Date

Page

Calculus

① $t = e^{2/\omega}$

$dt = e^{2/\omega} \cdot \frac{d}{d\omega}(2/\omega)$

$dt = \frac{-2e^{2/\omega}}{\omega^2} d\omega$

$\frac{dt}{-2} = \frac{e^{2/\omega}}{\omega^2} d\omega$ ——— ①

$I = -\frac{1}{2}\int_{e^{100}}^{e} dt$

$= -\frac{1}{2}\left([t]\right)_{e^{100}}^{e}$

$= \frac{-1e^{100}}{2} + \frac{1e}{2}$

$= \frac{e}{2} - \frac{e^{100}}{2}$

② $I = \int \frac{2+3+1}{(\frac{4}{t}+2t)^3} dt$

Date ______ Page ______

$\cancel{y = 4+2t}$
$\cancel{dy = 4+}$

$y = t^4 + 2t$
$dy = (4t^3 + 2)\,dt$

$= 2\,(2+3+1]\,dt$

$\Rightarrow \quad I = \frac{1}{2}\int \frac{dy}{y^3}$

$= \frac{1}{2}\,\frac{y^{-3+1}}{-2} = \frac{1}{4} \cdot \frac{1}{y^2}$

$= \frac{-1}{4y^2} = \frac{-1}{4(t^4+2t)^2}$

(4) $f(x) =$

$\int_{-2}^{3} f(x)\,dx$

$\underline{I,}\quad \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,d(x)$

$= I = \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

Date ____ Page ____

$$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= 1-(-8)+6(3)-6(1)$$

$$= 1+8+18-6$$
$$= 21$$

(5) $t = e^{4y^2-y}$

$dt = e^{4y^2-y}(8y-1)dy$

$\therefore \quad I = 3\int dt = 3t + c$

$= 3(e^{4y^2-y}) + c$

(6) $\int f''(x) = f'(x)$

$$= \int 15x^{1/2} + 5x^3 + 6\,dx$$

$$= \frac{2}{3} \cdot 15x^{3/2} + \frac{5x^4}{4} + 6x$$

$$f(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + c$$

$$= f(x) = \frac{10x}{5/2} + \frac{5}{4}\frac{x^5}{5} + \frac{6x^2}{2} + cx + c'$$

Date ________ Page ________

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + cx + c'$

$= -\frac{5}{4} = 4 + \frac{1}{4} + 3 + c + c'$

$= -\frac{3}{2} = 7 + c + c'$

$c + c' = -\frac{17}{2}$

$= f(4) = 4 \cdot 4^{5/2} + (4)^4 + 3(4)^2 + 4c + c'$

$= 404 = 4^{7/2} + 4^{(4)} + 48 + 4(c + c)$

$= -28 = 4c + c'$

$= c' = -4c - 28$

$c + (-4c) - 28 = -8.5$

$-3c = 28 - 8.5$

$\underline{c = -6.5}$

$c' = -8.5 + 6.5 = -2.5$

$= \underline{\underline{-2.5}}$

Date

Page

$$\left[f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13x}{2} - \frac{5}{2} \right]$$

(9) $$\frac{dv}{9.8 - 0.196v} = dt$$

$$= \frac{dv}{98/10 - 196/1000 v} = dt$$

$$= \int \frac{dv \, (100}{9800 - 196v} = \int dt.$$

$$= \frac{50 \cdot 100 \cancel{0} \log (9800 - 196)}{-196} = t + c$$

$$= \frac{-50 \log (9604}{98} = \underline{\underline{t + c}}$$

(10) $$\frac{t\,dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{t\,dy}{dt} = t^2 - t + 1 - 2y = x$$

Date ________ Page ________

$$\frac{dx}{dt} = 2t - 1 - \frac{2dy}{dt}$$

$$= \frac{2dy}{dt} = 2t - 1 - \frac{dx}{dt}.$$

$$\frac{dy}{dt} = t - 1/2 - 1/2\frac{dx}{dt}.$$

$$= \frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dy}{dt}.$$

$= y = 1/2$ at $t = 1$

$\cdot\ t\,(t - 1/2 - 1/2\, dx/dy\,t) = x$

$$= t2dt - \frac{t}{2}dt - \frac{t}{2}dx = xdt.$$

$$= \frac{t^3}{3} - \frac{t^2}{4} - \frac{t^\alpha}{\alpha} = x^t + c$$

$$= \frac{t^3}{3} - \frac{t^\alpha}{4} = \frac{3xt}{2} + c$$

$$= \frac{t^3}{3} - \frac{t^\alpha}{4} = \frac{3t}{\alpha}(t^\alpha - t + 1 - 2y) + c$$

$$= \frac{-7t^3}{6} + \frac{101}{8} - \frac{3t}{2} + 3yt = c.$$

Date ____ Page ____

$= \frac{-28+30}{24} = c$

$= \rightarrow \ ^{2}/_{24} = \frac{1}{12} = c$

(11) $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x\alpha}}$

$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^\alpha}} \, dy.$

$= \frac{y^{-2}}{-2} = \frac{1}{\alpha} \frac{t^{1/\alpha}}{1/\alpha} + c$

$= \frac{-1}{2y^2} = t^{1/\alpha} + c$

$= \frac{-1}{2(-1)} = c$

$= c = 1/2$

$= \cancel{1+\alpha} \ 1+x^\alpha > 0$ for all $x \in R$

Date

Page

(12) $f'(x) = 3x^2 - 20x$

$f''(x) = 6x$

$f'''(x) = 6$

$f''''(x) = 0$

$= f'(3) = 3(3)^2 - 20(3)$
$= 27 - 60$
$= -33$

$= f''(3) = 18$
$f'''(3) = 6$
$f''''(3) = 0$

$\Rightarrow$ Taylor Expansion $= -54 + (x-3)(-33)$
$+ \frac{18}{2!}(x-3)^2 + \frac{6}{3!}$
$(x-3)^3 + 0.$

$= -57 + 99 - 33x + 9(x-3)^2 + (x-3)^3$
$= 42 - 33x + 9(x^2 - 6x + 9) + (x^3 - 27$
$9x^2 + 27x$

$= x^3 - 60x + 123 - 27$
$= x^3 - 60x + 96$

Page

Date

(10)

(13) $f(0) = 1^4 = 1$

$f'(0) = M(1+x)^{M-1}$
$= M(1-0)^{M-1}$
$= M(1) = M$

$= f''(0) = M(M-1)(1+0)^{M-1-1}$
$= M^2 - M$

$= f'''(0) = M(M-1)(M-2)(1+0)^{M-3}$
$= M(M-1)(M-2)$

· Melarin Series:

$f(0) + f'(0) + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} \ldots$

$= 1 + Mx + \frac{(M^2-M)}{2!}x^2 + \frac{M(M-1)(M-2)}{3!}$

$+ \frac{M(M-1)(M-2)(M-3)x^4}{4!} \ldots$

Date ______ Page ______

(14) $f(0) = 1$

$f'(0) = \frac{1}{2} (1+x)^{-1/2} = \frac{1}{2} (1)^{-1/2}$

$f''(0) = \frac{-1}{2} \frac{1}{2} (1+x)^{-3/2} = -\frac{1}{4} (1)$

$= \frac{-1}{4}$

$\Rightarrow \quad 1 + \frac{1}{2}(x) + -\frac{1}{4}\left(\frac{x^2}{2!}\right) + \frac{1}{8}\left(\frac{x^3}{3!}\right) -$

$\frac{1}{16}\left(\frac{x^4}{4!}\right) \dots\dots \frac{-1}{2^n}\frac{x^n}{n!}$

(15) $f(-4) + (x-(-4))f'(-4) + (x-(-4))^2$

$f''(-4) + \frac{(x-(-4))^3}{3!} f'''(-4)$

$= \quad f(x) = e^{-6x}$

$f'(x) = \frac{-1}{6} e^{-6x}$

$x = -4$

Page

Date

$$f'(-4) = \frac{-1}{6} e^{24}$$

$$f''(-4) = \frac{1}{36} e^{-6x} = \frac{1}{36} e^{24}$$

$$\Rightarrow e^{24} + (x+4) - \tfrac{1}{6} e^{24} + \frac{(x+4)}{2!} \frac{1}{36} e^{24}$$

$$- \frac{1}{216} \frac{e^{24}(x+4)^3}{3!} \ldots$$

(1.6) $f(0) + (x-0) f'(0) + (x-0)^2 f''(0) +$

$(x-0) f'''(0)$

$= f(x) = \ln(3+4x)$

$x = 0 \to \ln(3)$

$\cancel{x=0}$

$$f'(x) = \frac{4}{3+4x}$$

$\cancel{x \Rightarrow 0}$ $\cdot$ $x = 0 \to 4/3$

$$f''(x) = \frac{(4x+3)\cdot(0) - (4)(4)}{(3+4x)^2}$$

Date ______ Page ______

$= \frac{-16}{(3+4x)^2} \rightarrow x=0$

$= f'''(x) = \frac{3^2}{(3+4x)^3} = \frac{32}{24}$

$\therefore \ln(3) + \frac{4x}{3} + (x-0)^2 \frac{(-16)}{9(2!)} + \frac{32}{27}$

$\frac{(x-0)^3}{\cancel{2}\,3!} + \dots - \frac{64}{81(4!)}(x-0)^4 \dots$

$= \ln 3 + \frac{4x}{3} - \frac{16}{9(2!)} x^2 + \frac{32}{27} \frac{x^3}{3!} -$

$\frac{64}{81(4!)} x^{4-1} \dots$

③ $f(x) = \int f'(x)\,dx$

$= \int (x^4 + 3x - 9)\,dx$

$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

Date ____ Page ____

(7) $z = f(x+iy) + g(x-iy)$

- $p = z' =$

$f'(x+iy) + g'(x+iy)$

- $m = z' =$

$f'(x+iy)(i) + g(x-iy)(-i)$

- $q = z'' =$

$f''(x+iy) + g(x-iy)$ —— (a)

- $n = z'' = f''((x+iy\ i^2) + g'(x-iy)$
$(-i)^2$

$n = f''(x+iy)(i^2) + g'(x-iy)(i^2)$

$n = i^{(2)}\{q\}$

$n = i^{(2)} q$

$n = i^2 q$ (from (a))

Date ______ Page ______

(8) integrating on both sides:

$$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$$

$$\frac{r^{-2+1}}{-1} = \log\theta + c$$

$$-r^{-1} = \log\theta + c$$

$$-1/r = \log\theta + c$$

$= r(1) = 2$

$$= \frac{-1}{r} = \log(1) + c$$

$$-\frac{1}{2} = 0 + c$$

$$c = -1/2$$

$$= -1/r = \log\theta - 1/2$$

$$\log\theta = 1/2 - 1/r$$