Assignment

Q1)

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Mean $= \dfrac{4+7+9+9+11+15+18+18+18+25}{10}$

$= 13.4.$

Mode $= 18.$

Median $= \left(\dfrac{n+1}{2}\right)^{th}$ term

$= 5.5^{th}$ term.

$= 13.$

IQR $\Rightarrow$ $q_1 = \left(\dfrac{n}{4}\right)^{th}$ term

$= 2.5^{th}$ term

$= 8.$

$q_3 = \left(\dfrac{3n}{4}\right)^{th}$ term

$= 7.5^{th}$ term

$= 18.$

IQR $= q_3 - q_1$

$= 10.$

2) N.o H | 0 | 1 | 2 | 3 | 4
N. of | 5 | 11 | 26 | 15 | 3

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{120}{60}$

$= 2.$

f	x	fx
0	5	0
1	11	11
2	26	52
3	15	45
4	3	12

Median = ~~60~~

$n = 60.$

Median $= \left(\frac{n+1}{2}\right)^{th}$ term.

$= 30.5^{th}$ term.

$\therefore = 2.$

Mode = 2.

IQR ⇒ $q_1 = \left(\frac{n}{4}\right)^{th}$ term

$= 15^{th}$ term

$= 1.$

$q_3 = \left(\frac{3n}{4}\right)^{th}$ term.

$= 45^{th}$ term

$= 3$

~~I~~QR $= q_3 - q_1$

$= 2.$

Standard deviation.

$$s = \sqrt{\frac{\Sigma f (x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{58}{60}}$$

$$= 0.98319208.$$

Q2) 1) $X \sim P(0.5)$.

$$P(X=0) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \frac{e^{-0.5} \lambda^{0}}{0!}$$

$$= e^{-0.5}$$

$$= 0.6065$$

2) Y = no. of years with a claim

the $Y \sim B(3, 0.3935)$.

$$P(Y=1) = {}^{3}C_{1} \times (0.3935)^{1} \times (0.6065)^{2}$$

$$= 0.434.$$

3) $X \sim \exp(0.5)$

$$P(x>2) = \int_{2}^{\infty} \lambda e^{-0.5x} \, dx.$$

$$= \left[e^{-0.5x}\right]_{2}^{\infty}.$$

$$= e^{-\infty} - e^{-1}$$

$$= \frac{1}{e} = 0.368$$

Q3) $X \sim \text{Gamma}(\alpha, \lambda)$

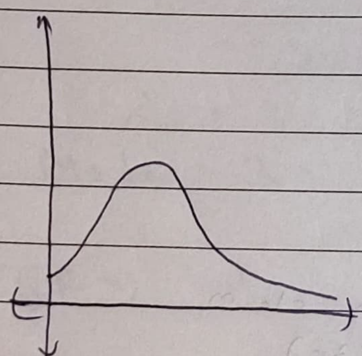
$\alpha = 2 \quad \lambda = 1/2 \; ; \; 0 < x < \infty$

① $E(x) = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$

$V(x) = \frac{\alpha}{\lambda^2} = \sigma^2 \Rightarrow \sigma = \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{0.25}}$

$= \sqrt{8} = 2.828$

②



Increases in the start and then decrease when $\alpha = 2$ $\lambda = 1/2$.

Q4) $P(H) = \frac{10}{18}$, $P(F) = \frac{8}{18}$, $P(HQ) = \frac{6}{18}$

$P(FQ) = \frac{7}{18}$

$P(X) = P(FF) + P(FM) + P(MH)$

$= \frac{80}{306} + \frac{56}{306} + \frac{80}{306}$

$= \frac{216}{306}$

$P(Q) = \frac{216}{306} \times \frac{7}{18} = 0.2745$

Q. $X \sim U(1, 2)$.

$a = 1, \quad b = 2$.

$f_X(x) = \frac{1}{2-1} = 1$.

$f_Y(y) = f_X(w^{-1}(y)) \left| \frac{d}{dy}(w^{-1}(y)) \right|$

$= f_X\left(6 - \frac{3}{y}\right) \cdot \frac{3}{y^2}$

$= \frac{3}{y^2} \cdot \frac{3}{y^2}$.

$y = \frac{3}{6-x}$.

$6 - x = 3y$

$x = 6 - \frac{3}{y} = w^{-1}(y)$.

$\frac{dy}{dx} = \frac{3}{y^2}$

Q7) (1) $F(3) = P(x \le 3)$.

$= 0.05 + 0.15 + 0.35$

$= 0.55$.

(2). $E(x) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.04) + 5(0.05)$

$= 3.25$

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(3) $E(x^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$.
$= 11.45$.

$V(x) = E(x^2) - [E(x)]^2$
$= 11.45 - (3.25)^2$
$= 0.8875$

(4) Coefficient of skewness
$= E(x - \bar{x})^3$

Q8) $x \sim B(15, 0.2)$.

$P(x < 3) = P(x=0) + P(x=1) + P(x=2)$
$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)(0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$
$= 0.39802$.

Q10) $x \sim P(0.4)$

$P(x > 2) = 1 - P(x \leq 2)$
$= 1 - 0.99207$
$= 0.00793$.

Q12) $X \sim Exp(10)$.

$\lambda = 10$
$x = 0.1$ days.
$E_x(0.1) = \lambda e^{-\lambda x} = 10 e^{10(0.1)} = 3.67879$.

Q13) $X \sim U(75, 120)$ $\quad a = 75, \quad b = 120$

$$f_x(x) = \frac{1}{b+a} = \frac{1}{120-75} = \frac{1}{45}$$

$$P(80 < x < 100) = F_x(100) - F_x(80)$$
$$= \frac{100}{45} - \frac{80}{45}$$

Q14) $X \sim \text{Gamma}(5, 0.4)$ $\quad \lambda = 0.4 \quad \alpha = 5$

$P(x < 22.89)$

$$2\alpha x \sim \chi^2_{10} = 0.8x \sim \chi^2_{10}.$$

$$P(x < 22.89) = P(0.8 \times 18.312)$$
$$= P(\chi^2_{10} \times 18.312)$$
$$= 0.9499296.$$