Name ____ R. No.____ Class / Div.____ Date 23-11-21

Subject Calculus Topic chp 1, 2, 3, 4 Page No. ____

Assignment 1

1. $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$\lim_{h \to 0} \dfrac{2(h^2 - 6h + 9) - 18}{h}$

$\lim_{h \to 0} \dfrac{2h^2 - 12h + 18 - 18}{h}$

$\lim_{h \to 0} \dfrac{h(2h - 12)}{h}$

$= 2(0) - 12$

$= -12$

2. To find if the function is continuous or Discontinuous :

a) at $x = 4$

step 1:- $g(4) = 2(4) = 8$

step 2:- $\lim_{x \to 4^-} (2x) = 2(4) = 8$ | $\lim_{x \to 4^+} (2x) = 2(4) = 8$

step 3:- $\lim_{x \to 4} = f(4)$

$\therefore$ ~~The function~~ $F(x)$ is continuous at $x = 4$

b) at $x = 6$

step 1:- $g(6) = 6 - 1 = 5$

step 2:- $\lim_{x \to 6^-} (2x) = 2(6) = 12$ | $\lim_{x \to 6^+} (x-1) = (6-1) = 5$

step 3:- $\lim_{x \to 6} \neq f(6)$

$\therefore F(x)$ is discontinuous at $x = 6$

3. $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{(3-\sqrt{x})(\cancel{\sqrt{}}3+\sqrt{x})}$

$\lim_{x \to 9} (-\sqrt{x} - 3)$

$= -\sqrt{9} - 3$

$= -6$

4. To find if the function is continuous or discontinuous:

a) at $x = -1$

step 1: $f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12} = 0.0833$

step 2: $\lim_{\cancel{f}x \to -1} = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12} = 0.0833$

step 3: $\lim_{x \to -1} = \cancel{\lim} f(-1)$

$\therefore$ The function is continuous

b) at $x = 0$

step 1: $f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9} = 0.556$

step 2: $\lim_{x \to 0} = \frac{4(x)+5}{9-3(x)} = \frac{4(0)+5}{9-3(0)} = \frac{5}{9} = 0.556$

step 3: $\lim_{x \to 0} = f(0)$

$\therefore$ The function is continuous

c) at $x = 3$

step 1: $f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{9-9} = \frac{17}{0} = 0$

~~step 2:~~ $\cancel{f \lim_{x \to 3} \frac{4x+5}{9-3x} = \lim_{x \to 3} \frac{3x+x+9-4}{3(3-x)} = \lim_{x \to 3} \frac{(3x+9)+(x-4)}{3(3-x)} = \lim_{x \to 3} \frac{(3(3+x)}{3(3}}$

step 2: $\lim_{x \to 3} \frac{4x+5}{-3x+9} = \left(\lim_{x \to 3} 4x+5\right) \lim_{x \to 3} \frac{1}{\cancel{5}-3x+9}$

$= \cancel{4(3)+5}\ 4(3)+5 \times \frac{1}{-3(3)+9} = 17\infty = \infty$

step 3: $\lim_{x \to 3} \neq f(3)$

$\therefore$ The Function is discontinuous

5. $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \infty$

6. a) $\lim_{y \to 6} g(y)$

$\lim_{y \to 6} 1 - 3y$

$= 1 - 3(6)$

$= -17$

b) $\lim_{y \to -2} g(y)$

$\lim_{y \to -2} 1 - 3y$

$= 1 - 3(-2)$

$= 7$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Diff w.r.t 't'

$F'(t) = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 4t^2(3t^2 - 16t) - t(3t^2 - 16t) + \cancel{8}\ 8t(t^3 - 8t^2 + 12) - 1(t^3 - 8t^2 + 12)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$

$= 12t^4 + 8t^4 - 64t^3 - 3t^3 - 64t^3 - t^3 + 16t^2 + 8t^2 + 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

diff w.r.t 'w'

$R'(w) = \dfrac{2w^2 + 1\frac{d}{dw}(3w + w^4) - 3w + w^4\frac{d}{dw}(2w^2 + 1)}{(2w^2 + 1)^2}$

$= \dfrac{(2w^2 + 1)(4w^3 + 3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$

$= \dfrac{2w^2(4w^3 + 3) + 1(4w^3 + 3) - (\cancel{3w}\ 12w^2 + 4w^5)}{(2w^2 + 1)^2}$

$= \dfrac{8w^5 + 6w^2 + 4w^3 + 3 - 12w^2 + 4w^5}{(2w^2 + 1)^2}$

$= \dfrac{12w^5 + 4w^3 - 6w^2 + 3}{(2w^2 + 1)^2}$

9) $F(x) = 2e^{x} - 8^{x}$

Diff w.r.t 'x'

$F'(x) = 2e^{x} - 8^{x} \log 8$

10) $y = z^{5} - e^{z} \log \ln(z)$

Diff w.r.t 'z'

$f'(z) = 5z^{4} - e^{z} \frac{d}{dx}(\log z) + \log z \frac{d}{dx}(e^{z})$

$= 5z^{4} - \left(e^{z} \cdot \frac{1}{z} + \log z \cdot e^{z}\right)$

$= 5z^{4} - \left(\frac{e^{z} + z\log z\, e^{z}}{z}\right)$

$= \frac{5z^{5} - e^{z} \cancel{+\log z} - z\log z\, e^{z}}{z}$

11) a) $F(x) = (6x^{2} + 7x)^{4}$

Diff w.r.t 'x'

$F'(x) = 4(6x^{2}+7x)^{3} \, d/dx \, (6x^{2}+7x)$

$= 4(6x^{2}+7x)^{3}(12x+7)$

b) $F(t) = 5 + e^{4t+t^{7}}$

Diff w.r.t 't'

$F'(t) = 0 + e^{4t+t^{7}} \, d/dt \, (4t + t^{7})$

$= e^{4t+t^{7}}(4 + 7t^{6})$

12) a) $z = \ln(7 - x^{3})$

Diff w.r.t 'x'

$F'(z) = \frac{1}{7-x^{3}} \, d/dx \, (7-x^{3})$

$= \frac{1}{7-x^{3}} \cdot (-3x^{2})$

$= \frac{-3x^{2}}{7-x^{3}}$

Again Diff w.r.t 'x'

$\cancel{F''(z)}\ F''(z) = \frac{(7-x^{3}) \, d/dx(-3x^{2}) - (-3x^{2}) \dots}{(7-x^{3})^{2}}$

$= \frac{(7-x^{3})(-6x) - (-3x^{2} \cdot -3x^{2})}{(7-x^{3})^{2}}$

$= \frac{(7-x^{3})(-6x) - (9x^{4})}{\cancel{7-3} \cancel{(7-x)} (7-x^{3})^{2}}$

b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

Diff w.r.t 'v'

$$Q'(v) = \frac{(6+2v-v^2)^4 \, d/dv(2) - 2 \, d/dv (6+2v-v^2)^4}{((6+2v-v^2)^4)^2}$$

$$= \frac{\cancel{-2(} -2(4(6+2v-v^2)^3) \, d/dv (6+2v-v^2)}{(6+2v-v^2)^8}$$

$$= \frac{-8((6+2v-v^2)^3 (2-2v))}{(6+2v-v^2)^8}$$

$$= -\frac{16(1-v)}{(6+2v-v^2)^5}$$

Diff w.r.t 'v'

$$Q''(v) = \cancel{\frac{-16(1-v)}{(6+2v-v^2)^5}} \quad -16 \frac{d}{dv}\left[\frac{v-1}{(v^2-2v-6)^5}\right]$$

$$= -16.\left[\frac{(v^2-2v-6)^5 \, d/dv (v-1) - (v-1) \, d/dv (v^2-2v-6)^5}{((v^2-2v-6)^5)^2}\right]$$

$$= -16\left[\frac{(v^2-2v-6)^5(1) - (v-1)(5(v^2-2v-6)^4 \, d/dv (v^2-2v-6}{(v^2-2v-6)^{10}}\right.$$

$$= -16\left[\frac{(v^2-2v-6)^5 - 5(v-1)(v^2-2v-6)^4(2v-2)}{(v^2-2v-6)^{10}}\right]$$

$$= \cancel{-16}\Big[= \frac{-16((v^2-2v-6)^5 - (5(v-1)(2v-2)(v^2-2v-6)^4))}{(v^2-2v-6)^{10}}$$

c) $F(t) = \ln(1+t^2)$

Diff w.r.t 't'

$$F'(t) = \frac{1}{1+t^2} \frac{d}{dt}(1+t^2)$$

$$= \frac{1}{1+t^2} \cdot (2t)$$

$$F'(t) = \frac{2t}{1+t^2}$$

Again Diff w.r.t 'x'

$$F''(t) = \frac{(1+t^2)\, d/dt\, (2t) - (2t)\, d/dt\, (1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

$$= \frac{2(1-t^2)}{(1+t^2)^2}$$

13) $F(x) = x^3 - 3x^2 - 9x + 12$

Diff w.r.t 'x'

$F'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$F'(x) = 3(x+1)(x-3)$

$\therefore x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad x = 3$

Again, Diff w.r.t 'x'

$F''(x) = 6x - 6$

$= 6(x-1)$

$F''(-1) = 6(-1-1) = 6(-2) = -12$ = Negative

F^4 ∴ The function is maxima

$F''(3) = 6(3-1) = 6(2) = 12$ = Positive

∴ The function is minima

~~∴ The maximum value is -17.~~

~~∴ The minimum value is 13.~~

$F(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

∴ The maximum value is 17

$F(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$

∴ The minimum value is -15

~~14~~

14) $F(x) = 4x^3 - 18x^2 + 24x - 7$

Diff w.r.t 'x'

$F'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x-1)(x-2)$

$\therefore x - 1 = 0$ or $x - 2 = 0$

$x = 1$ $\quad x = 2$

$F'(x) = 12x^2 - 36x + 24$

Again, Diff w.r.t 'x'

$F''(x) = 24x - 36$

$= 12(2x - 3)$

$F''(1) = 12(2(1) - 3) = 12(-1) = -12 =$ Negative

$\therefore$ The Function is maxima

$F''(2) = 12(2(2) - 3) = 12(1) = 12 =$ Positive

$\therefore$ The Function is minima

$\therefore F(x) = 4x^3 - 18x^2 + 24x - 7$

$\therefore F(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3$

$\therefore$ The maximum value is 3

$\therefore F(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7$

$= 1$

$\therefore$ The minimum value is 1

15) $F(x) = x^2(x+3)$

$F(x) = x^3 + 3x^2$ ~~Diff w.r.t 'x'~~

~~$F'(x) =$~~ Diff w.r.t 'x'

$F'(x) = 3x^2 + 6x$

Find Increasing or Decreasing

$3x^2 + 6x$ ~~$= 0$~~ $= 0$

$3x(x+6) = 0$

$\therefore$ $3x = 0$ or $x + 6 = 0$

$x = 0$ $x = -6$

$F'(-1) = 3 \times -1(-1+6) = -3(5) = -15$

Decreasing | Increasing | Decreasing

0   6

16) $F(x) = 3x^2 - 6x$

Diff w.r.t 'x'

$F'(x) = 6x - 6$

$= 6(x-1)$

$\therefore$ ~~$x =$~~ $x - 1 = 0 \rightarrow x = 1$

Decreasing | Increasing

6

Again Diff w.r.t 'x'

$F''(x) = 6$