Page No.:
Date:

Probability and Statistics Assignment

Q1 Given : Home insurance policy

Mean : 800

S.D : 100

Car insurance policy

Mean : 1200

S.D : 300

$P(x > 800)$ $= pq^{n-1}$

$= \frac{e^{-\lambda} \lambda^{x}}{x!}$ $= \cancel{1200}\ \cancel{800}^{\cancel{1200}-1}$

$=$

$: \frac{e^{-1200}\ 1200^{800}}{800!}$

Q2 1] MGF of Negative Binomial distribution

$\cancel{\left[\frac{pt}{1-qt}\right]}^{r}$ $(pe^{t} + q)^{n}$

MGF of gamma distribution

$\left[\frac{\lambda - t}{\lambda}\right]^{-\alpha}$

2] $Var(x) : M''x(t) - E(x^2)$

$Var(x) : M''(x)(t) - [M'x(t)]^2$

$S.D(x) : \sqrt{M''(x)(t) - [M'x(t)]^2}$

Page No.:
Date:

Q3 1] $f(x) = \lambda e^{-\lambda(x-a)}$

let $x = \lambda e^{-\lambda(x-a)}$

$M_X(t) = E(e^{tx})$

$M_X(t) = E(e^{t[\lambda e^{-\lambda(x-a)}]}$

2] $E(X) = \lambda e^{-\lambda(x-a)}$

$= \lambda$

$V(X) = \lambda$

since it is MGF of poission distribution

Q4 $g_V(t) = \left[\frac{p}{1-qt}\right]^K$

$g_V(t) = p^K (1-qt)^{-K}$

$g_V(t) = E(t^V)$

$= E(t^{p^K(1-qt)^{-K}})$

$E(t^{p^K}\ t^{(1-qt)^{-K}})$

$t^{(1-qt)-K}$ $\quad g_V \cancel{t} ^{PK}\ (t^{pk})$

$t^{(1-qt^{-K})}$ $\quad g_V(t)\ (t^P)$

$\therefore g_4(t)(t^P)$

Q5 $f(x,y) = ax(x+y)$

$P(y > \frac{1}{3}x + 10)$ $\quad E(y/x)$

$E(y/x) = \frac{P(x,y)}{P(y)}$

Page No. :
Date :

$$E(y/x) = \frac{ax(x+y)}{1/3\, x + 10}$$

$$P\left(y > \frac{1}{3}x + 10\right)$$

$$P\left(10 > \frac{1}{3}\, 5 + 10\right)$$

Q6 i] $X \sim Poi\ \lambda$ $\quad Y \sim Poi\ (\mu)$

$X - Y \sim Poi\ (\lambda - \mu)$

MGF of Poission distrubution is

$e^{\lambda(e^t - 1)}$

$\therefore X \sim Poi\ \lambda$

$M_x(t) = E(e^{tx})$

$M_x(t) = E(e^{t\lambda})$

$M_x(t) = E(e^t)(e^\lambda)$

$M_x(t) = e^\lambda E(e^t)$

$Y \sim Poi\ \mu$

$M_y(t) = E(e^{ty})$

$M_y(t) = E(e^{t\mu})$

$M_y(t) = E(e^t)(e^\mu)$

$M_y(t) = e^\mu E(e^t)$

$\therefore X - Y \sim Poi\ (\lambda - \mu)$

$= M_x(t) - M_y(t)$

$= e^\lambda \cancel{E(e^t)} - e^\mu \cancel{E(e^t)} = e^\lambda - e^\mu$

Page No. :
Date :

a) $x \sim Poi\ \lambda$

MGF :

$M_X(t) = E(e^{tx})$

$M_X(t) = E(e^{t\lambda})$

$M_X(t) = e^{\lambda}\ E(e^{t})$

$M_X(t) = e^{\lambda}\ E(e^{t})$

$y \sim Poi\ \mu$

MGF :

$M_Y(t) = E(e^{ty})$

$M_Y(t) = E(e^{t\mu})$

$M_Y(t) = e^{\mu}\ E(e^{t})$

$\therefore\ x + y \sim Poi\ (\lambda + \mu)$

$= M_X(t) + M_Y(t)$

$= e^{\lambda}\ E(e^{t}) + e^{\mu}\ E(e^{t})$

$= E(e^{t}) + (e^{\lambda} + e^{\mu})$

Q4 1] $Var\ (x / y = 2)$

$Var\ (x/y) = E\ (v\ (x/y) + v\ (E\ (x/y)))$

$= E\ (2 + v\ (E\ (x/y))$

2] $f\ (u, v) = \frac{48}{67}\ (2uv - u^2)$

$E\ (u/v = v)$

Page No. :
Date :

Q8 $x \sim \text{gamma}(3, 2)$

$E(y/x = x)\ 3x + 1$

$V(y/x = x) = 2x^2 + 5$

$E(x/y) = \sum_{x} P(X = x / y = y)$

$E(x/y) = 3x + 1\ P(X = x / y = y)$

$S.D = \sqrt{2x^2 + 5}$

$z = x + y$

Q9 $E(x) = 3$ $\quad E(y) = 4$

$S.D(x) = 2$ $\quad S.D(y) = 1$

i) $\text{cov}(x, z)$

$= E(xz) - E(x)\ E(z)$

$\text{Var}(z) = \text{Var}(x + y)$

Q10 $Kx^{-\alpha}\ e^{\frac{y}{\beta}}$

$$\int_0^\infty Kx^{-\alpha}\ e^{\frac{y}{\beta}}$$

$$= \left[\frac{Kx^{-\alpha+1}}{\alpha+1} \frac{e^{\frac{y}{\beta}+1}}{y/\beta+1} \right]_0^\infty$$

$= Kx\ e$

$K = \frac{e}{x}$