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Numerical Methods & Algebra
Assignment 2.

A1) Area of circular path $A(r) = \pi r^2$

$A'(r) = 2\pi r$

Change in area $= A'(r)$

$= A'(12.5)(0.15)$

$= 3.75\pi$

$A(12.65) - A(12.5)$

$\Rightarrow \pi(12.65)^2 - \pi(12.5)^2$

$= 160.022\pi - 156.25\pi$

$= 3.7725\pi \text{ cm}^2$.

i) Absolute error $= 3.7725\pi \text{ cm}^2 - 3.75\pi \text{ cm}^2$

$= 0.0225\pi \text{ cm}^2$

ii) Relative error $= \dfrac{3.7725\pi - 3.75\pi}{3.7725\pi}$

$= \dfrac{0.0225\pi}{3.7725\pi}$

$= 0.006 \text{ cm}^2$

iii) Percentage error $= 0.006 \times 100$

$= 0.6\%$.

$$f(x) = f(x_0) + (x - x_0)\, f[x_1, x_0]$$

$x_0 = 4 \quad x_1 = 6$

$$f[x_1, x_0] = \frac{[f(6) - f(4)]}{6 - 4}$$

$= 0.088046.$

$$f(5) \approx f(4) + (5-4)\, 0.88046$$

$= 0.690106$

$x_0 = 4.5 \quad x_1 = 5.5$

$$f[x_1, x_0] = \frac{[f(5.5) - f(4.5)]}{(5.5 - 4.5)}$$

$= 0.0871502$

$$f(5) \approx f(4.5) + (5 - 4.5)\, 0.0871502$$

$= 0.696788.$

$x^4 - x - 10 = 0$

$\therefore f(x) = x^4 - x - 10$

$a = 1.5 \quad b = 2.$

$f(1.5) = 5.0625 - 1.5 - 10$

$= -6.4375$

$f(2) = 16 - 2 - 10$

$= 4.$

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The function is continuous and its root lies in the interval $[1.5, 2]$

Let 't' be the mid-point of the interval

$$t_1 = \frac{1.5+2}{2} = 1.75.$$

$$\therefore f(1.75) = 9.3789 - 1.75 - 10$$
$$= -2.371 \text{ which is } < 0.$$

Since $f(t) < 0$ we will assume $a = t$
$f(t)$ is negative so it is replaced with a.

$$\therefore t_2 = \frac{a+b}{2}$$
$$= \frac{1.75+2}{2}$$
$$= 1.875.$$

$$f(t_2) = (1.875)^4 - 1.875 - 10$$
$$= 0.684 \text{ which is } > 0.$$

Since $f(t_2) > 0$ $t_2 = b$

$$t_3 = \frac{a+b}{2} \Rightarrow \frac{1.75+1.875}{2}.$$

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$t_3 = 1.8125$

$f(t_3) = 10.7923 - 1.8125 - 10$
$= -1.0202$ which is < 0.

Since $f(t_3) < 0 \quad a = t_3$.

$$t_4 = \frac{1.8125 + 1.875}{2}$$
$= 1.844$.

$f(t_4) = 11.556 - 1.844 - 10$
$= -\cancel{4}0.288 < 0$

$\therefore t_4 = a$.

$$t_5 = \frac{1.844 + 1.875}{2}$$
$= 1.859$

$f(t_5) = 0.097 > 0$.

$\therefore$ Interval after fifth iteration is.

$[1.844, 1.859]$

Hence 1.859 is the final solution

1). $f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

x	0	1	2
$f(x)$	-1	-1	5

$f(1) = -1$ which is < 0
$f(2) = 5$ which is > 0.

$\therefore$ The root lies between $[1, 2]$

$t_0 = \frac{1+2}{2} = 1.5$

$f(t_0) = f(1.5) = 0.875$
$f'(t_0) = f'(1.5) = 5.75.$

$t_1 = t_0 - \frac{f(t_0)}{f'(t_0)}$

$t_1 = 1.3478$

$f(t_1) = f(1.3478) = 0.10071$
$f'(t_1) = 4.4497$

$t_2 = t_1 - \frac{f(t_1)}{f'(t_1)}$
$= 1.3252$

$f(t_2) = f(1.3252) = 0.00206$
$f'(t_2) = 4.2685$

$$t_3 = t_2 - \frac{f(t_2)}{f'(t_2)}$$

$= 1.3247$

$f(t_3) = 0$
$f'(t_3) = 4.2645$

$$t_{(4)} = t_3 - \frac{f(t_3)}{f'(t_3)}$$

$= 1.3247$

$\therefore$ the approximate root for the equation $x^3 - x - 1 = 0$ using Newton Raphson method $= 1.3247$

5) $7A - (I+A)^3$

$\Rightarrow 7A - (I + A^3 + 3A^2 + 3A)$

$\Rightarrow 7A - (I + A.A^2 + 3A + 3A)$

[Since $A^2 = A$].

$7A - (I + A.A + 6A)$

$7A - (I + A^2 + 6A)$

$7A - (I + A + 6A)$

$7A - (I + 7A)$

$7A - I - 7A$

$\Rightarrow -I$

$\Rightarrow$ ~~-1 0 0 0 1~~ $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, [sol]

6). $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} (adj\ A)$

$|A| = 1(-0-6) - (-1)(-4-0) + 2(4-0)$
$= -6 + 1(-4) + 2(4)$
$= -7 - 4 + 8$
$= -2.$

$a_{11} = 1(-0-6) = -6$
$a_{12} = (-1)(-4-0) = 4$
$a_{13} = (4-0) \Rightarrow 4$
$a_{21} = (-1)(1-2) = 1$
$a_{22} = (-1-0) = -1$
$a_{23} = (-1)(1-0) = -1$
$a_{31} = (-6-0) = -6$
$a_{32} = (-1)(6-8) = 2$
$a_{33} = (0-(-4)) = 4$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -7 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}^{t}$$

$$= \begin{bmatrix} -7 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix} \frac{-1}{3}$$

$$\Rightarrow \begin{bmatrix} \frac{7}{3} & \frac{-1}{3} & 2 \\ \frac{-4}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-4}{3} & \frac{1}{3} & \frac{-4}{3} \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix} \quad (\text{ans})$$

7) $A = 8i - 9j$
$B = 7i - 9j$

$a.b = (8, -9)\ (7, -9)$
$= \cancel{56 + 81}\ 56 + 81$
$= 137$

$|a| = \sqrt{8^2 + 9^2}$
$= 12.042$

$|b| = \sqrt{7^2+9^2}$
$= 11.4018$

$$\theta = \cos^{-1} \frac{\Sigma(a.b)}{|a||b|}$$

$$= \cos^{-1} \frac{137}{11.402 \times 12.042}$$

$$= \cos^{-1} \frac{137}{137.3029}$$

$$= \cos^{-1} 0.998$$

$$\theta = 3.624^\circ$$

A8) $R = \sqrt{A^2+B^2+2AB\cos\theta}$

$R = \sqrt{75^2+25^2+2(75)(25)\cos 30^\circ}$

$= 97.405$

A9) All 3 digit numbers which leave a remainder of 2 when divided by 3 form an A.P

101, 104, 107, ... 998

~~i.e.~~ $a = 101 \quad d = 3$

$a_n = 998$

$998 = 101 + (n-1)3$

$299 = n-1$

$300 = n$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_n = \frac{300}{2}\left(2(101) + (300-1)3\right)$$

$$= 150\left(202 + 299(3)\right)$$

$$= 150(1099)$$

$$= 164850 \quad \text{(ans)}$$

1) $a_n = ar^{n-1}$

$32 = ar^5$

$128 = ar^7$

$$\frac{128}{32} = \frac{ar^7}{ar^5}$$

$\Rightarrow 4 = r^2, \quad r = \pm 2$

11) $(4+3x)^5$

$${}^5C_0\, 4^5 + {}^5C_1\, 4^{(4)}\, 3x + {}^5C_2\, 4^{(3)}\, 3x^2 + {}^5C_3\, 4^{(2)}(3x)^3 + {}^5C_4\, 4.(3x)^4 + {}^5C_5 (3x)^5$$

$$\Rightarrow 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$$

12) $$\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\det(\lambda I - A) = 0$.

$$\det\left(\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \begin{bmatrix}2&-3&0\\2&-5&0\\0&0&3\end{bmatrix}\right) = 0$$

$$\det\begin{bmatrix}\lambda-2&3&0\\-2&\lambda+5&0\\0&0&\lambda-3\end{bmatrix} = 0$$

$$(\lambda-2)\left((\lambda+5)(\lambda-3)-0\right) = 0$$
$$(\lambda-2)(\lambda^2+5\lambda-3\lambda-15) = 0$$
$$(\lambda-2)(\lambda^2+2\lambda-15) = 0$$
$$(\lambda-2)\left(\lambda(\lambda+5)-3(\lambda+5)\right) = 0$$
$$(\lambda-2)(\lambda-3)(\lambda+5) = 0$$

Eigen values are 2, 3, -5

13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{f(5)}{f'(5)}$$

$$= 5 - \frac{(-13)}{13}$$

$= 6$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 6 - \frac{f(6)}{f'(6)}$$

$$= 6 - \frac{9}{32}$$

$$= 5.71875$$

$\therefore$ ans $= 5.71875$.

14) $(1 - x + x^2)^4$

$[1 + (x^2 - x)]^4$

Let $x^2 - x = y$.

$\therefore (1+y)^4 = {}^4C_0 1^4 + {}^4C_1 y + {}^4C_2 y^2 + {}^4C_3 y^3 + {}^4C_4 y^4$.

$= 1 + 4y + 6y^2 + 4y^3 + y^4$.

$= 1 + 4(x^2 - x) + 6(x^2 - x)^2 + 4(x^2 - x)^3 + (x^2 - x)^4$

$\Rightarrow 1 + 4x^2 - 4x + 6x^4 + 6x^2 - 12x^3 + 4[{}^3C_0 x^6 - {}^3C_1 x^5 + {}^3C_2 x^4 - {}^3C_3 x^3] + [{}^4C_0 x^8 - {}^4C_1 x^7 + {}^4C_2 x^6 - {}^4C_3 x^5 + {}^4C_4 x^4]$

$\Rightarrow 1 + 4x^2 - 4x + 6x^4 + 6x^2 - 12x^3 + 4x^6 - 12x^5 + 12x^4 - 4x^3 + x^8 - 4x^7 + 6x^6 - 8x^5 + x^4$

$\Rightarrow 1 + 10x^2 - 16x^3 + 19x^4 - 4x - 20x^5 + 10x^6 - 4x^7 + x^8$

15) $e^{-x} = 3\log(x)$

Let $a = 0.5$ and $b = 1.5$.

$\frac{a+b}{2} = \frac{0.5+1.5}{2} = 1.$

$t = 1$

$f(t) = e^{-1} - 3\log 1$ zero.

$= 0.3679.$

which is positive so. $a = t_1$

$a = \cancel{0.5}$ $b = 1.5$.

$t_1 = \frac{0.5+1}{2} = \cancel{0.25}$ 1.25

$f(t_1) = e^{1.25} - 3\log(1.25)$

$= \cancel{0.3768}$

which is positive so, $t_1 = b$.

$\therefore$ $a = 0.5$ $b = 0.75$.

$t_2 = \frac{a+b}{2} = \frac{0.5+0.75}{2}.$

$= 0.625$

$f(t_2) = 2.0728.$