Financial Mathematics

(1) $d^{(4)} = 8\%$

$$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$= 1 - (1 - 2\%)^4$$

$d = 7.76318\%$

(2) i) $\delta = -\ln(1-d)$

$= 8.08108\%$

(3) (ii) $i = \frac{d}{1-d}$

$$= \frac{7.76318}{1 - 8.08108}$$

$= 8.41657\%$

iii) $d^{(12)} = 12\left[(1 - (1-d)^{1/12}\right]$

$= 8.053934\%$

Q2) $\delta(t) = 0.004t + 0.0002t^2$

Amount due $= ₹\,1000$ (C)

$$PV_0 = C \cdot v(t)$$

$$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2}$$

$$= 1000 \times e^{-\left[0.002t^2 + 0.0002t^3/3\right]_0^{10}}$$

$$= 1000 \times e^{-0.2666666}$$

$= 765.93$

(ii) $(1+i)^{10} = e^{-0.2666666}$

$i = 2.702540\%$

(Q3) Interest earned on the investment, paid at beginning is 'd' whereas interest earned on investment paid at the end period is i.
Therefore, on discounting 'i' from end of period using discounting factor 'v', gives 'd'.

Hence,

$$d = iv.$$

ii) a) $\frac{d^{(4)}}{4} = 2.25\%$

$$d = 1-(1-2.25)^4$$
$$d = 8.7007806\%$$
$$d^{(2)} = 2(1-(1-d)^{1/2})$$
$$= 8.89875\%$$

(b) $d^{(1/2)} = 5\%$

$$d = 1-\left(1-\frac{d^{(1/2)}}{(1/2)}\right)^{1/2}$$

$$d = 1-(1-10\%)^{1/2}$$
$$d = 5.1316702\%$$
$$d^{(2)} = 2(1-(1-d)^{1/2})$$
$$d^{(2)} = 5.1992507\%$$

iii) $v\left(0, \frac{91}{365}\right) = \frac{1}{(1+i)^{91/365}}$

Q4) iii) $i = 0.08$

a) $d = \frac{i}{1+i} = 7.40740\%$

$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$
$d^{(12)} = 7.671478\%$

b) $i^{(365)} = 365\left[(1+i)^{1/365} - 1\right]$
$= 7.696915\%$

c) $\delta = \ln(1+i)$
$= 7.696104\%$

d) $i^{(0.5)} = \frac{1}{2}\left((1+i)^{1/1/2} - 1\right)$
$= \frac{1}{2}\left((1+i)^{2} - 1\right)$
$= 8.32\%$

Q5) i) $d^{(4)} = 6\%$
$d = 1 - (1 - 1.5\%)^4$
$d = 5.866344937\%$

a) $i = \frac{d}{1-d}$
$= 6.231931\%$

b) $i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$
$= 6.137752\%$

b)

b) $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$d^{(12)} = 6.030253\%$

(ii) $i = \dfrac{20,000}{2,00,000} \times 100$

$= 10\%$

No. of days $= 93$.

$AV_{93} = 2,00,000 \times (1.1)^{93/365}$

$= 204,916.3564$

6) —

$d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$d^{(12)} = 13.719544\%$

b) $i = \dfrac{d}{1-d} = 14.79591837\%$

$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$

$= 14.285714\%$

c) $\delta = -\ln(1-d)$

$\delta = 13.798574\%$

Q7) Amount invested (C) = 20,000

$AV_{17} = 26,000$

$17 \text{ months} = 1 + \frac{5}{12} = 1.41664 \text{ yrs.}$

$AV = C(1+i)^n$

${}^{13}\cancel{26,000} = {}^{10}\cancel{20,000}(1+i)^{1.416666}$

$i = 20.34570672\%$

i) $i^{(4)} = 4((1+i)^{1/4} - 1)$

$i^{(4)} = 18.955255\%$

(ii) $d = \frac{i}{1+i} = 16.9060511\%$

$d^{(2)} = 1 - (1-d)^{1/2}$

$d^{(2)} = 17.688235\%$

III) $26,000 = 20,000(1 + 41666667 i_s)$

$i_s = 21.1764706\%$

Q8] $i = 8\%$

$i^{(P)} = P[(1.08)^{1/P} - 1]$

P	$i^{(P)}$
1	8%
2	7.8406%
3	7.79567%
4	7.770621%
12	7.72084%
100	7.698907%
365	7.69691%
∞	7.696904%

→ when continuous compounding occurs, when p approaches infinity then $i^{(p)}$ has limiting value known as force of interest or δ

Q9) i) Force of Interest : The measure of interest at individual moments of time or at any instant of time is called force of interest.

(ii) $i^{(2)} = 0.0775$

Calculate, $i, \delta, d^{(12)}, i^{(1/2)}$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$$

$$i = 7.900156\%$$

b) $\delta = \ln(1+i)$

$= \ln(8.900156)$

$= 7.603613\%$

c) $d = \dfrac{i}{1+i}$

$= 7.321728\%$

$$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$

$= 7.579575\%$

d) $i^{(1/2)} = \dfrac{1}{2}\left((1+i)^2 - 1\right)$

$= 8.212219\ldots\%$

Q10)
$$\delta(t) = \begin{cases} 0.08, & 0 \le t \le 5 \\ 0.06, & 5 \le t < 10 \\ 0.04, & t \ge 10 \end{cases}$$

Case 1, $0 \le t \le 5$

$V(t) = e^{-\int 0.08}$

$= e^{[0.08t]}$

when, $5 \le t \le 10$

$$v(t) = e^{-\int_0^5 0.08 - \int_5^t 0.06}$$

$$= e^{-[0.08t]_0^5 + [0.06t]_5^t}$$

$$= e^{-0.4+0.3-0.06t}$$

$$= e^{-0.06t-0.1}$$

$t \ge 10$

$$v(t) = e^{-\int_0^5 0.08 - \int_5^{10} 0.06 - \int_{10}^t 0.04}$$

$$= e^{-[0.08]_0^5 - [0.06t]_5^{10} - [0.04t]_{10}^t}$$

$$= e^{-\cancel{0.4}+0.3-0.6+\cancel{0.4}-0.04t}$$

$$e^{-0.07t-0.3}$$

Q11) a) $t = \frac{9}{12}$ months

$= 3/4$ years.

$C = 50,000$ $d = 18\%$

$PV_0 = C(1-d)^t$

$= 50000(1-0.18)^{3/4}$

$= 43085.43$

b) $\delta = 6\%$, $d^{(12)} = ?$

$d = 1 - e^{-\delta}$

$= 5.82354660\%$

$d^{(12)} = 12[1-(1-d)^{1/12}]$

$= 5.985025\%$

(ii) $d^{(}$ $d =$

(ii) $d^{(4)} = 6\%$.

$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= \cancel{5.86686}\ 5.8663449\%$

$i = \frac{d}{1-d}$

$i = 6.231931537\%$.

Now,

$i^{(4)} = 4\left((1+i)^{1/4} - 1\right)$

$i^{(4)} = 6.0913705\%$.

$i^{(12)} = 12\left[(1+i)^{1/12} - 1\right)$

$i^{(12)} = 6.060709\%$.

d) Interest earned on an investment that is paid at the beginning is 'd' whereas an interest earned on an investment is paid at the end is 'i'.

So, if we discount 'i' from the end of the year using discounting factor 'v' we get,

$$d = iv.$$

Therefore, this relation holds true.

Q12) $C = 7049$

$AV_6 = C \cdot A(0,6)$

$= 7049 \cdot \left[A(0,2) \cdot A(2,4.5) \cdot A(4.5,6)\right]$

$A(0,2) =$

$i^{(12)} = 6\%$.

$i^{(12)}/12 = 0.5\%$.

$A(0,2) = (1.005)^{24}$

$\Rightarrow$ $A(2, 4.5)$,
i is calculate for each quarter $= 15\%$.
$A(2, 4.5) = (1 + 0.15)$
$= (1.15)$

$\Rightarrow$ $A(4.5, 6)$, $d^{(12)} = 6\%$
$d^{(12)}/12 = 0.50\%$
$A(4.5, 6) = \dfrac{1}{(1-0.005)^{18}}$
$\therefore AV_6 = 7049 \times (1.005)^{24} \times (1.15) \times \dfrac{1}{(1-0.005)^{18}}$

$AV_6 = 9999.894$.

Q13] Let $C = x$
$AV_n = 2x$
Time $= n$ years.
i) $AV_n = C(1+i)^n$
$i = 10\%$
$2x = x(1.1)^n$
$(1.1)^n = 2$
$\ln 2 = n \ln(1.1)$
$n = \dfrac{\ln(2)}{\ln(1.1)}$

$n = 7.27$ yrs.

(ii) $AV_n = \dfrac{C}{(1-d)^n}$
$d^{(12)} = 10\%$
$d^{(12)}/12 = 0.83333\%$
$AV_n = \dfrac{C}{\left(1 - \dfrac{d^{(12)}}{12}\right)^{12n}}$

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$$2x = \frac{x}{(1-0.83333)^{12n}}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12n} = \frac{1}{2}.$$

$$12n \log\left(1-\frac{d^{(12)}}{12}\right) = \log\left(\frac{1}{2}\right)$$

$n = 6.902$ years.