664$$

(ii) EPV of benefits $= 200000 \times x\% \cdot (I\bar{A})_{[40]} +$
$200000\,(1 - x\%)\,\bar{A}_{[40]}$
$= (1.04)^{0.5}\left[200000 \times x\% \cdot (IA)_{[40]} +\right.$
$\left.200000\,(1 - x\%)\,A_{[40]}\right)$
~~EPV of benefits =~~
$= (1.04^{0.5})\left[200000 \times x\% \cdot (IA)_{[40]} + 200000\right.$
$\left.(1 - x\%)\,A_{[40]}\right] = 196116.14$
$x\% = 9\%$ is simple reversionary rate.

(iii) $P\,\ddot{a}_{40:\overline{24}} = 200000\,\bar{A}_{40}$
$P = 2960.54$

$${}_{10}V = 300000\,\bar{A}_{50} - P\,\ddot{a}_{50:\overline{15}}$$
$$= 300000 \times (1.04)^{0.5} \times 0.32907 - 2960.54 \times 11.253$$
$$= 67361.10$$

6) Death = μ_x

Cancer = σ_x

Value of benefits on death & cancer

$$V = 100000 \int_0^{20} v^t \, {}_tp_{45}^{hh} \left(\mu_{45} + \sigma_{45+t}\right) dt.$$

$$= 100000 \int_0^{20} e^{-\ln(1.05)t} \, {}_tp_{45+t}^{hh} \, 0.008 \, dt.$$

$$\cancel{e^{-0.008t}}$$

$$= 100000 \int_0^{20} e^{-\ln(1.05)t} \times e^{-0.008t} \times 0.008 \, dt$$

$$= 800 \int_0^{20} e^{-0.05679t} \, dt.$$

$$= 800 \left[\frac{-e^{-0.05679t}}{0.05679} \right]_0^{20}$$

$$= 9562.8$$

Value of Return of Premium

$$= P e^{-\int_{45}^{65} (0.006 + 0.002)dx} \, v^{20}$$

$$P e^{-0.008(65-45)} \, v^{20}$$

$$= P \cdot e^{-0.16} \times 0.37689$$

Premium = Value of Benefits on Death & Cancer + Value of Return on Premium + Commission + Exps + Profit Margin.

$P = 9562.8 + 0.321164 \times P + 0.02P + 0.02P + 0.1P$

$P = 17747.15$

7) PV of Net CF = PV of premium - PV of benefits - PV of Expenses - PV of Commission.

PV of Premium $= 30000\ \ddot{a}_{[45]:20}$

$= 30{,}000\ (11.888)$

$= 356640$

PV of Benefits $= 1000000\ A_{[45]:20]}$

$= 1000000 \times 0.32711$

$= 327110$

PV of expenses $= 5000 + 2.5\%\ (30000)$
$(\ddot{a}_{[45]:20} - 1) + 500\ p_{[45]}\ v\ (1 + 1.019231\ v\ {}_1p_{[46]} +$
$1.019231^2\ v^2\ {}_2p_{[46]} + \cdots 1.019231^{18}\ v^{18}\ {}_{18}p_{[46]}]$

$= 19438$

PV of Renewal commission $= 2\%$ of 30000
$(\ddot{a}_{[45]:20} - 1)$

$= 65333$

PV of Net CF $= 356640 - 327110 - 19438 -$
6533.1

$= 3559.1$

Margin = PV of net CF / Annual Prem

$= \frac{3559-1}{30000}$

$= 10\%$

$1 = 559$

8) i) Assumption of independence of decrements is used to derive single decrement table from multiple decrement table.

$(a\mu)^j_x = \mu^j_x$ for all j & all x.

ii) In case of term assurance, it is likely that people who lapse their policies have lower than average mortality.

iii) a) $1000 \int_0^5 e^{-\delta t} \left({}_tp^{P1}_{45}\, \mu_{45+t} + {}_tp^{PP}_{45}\, \nu_{45+t} \right) dt$

b) $250 \int_0^5 e^{-\delta t}\, {}_tp^{IP}_{45}\, \nu_{45+t}\, dt$.

c) $150 \int_0^5 e^{-\delta t}\, {}_tp^{II}_{45}\, dt$.

d) $250 \int_0^5 e^{-\delta t}\, {}_tp^{PP}_{45}\, \theta_{45+t}\, dt$

(iv) lets assume decrements are independent. Hence given the independent forces of decrement can be assumed to apply when both decrements occur together in same pop.

$$(aq)^{c}_{45} = \frac{0.05}{0.18}\left(1-e^{-0.18}\right) = 0.045758$$

$$(aq)^{s}_{45} = \frac{0.13}{0.18}\left(1-e^{-0.18}\right) = 0.118972$$

$$(ap)_{45} = 1-0.045758-0.118972$$
$$= 0.835270$$

$$(aq)^{c}_{46} = \frac{0.06}{0.16}\left(1-e^{-0.16}\right)$$
$$= 0.055446$$

$$(aq)^{s}_{46} = \frac{0.1}{0.16}\left(1-e^{-0.16}\right) = 0.092410$$

$$(ap)_{46} = 1-0.055446-0.092410$$
$$= 0.852144$$

probab being in force $= 0.835270 \times 0.852144$
$= 0.711770$.

9) i) In with profit policies, surplus on the underlying fund is shared between policyholders & shareholders in a pre-decided manner. Hence the surplus fund arising in contract is shared in form of bonuses and hence the need of bonuses.

ii) Investment Returns
Expenses incurred & surplus
Mortality surplus
Policies that lapsed or surrended
Fluctuation in interest rates surplus
Reinsurance surplus

(iii) Terminal bonus are typically a percentage of basic sum assured where percentage varies according to the term of policy at the date of payment. This can also be a strategy to build lower guarantees. Companies would have greater freedom in choosing investment assets. They can invest in high risk assets that would provide higher returns that could be distributed to PH in terms of higher Terminal bonus.

Endowment

SA = 1000000

10)

50 — 51 — 60 — 65

premium

com 20% of $12P$ $\quad$ 2% of $12P$

$2500 + 35\% \times 12P$

$e = 400 + 1.5\%$ of RP

FRB = 1.92308% of SA → compounded @ 6%

(i) ~~A~~ Let Monthly Premium be P

EPV of Premium = $12P\, \ddot{a}^{(12)}_{50:\overline{10}|}$ @ 6%

Death

EPV of Benefits = $1000000\left(q_{50} v + 1.019231 v^2\, {}_{1|}q_{50}\right.$ $\left. + 1.019231^2 v^3\, {}_{2|}q_{50}\right) \ldots$

$$= \frac{1000000}{1.019231}\left(q_{50}\, v(1.019231) + {}_{1|}q_{50}\, v^2 \times (1.019231)^2 + \ldots + {}_{14|}q_{50}\, v^{15} \cdot 1.019231^{15}\right)$$

v @ 6%

EPV of ~~Moto~~ $= \dfrac{1000000}{1.019231}\; A_{50:\overline{15}|}$ @ 4%

EP

as $\dfrac{1.019231}{1.06} = \dfrac{1}{1.04}$

EPV of maturity benefit = $1000000\, v^{15}\, {}_{15}p_{50}$ @6%
$\times\ 1.079231^{15}$

$\cancel{\text{EPV of expense \& Com} = (0.2+0.35) \times 12P + (0.02+0.015)\ {}_{1}p_{50}\ (1.06)^{-1} \times 12P \times a}$

EPV of Expense = $(0.2+0.35)\, 12P +$
$(0.02+0.015)\ v\ {}_{1}p_{50} \times 12P\ \ddot{a}^{(12)}_{51:\overline{9|}} +$
$2500 + 400\ v\ {}_{1}p_{50} \times \ddot{a}_{51:\overline{14|}}$ @6%

$= 9.333P + 6115.061$

EPV of premium = $12 \times 7.4804 \times P$
$= 89.7648\,P$

EPV of Death B = $\dfrac{1000000}{1.079231}\ (0.56779)$

$= 556488.1759$

EPV of Maturity = 9515.924100

Equating,

$89.7648P = 566004.1 + 9.333P + 6115.06$

$\boxed{P = 7113.12}$

ii) bonus @ constant rate of 2%.

SA + Bonus at end of 10^{th} y $= 1000000 (1.02)^{10}$
$= 1218994$

PV of Benefits at end of 10^{th} year

$= \cancel{\frac{1218994}{1.05}}\, q_{60} +$

$= 1218994 \big(v\, q_{60} + v^2\, {}_{1|}q_{60} (1.04) + v^3\, {}_{2|}q_{60} (1.04)^2 + \dots + v^5\, 1.04^4\, {}_{4|}q_{60} + 1.04^5\, {}_5p_{60} \big)$

$= 0.952948 \times 1218994$
$= 1160774$

PV of exps $= 400 \times \ddot{a}_{60:\overline{5|}}$ @ 5%
$= \cancel{1.78}\ 1787.39$

Reserves $= 1160774 + 1787.39$
$= 1162561$

11)

30 —— 37 —— 50

$i = 6\%$

$SA = 400000$

Simple B = 40 per 1000 SA

Net premium prospective reserve at 7

EPV Net Premium $= P\ddot{a}_{30:\overline{20|}}$

EPV of benefit $= 400000\, A_{30:\overline{20|}}$

$\therefore \quad P\ddot{a}_{30:\overline{20|}} = 400000\, A_{30:\overline{20|}}$

$$P = \frac{400000\,(0.315817)}{12.08705}$$

$$\boxed{P = 10451.42}$$

Total bonus at time 7 $= 7 \times 0.04 \times 400000$

$= 112000$

SA at time 7 $= 400000 + 112000 = 512000$

Reserve at time 7

$= 512000\, A_{37:\overline{13|}} - 10451.42\, \ddot{a}_{37:\overline{13|}}$

$= 512000\,(0.471638) - 10451.42\,(9.333835)$

$= 143926.89$