ASSIGNMENT 2

1) $$I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

Let $e^{2/w} = t$

Differentiating wrt w on both sides,

$$e^{2/w} \times \frac{-2}{w^2} = \frac{dt}{dw}$$

$$\Rightarrow \frac{e^{2/w}}{w^2}\, dw = \frac{-1}{2}\, dt$$

$$\therefore I = \int_{e^{100}}^{e} \frac{-1}{2}\, dt$$

$$= \left[\frac{-1}{2} t\right]_{e^{100}}^{e}$$

$$= \frac{-e}{2} + \frac{e^{100}}{2}$$

$$= \frac{e^{100} - e}{2}$$

2) $$I = \int \frac{2t^3 + 1}{(t^4 + 2t)^3}\, dt$$

$$= \frac{1}{2} \int \frac{4t^3 + 2}{(t^4 + 2t)^3}\, dt$$

Let $(t^4 + 2t) = u$

Differentiating wrt t on both sides,

$$4t^3 + 2 = \frac{du}{dt}$$

$\Rightarrow (4t^3 + 2)\,dt = du$

$\Rightarrow I = \frac{1}{2} \int \frac{1}{u^3}\,du$

$= \frac{1}{2} \times \frac{u^{-2}}{-2}$

$= \frac{-1}{4u^2} = \frac{-1}{4(t^4 + 2t)^2}$

3) $f(x) = \int f'(x) = \int (x^4 + 3x - 9)\,dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x$

4) $\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= \left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= (1 + 8) + (18 - 6)$

$= 9 + 12$

$= 21$

5) $I = \int 3(8y-1)\, e^{4y^2-y}\, dy$

Let $e^{4y^2-y} = x$

Differentiating both sides wrt y,

$$e^{4y^2-y} \times (8y-1) = \frac{dx}{dy}$$

$$\Rightarrow (8y-1)\, e^{4y^2-y}\, dy = dx$$

$$\Rightarrow I = \int 3\, dx$$

$$= 3x$$

$$\Rightarrow I = 3e^{4y^2-y}$$

6) $f(x) = \iint f''(x)$

$$= \iint \left(15\sqrt{x} + 5x^3 + 6\right) dx\, dx$$

$$= \int \left(\frac{15 \times 2x^{3/2}}{3} + \frac{5x^4}{4} + 6x\right) dx$$

$$= \int \left(10x^{3/2} + \frac{5}{4}x^4 + 6x\right) dx$$

$$= \frac{10 \times 2x^{5/2}}{5} + \frac{5 \times x^5}{4 \quad 5} + \frac{6x^2}{2} + C$$

$$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c$$

$f(1) = -5/4$

$$\Rightarrow -5/4 = 4 + \frac{1}{4} + 3 + c$$

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$\Rightarrow -5/4 = \frac{29}{4} + c$

$\Rightarrow c = \frac{-34}{4}$

$\Rightarrow \cancel{c = \frac{-17}{2}}$

$f(4) = 404$

$\Rightarrow 404 = 4 \times 4^{5/2} + \frac{4^5}{4} + 3 \times 4^2 + c$

$\Rightarrow 404 = 4^{7/2} + 4^4 + 48 + c$

$\cancel{\Rightarrow 404 = 128 + 52 + c}$

$\cancel{\Rightarrow c}$

$\Rightarrow 404 = 128 + 256 + 48 + c$

$\Rightarrow c =$

8) $\cancel{d\theta}\ \frac{dr}{d\theta} = \frac{r^2}{\theta}$

$$\Rightarrow \frac{1}{r^2}\,dr = \frac{1}{\theta}\,d\theta$$

Integrating on both sides,

$$\int \frac{1}{r^2}\,dr = \int \frac{1}{\theta}\,d\theta$$

$$\Rightarrow \frac{-1}{r} = \log\theta + c$$

$$\Rightarrow -1 = r\log\theta + rc$$

$$\Rightarrow r\log\theta + rc + 1 = 0$$

$r(1) = 2$

$$\Rightarrow \log\theta + c + 1 = 2$$

$$\Rightarrow \log\theta + c = 1$$

$$\Rightarrow c = 1 - \log\theta$$

The solution is :- $r\log\theta$

$$r\log\theta + r(1 - \log\theta) + 1 = 0$$

$$\Rightarrow r\log\theta + r - r\log\theta + 1 = 0$$

$$\Rightarrow r + 1 = 0$$

$$\Rightarrow r = -1$$

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9) $\frac{dv}{dt} = 9.8 - 0.196\,v$

$\Rightarrow \frac{dv}{dt} + 0.196\,v = 9.8$

$I.F = e^{\int 0.196\,dt}$

$\Rightarrow IF = e^{0.196t}$

$v \times e^{0.196t} = \int 9.8 \times e^{0.196t}\,dt$

$\Rightarrow v \times e^{0.196t} = \frac{9.8\,e^{0.196t}}{0.196} + c$

~~$\Rightarrow v = 50 + c$~~

$\Rightarrow v\,e^{0.196t} = 50\,e^{0.196t} + c$

10) $ty' + 2y = t^2 - t + 1$

~~$\rightarrow ty' =$~~

11) $y' = \frac{xy^3}{\sqrt{1+x^2}}$

$\Rightarrow \frac{y'}{y^3} = \frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \frac{dy}{dx} \times \frac{1}{y^3} = \frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}} dx$

Integrating on both sides,

$\int \frac{1}{y^3} dy = \int \frac{x}{\sqrt{1+x^2}} dx$

$\Rightarrow \frac{-1}{2y^2} = \sqrt{x^2+1} + c$

$\Rightarrow \frac{-1}{2y^2} - \sqrt{x^2+1} - c = 0$

Given $y(0) = -1$

$\Rightarrow \frac{-1}{2\times(-1)^2} - \sqrt{0+1} - c = 0$

$\Rightarrow \frac{-1}{2} - 1 - c = 0$

$\Rightarrow \frac{-3}{2} - c = 0$

$\Rightarrow c = -3/2$

$\therefore$ the solution is

$$\frac{-1}{2y^2} - \sqrt{1+x^2} + \frac{3}{2} = 0$$

12) $f(x) = x^3 - 10x^2 + 6$

$f'(x) = 3x^2 - 20x$

$f''(x) = 6x - 20$

$f'''(x) = 6$

$f^{IV}(x) = 0$

$f(3) = 27 - 90 + 6 = -57$

$f'(3) = 3\times 9 - 20\times 3 = -33$

$f''(3) = 18 - 20 = -2$

$f'''(3) = 6$

$f^{IV}(3) = 0$

$$f(x) = f(a) + \frac{(x-a)^1}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a)$$
$$+ \frac{(x-a)^4}{4!} f^{IV}(a)$$

$$\Rightarrow f(x) = -57 + (x-3)\times -33 + \frac{(x-3)^2}{2}\times -2 + \frac{(x-3)^3}{6}\times 6$$
$$+ 0$$

$$\Rightarrow f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

13) $f(x) = (1+x)^{\mu}$

$f'(x) = \mu(1+x)^{\mu-1}$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$

$f(0) = 1$

$f'(0) = \mu \times 1 = \mu$

$f''(0) = \mu(\mu-1)$

$f'''(0) = \mu(\mu-1)(\mu-2)$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cancel{x} \dots$$

$$= 1 + (x \times \mu) + \frac{x^2}{2} \times \mu(\mu-1) + \frac{x^3}{6} \times \mu(\mu-1)(\mu-2) + \dots$$

$$= 1 + \mu x + \frac{\mu(\mu-1)}{2} x^2 + \frac{\mu(\mu-1)(\mu-2)}{6} x^3 + \dots$$

14) $f(x) = \sqrt{1+x} = (1+x)^{1/2}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$

$$f''(x) = \frac{1}{2} \times \frac{-1}{2}(1+x)^{-1} = \frac{-1}{4}(1+x)^{-1}$$

$$f'''(x) = \frac{1}{2} \times \frac{-1}{2} \times -1 \, (1+x)^{-2} = \frac{1}{4}(1+x)^{-2}$$

$f(0) = 1$

$f'(0) = \frac{1}{2}$

$f''(0) = \frac{-1}{4}$

$f'''(0) = \frac{1}{4}$

$$f(x) = 1 + x \times \frac{1}{2} + \frac{x^2}{2} \times \frac{-1}{4} + \frac{x^3}{6} \times \frac{1}{4} + \dots$$

$$\Rightarrow f(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} + \dots$$

15) $f(x) = e^{-6x}$

$f'(x) = -6e^{-6x}$

$f''(x) = 36e^{-6x}$

$f'''(x) = -216e^{-6x}$

$f(-4) = e^{24}$

$f'(-4) = -6 \times e^{24}$

$f''(-4) = 36e^{24}$

$f'''(-4) = -216e^{24}$

$$f(x) = e^{24} + (x+4) \times -6e^{24} + \frac{(x+4)^2}{2} \times 36e^{24} + \frac{(x+4)^3}{6} \times -216e^{24} + \dots$$

$$\Rightarrow f(x) = e^{24} - 6(x+4)e^{24} + 18(x+4)^2e^{24} - 36(x+4)^3e^{24} + \dots$$

16) $f(x) = \ln(3+4x)$

$f'(x) = \frac{1}{3+4x} \times 4 = 4(3+4x)^{-1}$

$f''(x) = 4 \times -1 \ (3+4x)^{-2} = -4(3+4x)^{-2}$

$f'''(x) = 4 \times -1 \times -2 \ (3+4x)^{-3} = 8(3+4x)^{-3}$

$f(0) = \ln(3)$

$f'(0) = 4 \times 3^{-1} = 4/3$

$f''(0) = -4 \times 3^{-2} = \frac{-4}{9}$

$f'''(0) = 8 \times 3^{-3} = 8/27$

$$f(x) = \ln 3 + x \times \frac{4}{3} + \frac{x^2}{2} \times \frac{-4}{9} + \frac{x^3}{6} \times \frac{8}{27}$$

$$\Rightarrow f(x) = \ln 3 + \frac{4x}{3} - \frac{2x^2}{9} + \frac{4}{81}x^3$$