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Numerical Methods and Algebra Assignment

1. Area of Circular Plate $= \pi r^2$
Actual Radius = 12.5 cm , Area = $\pi(12.5)^2 = 490.8738 \text{ cm}^2$
wrong radius = 12.65 cm, Area = $\pi(12.65)^2 = 502.7255 \text{ cm}^2$
Change in area = $A'(12.5)(0.15) = 11.78 \text{ cm}^2$
Change in Area = $\pi(12.65)^2 - \pi(12.5)^2 = 11.8516 \text{ cm}^2$

i) Absolute error = Actual value − Approximate value
= 0.0706275

ii) Relative error = $\dfrac{\text{Actual Value} - \text{Approximate value}}{\text{Actual Value}}$

$= \dfrac{11.8516 - 11.78}{11.8516}$

$= 0.0059642 \;/\; 0.006 \text{ cm}^2$

iii) Percentage error = Relative error × 100
= 0.006 × 100
= 0.6%

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2. a) Interpolate $\log(5)$ using $x=4$, $x=6$

4.0	0.60206
6.0	0.7781513

$h=2$

$u = \dfrac{x-x_0}{h} = \dfrac{5-4}{2} = 0.5$

$f(x) = f(x_0) + u\Delta f(x_0)$

$= 0.60206 + 0.5\,[0.7781513 - 0.60206]$

$= 0.690105$

b) Interpolate $\log(5)$ using $x = 4.5$, $x=5.5$

4.5	0.6532125
5.5	0.7403627

$F(x) = f(x_0) + u\Delta F(x_0)$ $\quad u = \dfrac{5.5-4.5}{2} = 0.5$

$= 0.6532125 + 0.5\,(0.7403627 - 0.6532125)$

$= 0.6967876$

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Bisection method

3. Root of $x^4 - x - 10 = 0$

x	y
1.5	-6.4375
2	4

- $x_3 = \frac{x_1 + x_2}{2} = \frac{1.5+2}{2} = 1.75$

  $y_3 = -2.371$

x	y
1.75	-2.371
2	4

  $x_4 = \frac{1.75+2}{2} = 1.875$

  $y_4 = 0.484619$

x	y
1.75	-2.371
1.875	0.484619

  $x_5 = \frac{1.75+1.875}{2} = 1.8125$

  $y_5 = -1.020248413$

x	y
1.8125	-1.020248413
1.875	0.484619

  $x_6 = \frac{1.8125+1.875}{2} = 1.84375$

  $y_6 = -0.287734$

x	y
1.84375	-0.287734
1.875	0.484619

  $x_7 = \frac{1.84375+1.875}{2} = 1.859375$

  $y_7 = 0.0933781$

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4. $f(x) = x^3 - x - 1$ Newton Raphson method

$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$f(1) = -1 < 0 \quad f(2) = 5 > 0$
$\therefore$ root lies b/w 1 and 2

$x_0 = \frac{1+2}{2} = 1.5$

$f(x_0) = f(1.5) = 0.875$
$f'(x_0) = f'(1.5) = 5.75$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 - \frac{0.875}{5.75} = 1.34783$

$x_1 = 1.34783$

$f(x_1) = 0.10068, \quad f'(x_1) = 4.44991$
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.34783 - \frac{0.10068}{4.44991} = 1.3252$

$x_2 = 1.3252$

$f(x_2) = 0.00206, \quad f'(x_2) = 4.26847$
$x_3 = 1.3252 - \frac{0.00206}{4.26847} = 1.32472$

$f(x_3) = 0, \quad f'(x_3) = 4.26463$
$x_4 = x_3 - \frac{0}{4.26463} = 1.32472$

$\boxed{\text{Root} \rightarrow 1.32472}$

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5. Given → $A^2 = A$, To Find: $7A-(I+A)^3$

$7A-(I+A)^3$
$7A-(I^3+A^3+3I^2A+3IA^2)$ $\qquad \because (A+B)^3 = A^3+B^3+3A^2B+3AB^2$
$7A-(I+A^3+3A+3A^2)$ $\qquad \because I^n = I$
$7A-(I+A^2.A+3A+3A^2)$
$7A-(I+A.A+3A+3A)$ $\qquad \because A^2=A$ (given)
$7A-(I+A^2+6A)$
$7A-(I+A+6A)$
$7A-(I+7A)$
$7A-I-7A$
$\Rightarrow -I$

6. $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{|A|} \text{adj} A$

$|A| = 1(0-6)-(-1)(-4-0)+2(4-0)$
$= 1(-6)+(-4)+8$
$= -2$

$\text{adj}.A = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}^T = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} \Rightarrow \frac{1}{-2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

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7. $A = 8i - 9j$, $B = 7i - 9j$

Angle b/w two vectors $= \theta = \cos^{-1}\left[\dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right]$

$$\theta = \cos^{-1}\left[\frac{(8i-9j)(7i-9j)}{|(8i-9j)||(7i-9j)|}\right]$$

$$= \cos^{-1}\left[\frac{(8)(7) + (-9)(-9)}{\sqrt{(8)^2+(-9)^2}\sqrt{(7)^2+(-9)^2}}\right]$$

$\Rightarrow 3.76^\circ$

8. Magnitude of Sabrina's displacement vector :

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30^\circ}$$

$= 97.46$ meters

9. Sum of all 3 digit numbers that leave a remainder of '2' when divided by 3 :

AP $\rightarrow$ 101, 104, 107 . . . 998

$a_n = a + (n-1)d \Rightarrow 101 + (n-1)3 = 998 \Rightarrow \boxed{n = 300}$

$$S_n = \left(\frac{101 + 998}{2}\right) \times 300 = \frac{1099 \times 300}{2} \Rightarrow \boxed{1,64,850}$$

$$\left[\because S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$$

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10. Let 'a' be first term of GP with common ratio 'r'

$\underline{a_n = ar^{n-1}}$

6^{th} term = 32 $\qquad$ 8^{th} term = 128

$ar^5 = 32$ — i) $\qquad$ $ar^7 = 128$ — ii)

Divide (ii) by (i)

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$\boxed{r = 2}$$

11. $(4+3x)^5$

$= 5c_0 . 4^5 + 5c_1 . 4^4 . (3x)^1 + 5c_2 . 4^3 . (3x)^2 + 5c_3 . 4^2 . (3x)^3 + 5c_4 . 4^1 . (3x)^4 + 5c_5 (3x)^5$

$= 1024 + (5)(256)(3x) + (10)(64)(9x^2) + (10)(16)(27x^3) + (5)(4)(81x^4) + 243x^5$

$\Rightarrow 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

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12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$(2-\lambda)[(-5-\lambda)(3-\lambda)] + 3(2)(3-\lambda)$

$-(2-\lambda)(5+\lambda)(3-\lambda) + 6(3-\lambda)$

$(3-\lambda)[-(2-\lambda)(5+\lambda) + 6] = 0$

$\lambda = 3 \quad \lambda = 1 \quad \lambda = -4$

Eigenvectors

For $\lambda = 3$, $(A - \lambda I)X = 0$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$
$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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$R_2 \to R_2/-14$ $\begin{bmatrix} -1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$R_1 \to R_1 + 3R_2$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$x_1 = 0$, $x_2 = 0$, $x_3 =$ indeterminate
$= \alpha$ (constant)

For $\lambda = 3$, eigenvectors $\Rightarrow \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

- For $\lambda = 1$, $\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$x_1 = 3x_2$
$x_3 = 0$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} x_2$

- For $\lambda = -4$ $\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$R_1 \to R_1 - 3R_2$ $\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

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$7x_3 = 0$ $\quad$ $2x_1 = x_2$,

$x_3 = 0$,

$$\text{Eigenvector} = \begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} x_1$$

13. $F(x) = x^3 - 7x^2 + 8x - 3$ $\qquad$ $x_0 = 5$

$F'(x) = 3x^2 - 14x + 8$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{(-13)}{13} = 6$$

$$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} = 6 - \frac{F(6)}{F'(6)} = 6 - \frac{9}{32} = 5.71875$$

$$\boxed{x_2 = 5.71875}$$

14. $(1 - x + x^2)^4$ $\qquad$ $(y + x^2)^4$

Put $1 - x = y$,

$(1 - x + x^2)^4 = (y + x^2)^4$

$= 4C_0\, y^4 (x^2)^0 + 4C_1\, y^3 (x^2)^1 + 4C_2\, y^2 (x^2)^2 + 4C_3\, y (x^2)^3 + 4C_4 (x^2)^4$

$= y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

$= (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$\Rightarrow 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$

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15. $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

x	y
1	0.3678
2	-0.7677

$x_1 = \frac{1+2}{2} = 1.5$, $y_1 = e^{-1.5} - 3\log(1.5) = -0.305143$

x	y
1	0.3678
1.5	-0.305143

$x_2 = \frac{x_0 + x_1}{2} = \frac{1+1.5}{2} = 1.25$

$y_2 = e^{-1.25} - 3\log(1.25) = -0.004225$

x	y
1	0.3678
1.25	-0.004225

$x_3 = \frac{1+1.25}{2} = 1.125$

$y_3 = 0.17119$

x	y
1.125	~~0.3678~~ 0.17119
1.25	-0.004225

$\boxed{x_4 = 1.187}$ $y_4 = 0.08108$