Date

Calculus: Unit 1 and 2 (Ch: 1, 2, 3, 4) Assignment.

Q1. $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$

$= \lim_{h\to 0} \frac{2(h^2+9-6h)-18}{h}$

$= \lim_{h\to 0} \frac{2h^2+\cancel{18}-12h-\cancel{18}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2h-12)}{\cancel{h}}$

$= -12$

Q2. $g(x) = \begin{cases} 2x & , x < 6 \\ x-1 & , x \geq 6 \end{cases}$

(a) $x = 4$

$\lim_{x\to 4^+} f(x) = 8$

$\lim_{x\to 4^-} f(x) = 8$

$f(4) = 8$

Funct^n is continuous at $x = 4$.

(b) $x = 6$

$\lim_{x\to 6^+} f(x) = 6-1 = 5$

$\lim_{x\to 6^-} f(x) = 2(6) = 12$

$f(6) = 5$

Funct^n is ~~not continuous~~ discontinuous at $x = 6$.

Date

Q3. $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x\to 9} \frac{\cancel{(x-9)}^{\{-1\}}(3+\sqrt{x})}{\cancel{(9-x)}}$

$= \lim_{x\to 9} -(3+\sqrt{x})$

$= -3-\sqrt{9}$

$= -3+3$ or $-3-3$

$= 0$ or -6.

Q4. $f(x) = \frac{4x+5}{9-3x}$

(a) $x=-1$

$\lim_{x\to(-1)} f(x) = \frac{(-4)+5}{9+3}$

$= \frac{1}{12}$

$f(-1) = \frac{1}{12}$

(b) $x=0$

$\lim_{x\to 0} f(x) = \frac{5}{9}$

$f(0) = \frac{5}{9}$

(C) $x=3$

$f(3) = \frac{17}{0}$, not a possible value, indeterminant value.

$\lim_{x\to 3} f(x) = \cancel{7x-4+4x}$ $\frac{9-3x+7x-4}{9-3x} = \cancel{1+\frac{7x-4}{9-3x}}$

Date

~~Hence, funct^n will be discontinuous at this pos~~

$\lim_{x\to 5^+} f(x) = -\infty.$

$\lim_{x\to 5^-} f(x) = +\infty$

Hence, funct^n is discontinuous at $x=3$. Infinite discontinuity

Q5. $\lim_{x\to\infty} \dfrac{6e^{7x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

Note: $e^{\infty} = \infty$, $e^{-\infty} = 0$

$\lim_{x\to\infty} = \lim_{x\to\infty} \dfrac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$

$= \lim_{x\to\infty} \dfrac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$

$= \lim_{x\to\infty} \dfrac{6-0}{8-0+0}$

$= \dfrac{6}{8}$

$= \dfrac{3}{4}$

~~Q6. (a) g~~

Q6. $g(y) = \begin{cases} y^2+5 & , \text{if } y < -2 \\ 1-3y & , \text{if } y \geq -2 \end{cases}$

(a) $\lim_{x\to 6^+} g(y) = -17$

$\lim_{x\to 6^-} g(y) = -17$

Date

(b) $\lim_{y \to -2^-} g(y) = 9$

$\lim_{y \to -2^+} g(y) = 7$

Q7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (8t-1)(t^3 - 8t^2 + 12) + (3t^2 - 16t)(4t^2 - t)$
$= 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 -$
$3t^3 - 64t^3 + 16t^2$
$= 20t^4 - 132t^3 + 24t^2 + 96t - 12.$

Q8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

~~$= 3 + 3w^3$~~

$= \dfrac{(3 + 4w^3)(2w^2+1) - (4w)(3w + w^4)}{(2w^2+1)^2}$

$= \dfrac{6w^2 + 3 + 8w^5 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$

$= \dfrac{4w^5 + 4w^3 - 6w^2 + 3}{4w^4 + 4w^2 + 1}$

Q9. $f(x) = 2e^x - 8^x$
$f'(x) = 2e^x - 8^x \log_e 8.$

Q10. $y = z^5 - e^z \ln(z)$
$\dfrac{dy}{dz} = 5z^4 - \left[e^z \ln(z) + \dfrac{e^z}{z} \right]$

~~$= 5z^4 - e^z \ln(z) + \dfrac{e^z}{z}$~~

Date

$$= 5z^4 - e^z \ln(z) - \frac{e^z}{z}$$

Q11. (a) $f(x) = (6x^2+7x)^4$

$$f'(x) = 4(12x+7)(6x^2+7x)^3$$

(b) $f(t) = 5+e^{4t+t^7}$

$$f'(t) = e^{4t+t^7}(4+7t^6)$$

Q12. (a) $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \cancel{\frac{-3x^3}{7-x^3}} \quad \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx^2} = \frac{-9x(7-x^3)+3x^2(-3x^2)}{(7-x^3)^2}$$

$$\cancel{= -63x^2+9x^5-9x^5}$$

$$= -63x+9x^4-9$$

Q12. (a) $z = \ln(7-x^3)$

$$z' = \frac{-3x^2}{7-x^3}$$

$$z'' = \frac{-6x(7-x^3)+3x^2(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x+6x^4-9x^4}{(7-x^3)^2}$$

$$= \frac{-42x-3x^4}{(7-x^3)^2}$$

Q12 (b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$Q'(v) = 2\frac{d}{dv}(6+2v-v^2)^{-4}$$

$$= \frac{-8}{(6+2v-v^2)^5}(2-2v)$$

$$= \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$Q''(v) = -16\frac{d}{dv}\left(\frac{(1-v)}{(6+2v-v^2)^5}\right)$$

$$= -16\left[\frac{-(6+2v-v^2)^5 - 5(6+2v-v^2)^4(2-2v)(1-v)}{(6+2v-v^2)^{5\times 2}}\right]$$

$$= -16\left[\frac{\cancel{(6+2v-v^2)^4}\left[-6-2v+v^2-5(2-2v)(1-v)\right]}{(6+2v-v^2)^{\cancel{10}\,6}}\right]$$

$$= -16\left[\frac{-6-2v+v^2-5\left[2-2v-2v+2v^2\right]}{(6+2v-v^2)^6}\right]$$

$$= \frac{-16}{(6+2v-v^2)^6}\left[-6-2v+v^2+20v-10v^2-10\right]$$

$$= \frac{-16\left[-9v^2+18v-16\right]}{(6+2v-v^2)^6}$$

$$= \frac{16(9v^2-18v+16)}{(6+2v-v^2)^6}$$

Date

(b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$ ...

$Q'(v) = 2\dfrac{d}{dv}\left[(6+2v-v^2)^{-4}\right]$

$= \dfrac{2(-4)}{(6+2v-v^2)^5}(-2v+2)$

$= \dfrac{-16(1-v)}{(6+2v-v^2)^5}$

$Q''(v) = -16\dfrac{d}{dv}(Q'v)$

$= -16\left[\dfrac{-(6+2v-v^2)^5 - 5(6+2v-v^2)^4(-2v+2)\cancel{(-2v+2)(1-v)}(1-v)}{(6+2v-v^2)^{10}}\right]$

(b) at last. $= -16\left[\dfrac{-6-2v+v^2 - 5\cancel{(6+2v}(-2v+2)(1-v)}{(6+2v-v^2)^6}\right]$

$= \dfrac{-16}{(6+2v-v^2)^6}\left[-6-2v+v^2-5(-2v+2v^2+2-2v\right.$

$= \dfrac{-16}{(6+2v-v^2)^6}\left[-6-2v+v^2+20v-10v^2-10\right]$

$9v^2$

$= \dfrac{-16}{(6+2v-v^2)^6}\left[18v-\cancel{11v^2}-16\right]$

$= \dfrac{16(Nv^2 \cancel{+16} - 18v + N6)}{(6+2v-v^2)6}$

(c) $f(t) = \ln(1+t^2)$

$f'(t) = \dfrac{2t}{1+t^2}$

$f''(t) = \dfrac{2(1+t^2) - \cancel{2}4t^2}{(1+t^2)^2}$

Date

$$= \frac{-2t^2+2}{(1+t^2)^2}$$

Q13. $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x^2 + x - 3x - 3)$

$= 3[x(x+1) - 3(x+1)]$

$= 3(x+1)(x-3)$

$f'(x) = 0$

$x = -1$ or 3

$f''(x) = 6x - 6$

$f''(-1) = -12$.

$f(x)$ is maximum at $x = -1$. $f(-1) = 17$

$f''(3) = 12$.

$f(x)$ is minimum at $x = 3$. $f(3) = -15$.

Q14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x^2 - x - 2x + 2)$

$= 12[x(x-1) - 2(x-1)]$

$= 12(x-1)(x-2)$

$f'(x) = 0$

$x = 1$ or 2

$\cancel{f''(1) =}$ $f''(x) = 24x - 36$

$f''(1) = -12$,

$f(x)$ is maximum at $x = 1$, $f(1) = 3$

Date : ______________

$f''(2) = 12.$

$f(x)$ is minimum at $x = 2$. $f(2) = 1$.

Date

Q15. $f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$\Rightarrow f'(x) = 3x^2 + 6x$

$= 3x(x+2)$

$f'(x) = 0,$

$x = 0$ or -2

$f''(x) = 6x+6$

$f''(0) = 6$, +ve value.

Thus, 0 is the point of minima.

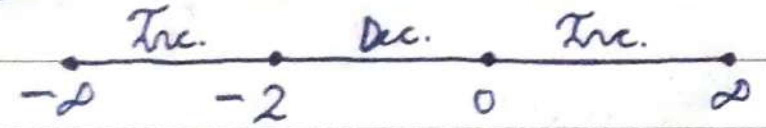
$f(0) = 0. \Rightarrow (0,0)$

$f''(-2) = -6$, −ve value

Thus, −2 is the point of maxima.

$f(-2) = 4 \Rightarrow (-2,4)$

$\Rightarrow$



$f'(-3) = 9$, +ve

$f(x)$ is increasing in $(-\infty, -2)$ and $(0, \infty)$

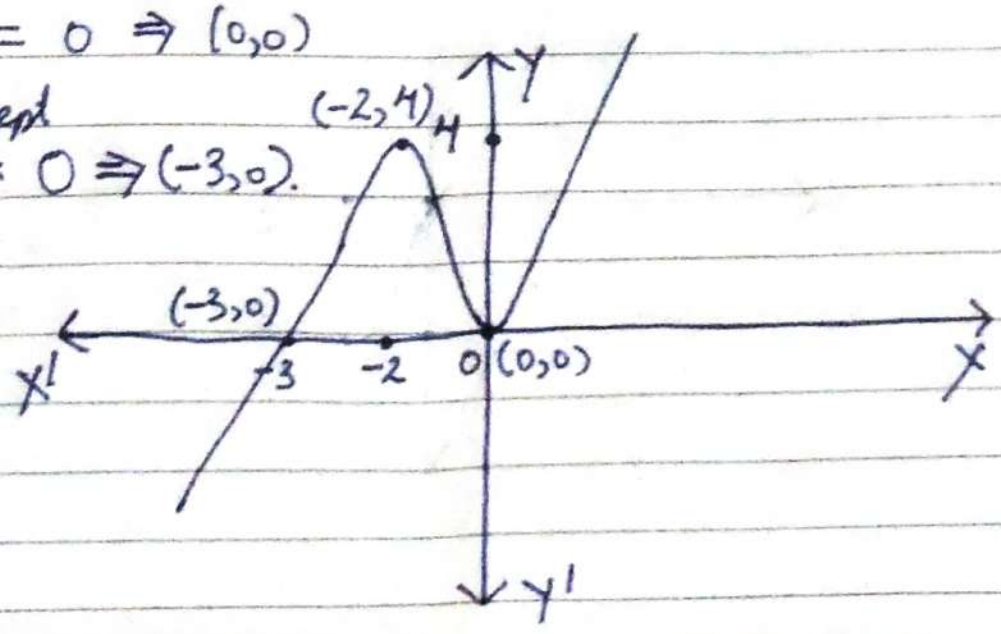
$f(x)$ is decreasing in $(-2, 0)$

$\Rightarrow$ y intercept:

$f(0) = 0 \Rightarrow (0,0)$

x intercept

$f(-3) = 0 \Rightarrow (-3,0)$.



Date :

Q16. $f(x) = 3x^2 - 6x$

$\Rightarrow f'(x) = 6x - 6$

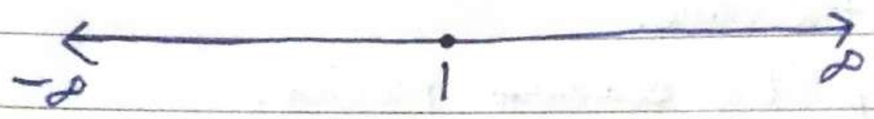
$f'(x) = 0, \quad x = 1$

$f''(x) = 6$, +ve

$f(x)$ is minimum at $x = 1$

$f(1) = -3 \Rightarrow (1, -3)$

$\Rightarrow$



$f'(0) = -6$

$f(x)$ is decreasing in $(-\infty, 1)$

$f'(2) = 6$

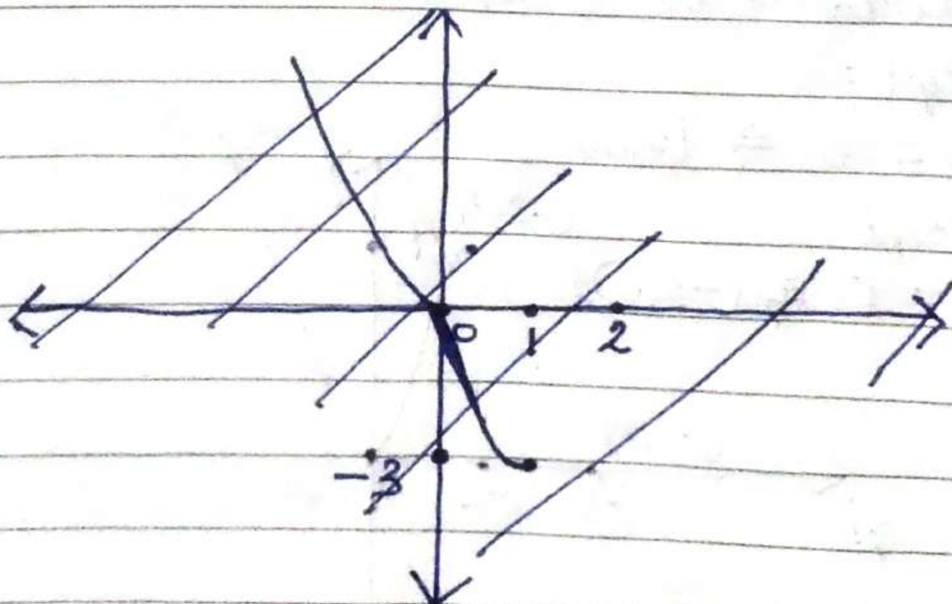
$f(x)$ is increasing in $(1, \infty)$

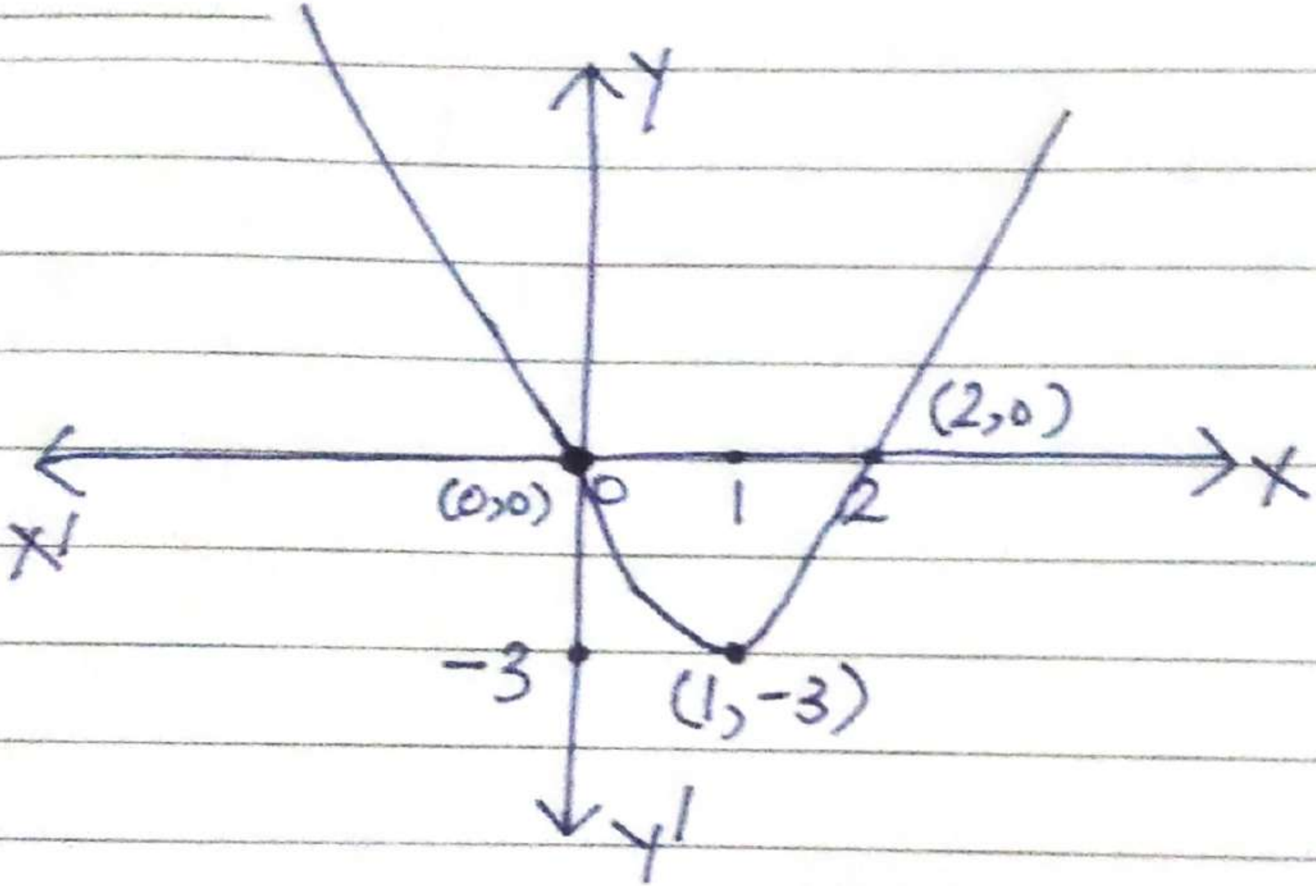
$\Rightarrow$ y intercept,

$f(0) = 0 \Rightarrow (0,0)$

x intercept

$f(2) = 0 \Rightarrow (2,0)$




Y
(2,0)
X
X'
(0,0)
O
1
2
-3
(1,-3)
Y'