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P&S Assignment - 1

1. Given Data: 4, 7, 9, 9, 11, 15, 18, 18, 18, 25
$n = 10$

$$\text{Mean} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= \frac{134}{10}$$

$$\boxed{\bar{x} = 13.4}$$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2} = \frac{11+15}{2} = 13$$

Mode = 18

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$\sigma = \sqrt{\frac{(-9.4)^2+(-6.4)^2+(-4.4)^2+(-4.4)^2+(-2.4)^2+(1.6)^2+(4.6)^2 + (4.6)^2+(4.6)^2+(11.6)^2}{9}}$$

$$\sigma = \sqrt{\frac{88.36+40.96+19.36+19.36+5.76+2.56+21.16+21.16+21.16+13}{9}}$$

$Q_2 = 13$ $Q_1 = 8$ $Q_3 =$

$$\sigma = \sqrt{41.648} = 6.4535$$

I.Q.R. = $Q_3 - Q_1 = 10$

ii.

No. of households	0	1	2	3	4
No. of people	5	11	26	15	3

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x	f	fx	$\bar{x}=2$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	5	0	2	4	20
1	11	11	2	1	11
2	26	52	2	0	0
3	15	45	2	1	15
4	3	12	2	4	12
	$\Sigma f = 60$	$\Sigma fx = 120$			$\Sigma f(x-\bar{x})^2 = 58$

Mean $= \bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{120}{60} = 2$

Median $= \dfrac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2} = \dfrac{2+2}{2} = 2$

Mode $= 2$

Standard Deviation $= \sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{n-1}} = \sqrt{\dfrac{58}{59}} = 0.99149$

$Q_1 = \dfrac{15^{th} + 16^{th}}{2} = \dfrac{1+1}{2} = 1$

$Q_2 = 2$

I.Q.R $= Q_3 - Q_1 = 2$

$Q_3 = \dfrac{3+3}{2} = 3$

② $X \sim P(0.5)$ $\quad \lambda = 0.5$

i. $P(X=0) = \dfrac{e^{-0.5}\, 0.5^{0}}{0!} = e^{-0.5} = 0.6065$

ii. Y- number of years with a claim

$Y \sim \text{binomial}\,(3, 0.3935)$ .... $(P(X \geq 1) = 1 - P(0) = 0.3935)$

$P(Y=1) = {}^{3}C_{1}\,(0.3935)^{1}\,(0.6065)^{2}$

$= 3(0.3935)(0.6065)$

$= 0.434$

iii. $X \sim \exp(0.5)$

$P(X > 2) = \displaystyle\int_{2}^{\infty} 0.5\, e^{-0.5(x)}\, dx$

$= \cancel{\tfrac{1}{2}}\, - \left[\dfrac{e^{-0.5x}}{\cancel{1/2}}\right]_{2}^{\infty}$

$\cancel{= e^{0} - e^{-1}}$ $\quad - (e^{-1}) = e^{-1}$

$\cancel{= \dfrac{e-1}{e}} =$ $\quad = 0.368$

③ $\alpha = 2$ $\quad \lambda = \dfrac{1}{2}$ $\quad 0 < x < \infty$

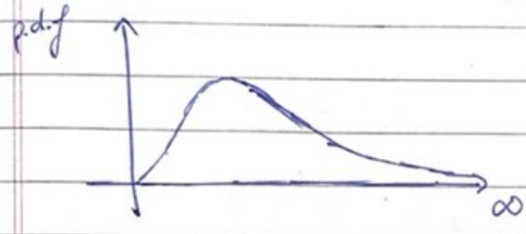
i. $X \sim \text{Gamma}\,(2, 1/2)$

Mean $= \cancel{\lambda = \tfrac{1}{2}} = \dfrac{\alpha}{\lambda} = \dfrac{2}{1/2} = 4$

Variance $= \dfrac{\alpha}{\lambda^{2}} = \dfrac{2}{1/4} = 8$

ii. $\alpha = 2 \qquad \lambda = 1/2$

p.d.f $= \int_0^{\infty} \frac{\cancel{\lambda^{\alpha}}\, x^{\alpha-1} e^{-\lambda x}}{\cancel{\lambda}\,\Gamma\lambda}$



as $\alpha = 2$ $\quad \underbrace{x^{\alpha-1}}_{} \lambda^{\alpha} e^{-\lambda x}$

increases in the beginning and then decreases.

iii

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma\alpha}$$

$$= \frac{\cancel{\lambda^{\alpha}} (\cancel{\Gamma\alpha})}{\cancel{\Gamma\alpha}} \frac{1}{\cancel{\lambda^{\alpha}}} = 1$$

c.d.f $\Rightarrow G(\alpha = 2, \lambda = \frac{1}{2}) = \begin{cases} 0 & , x < 0 \\ \frac{\lambda^{\alpha}}{\cancel{\lambda}\,\Gamma\alpha} \int_{0}^{x} x^{\alpha-1} e^{-\lambda x}\, dx & \end{cases}$

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(4) $m = 10$

$f = 8$

(5) $P(L_1) = 0.4 \quad P(L_2) = 0.0.5 \quad P(L_3) = 0.1$

$P(A|L_1) = 0.2 \quad P(A|L_2) = 0.4 \quad P(A|L_3) = 0.1$

$$P(L_2|A) = \frac{P(A|L_2)\cdot P(L_2)}{P(A|L_2)\cdot P(L_2) + P(A|L_1)\cdot P(L_1) + P(L_3)\cdot P(A|L_3)}$$

$$= \frac{0.4\times0.5}{0.4\times0.5 + 0.2\times0.4 + 0.1\times0.1}$$

$$= \frac{0.2}{0.2+0.08+0.01} = \frac{0.2}{0.29} = \frac{20}{29} = 0.6896$$

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(6) $X \sim U(1,2)$ $\qquad Y = \frac{3}{6-X}$

$F_Y(Y) = P(Y \le y)$

$F_Y(Y) = P\left(\frac{3}{6-x} \le y\right)$

~~$= P(3 \le y(6-x))$~~

$F_Y(Y) = P\left(\frac{6-x}{3} \ge y\right)$

$= P(6-x \ge 3y)$

$= P(x-6 \le -3y$

$F_Y(Y) = P(x \le 6-3y)$

$F_X(6-3y) = 6-3y$ $\qquad F_X(6-3y) = 6-3y$

$F_Y(y) = 6-3y$

$F_X(x) = \begin{cases} 0 & , x > 2 \\ 6-3x & , 1 < x \le 2 \end{cases}$ $\qquad F_Y(y) = \begin{cases} 0 & , y > 2 \\ 6-3y & , 1 < y \le 2 \end{cases}$

$f'_X(x) = \begin{cases} \\ -3 & , 1 < x \le 2 \end{cases}$

$f'_Y(y = \begin{cases} -3 & , 1 \le y \le 2 \end{cases}$

(7)

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

i. $F(3) = P(X \leq 3) = P(1) + P(2) + P(3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii. $E(x) = \sum_{0}^{5} x(p_x)$

$= 5(0.05) + 4(0.04) + 3(0.35) + 2(0.15) + 0.05$

$= 0.25 + \cancel{0.16}\ 1.6 + \cancel{0}\ 1.05 + 0.3 + 0.05$

$= 1.3 + 0.35 + 0.16$

$= 1.65 + \cancel{0.16}\ 1.6 = \cancel{1.81}\ 3.25$

iii. $V(x) = E(x^2) - (E(x))^2$

$= 0.05 + 0.6 + 3.15 + 6.4 + 1.25 - (3.25)^2$

$= 0.65 + 9.55 + 1.25 - 10.56$

$= 1.9 + 9.55 - 10.56$

$= 11.45 - 10.56$

$= 0.89$

iv. Coefficient of skewness $= E(x - \bar{x})^3$

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(8) $p = 0.2 \quad q = 0.8$

$x \sim \text{Binomial}(15, 0.2)$

$n = 15$

$P(x < 3) = P(0) + P(1) + P(2)$

$= {}^{15}C_0 p^0 q^{15} + {}^{15}C_1 p^1 q^{14} + {}^{15}C_2 p^2 q^{\cancel{8}13}$

$= 1(0.8)^{15} + 15(0.2)(0.8)^{14} + 105(0.2)^2(0.8)^{8}$

$= \cancel{0.035}\ 0.39802$

(9) $\cancel{P(x=7) = \left({}^{10-1}C \right.}$

$P(x=7) = {}^{6}C_2\, p^3 q^4$

$= \frac{\cancel{6}^{3} \times 5}{\cancel{2}} (0.01)^3 (0.99)^4$

$= 15(0.01)^3(0.99)^4$

$= 0.000014409$

(10) $\lambda = \frac{2}{5} = 0.4$

$x \sim P(0.4)$

$P(x > 2) = 1 - P(x \leq 2)$

$= 1 - 0.99207$

$= 0.00793$

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(11) $P(x=3) = \frac{1}{8}$

(12) $X \sim exp(10)$

$$P(x<0.1) = \int_0^{0.1} 10\,(e^{-10x})\,dx$$

$$= \cancel{10} \frac{\left[e^{-10x}\right]_0^{0.1}}{-\cancel{10}}$$

$$= -\left[e^{-1} - e^{-0}\right]$$

$$= \left[1 - e^{-1}\right] = 0.6321$$

(13) $P(80 < x < 100) = \int_{80}^{100} k\,dx$

$$= \int_{80}^{100} \frac{1}{120-75}\,dx$$

$$= \frac{1}{45}\left[x\right]_{80}^{100}$$

$$= \frac{20}{45} = \frac{4}{9}$$

(14) $X \sim gamma(5, 0.4)$ $\alpha = 5$ $\lambda = 0.4$

$X \sim gamma(\alpha, \lambda)$

$2\lambda x \sim \chi^2_{2\alpha = r}$

$0.8x \sim \chi^2_{10}$

$P(x < 22.89)$

$= P(2\lambda x < 2\lambda(22.89))$

$= P(\chi^2_{10} < 22.89 \times 0.8)$

$= P(\chi^2_{10} < 18.312)$

$18.0 \longrightarrow 0.945$

$18.5 \longrightarrow 0.9529$

$0.5 \longrightarrow 0.0079$

$1 \longrightarrow \dfrac{0.0079}{0.5}$

$0.31 \longrightarrow \dfrac{0.0079}{0.5} \times 0.31$

$18.31 \longrightarrow 0.945 + \dfrac{0.0079}{0.5} \times 0.31$

$18.31 \longrightarrow 0.949898$

$\therefore P(\chi^2_{10} < 18.312) = 0.949898.$

(15) $f(x) = \dfrac{k}{(x+5)^4}, \quad x > 0$ is p.d.f

1. We know that, $\displaystyle\int_{-\infty}^{\infty} \frac{k}{(x+5)^4}\, dx = 1$

$$1 = \int_0^{\infty} \frac{k}{(x+5)^4}\, dx$$

$$1 = \int_0^{\infty} k(x+5)^{-4}\, dx$$

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$$1 = k \left[\frac{(x+5)^{-3}}{-3}\right]_0^{\infty}$$

$$1 = \frac{k}{-3}\left(0 - 5^{-3}\right)$$

$$1 = \frac{k}{\cancel{-}3}\left(\frac{\cancel{-}1}{5^3}\right)$$

$$3(5^3) = k$$

$$\underline{k = 125 \times 3 = 375}$$

ii. $$F(10) = p(x \le 10) = \int_0^{10} f(x)\,dx$$

$$= \int_0^{10} \frac{375}{(x+5)^4}\,dx$$

$$= 375 \left[\frac{(x+5)^{-3}}{-3}\right]_0^{10}$$

$$= \frac{\overset{125}{\cancel{375}}}{\cancel{-3}}\left[15^{-3} - 5^{-3}\right]$$

$$= 125\left[5^{-3} - 15^{-3}\right]$$

$$= 125\left(\cancel{125}\ \frac{1}{125} - 0.000296296\right)$$

$$F(10) = 0.962963$$

iii. $$E(x) = \int_0^{\infty} x\, f(x)\,dx$$

$$= \int_0^{\infty} x \frac{375}{(x+5)^4}\,dx \quad \ldots \quad \cancel{(x+5)^4 = t}$$

$$\cancel{\frac{dt}{dx} = 3(x+5)^3}$$

$$\ldots \quad x+5 = t \qquad t^{-4}(t-5)$$

$$dt = dx \qquad t^{-3} - 5t^{-4}$$

$$x = t-5$$

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$$= \int_0^\infty (t-5)(375)(t)^{-4}\, dt$$

$$= 375 \int_0^\infty t^{-3} - 5t^{-4}\, dt$$

$$= 375 \left[\frac{t^{-2}}{-2} + \frac{5t^{-3}}{+3} \right]_0^\infty$$

$$= 375 \left(\frac{(x+5)^{-2}}{-2} + \frac{5(x+5)^{-3}}{+3} \right)_0^\infty$$

$$= \frac{375}{6} \left[-10(x+5)^{-3} + 3(x+5)^{-2} \right]_0^\infty$$

$$\cancel{= \frac{3\ 125}{2} \left(0 - 0 - 5^{-2} + 5^{-2} \right)}$$

$$\cancel{= 0}$$

$$= \frac{125}{2} \left(-2 \times 5^{-2} + 3 \times 5^{-2} \right)$$

$$= \frac{125}{2} \left(+1 \times 5^{-2} \right)$$

$$= \frac{\cancel{125}\ 5}{\cancel{25}(2)} = 2.5$$