

Homework 1Lagrange's Interpolation

- Q2. The function $y=f(x)$ is given by points (7,3) (8,1) (9,1) and (10,9). Find the value of y at $x=9.5$ using Lagrange's interpolation formula.

x	$f(x)$	$h=1$	value of $y=f(x)$ at $x=9.5$
7	3		$u = \frac{9.5-10}{1} = -0.5$
8	1		
9	1		
$x_n \leftarrow 10$	$\frac{9}{\downarrow}$		
	$\downarrow f(x_n)$		

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
7	3			
8	1	-2		
9	1	0	+2	
10	9	8	8	6
x_n	$\frac{9}{\downarrow}$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$

$$f(x) = f(x_n) + u \nabla f(x_n) + \frac{u(u-1)}{2!} \nabla^2 f(x) + \frac{u(u-1)(u-2)}{3!} \nabla^3 f(x)$$

$$= 9 + (-0.5)(8) + \frac{(-0.5)(-1.5)}{2} (8) + \frac{(-0.5)(-1.5)(-2.5)}{6} (6)$$

$$= 9 - 4 + 3 - 1.875$$

$$= 6.125$$

2. The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the independent variable. Find the estimate of the function at the position 6 of the independent variable.

x	y	$x_k = 6$	$f(x_k) = \frac{(6-7)(6-9)(6-10)(168)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)} + \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)}$
3	168		
7	120		
9	72		
10	63		

147

$$f(1)=4, f(2)=5, f(7)=5, f(8)=4$$

n	$f(n)$
-----	--------

1 4

2 5

7 5

8-4

$$= \frac{4 \cdot (-1) \cdot (+2)}{(-1) \cdot (+6) \cdot (-7)} + \frac{5 \cdot (+1) \cdot (+2)}{(1) \cdot (+5) \cdot (+6)} + \frac{1 \cdot (-1) \cdot (-2)}{(-1) \cdot (-6) \cdot (-5)}$$

$$\frac{(5)(4)(-2)}{(3)(8)(-1)} 5 + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$= \frac{-16}{21} + \frac{10}{6} + \frac{10}{3} - \frac{40}{21}$$

$$= -\frac{56}{21} + \frac{305}{6}$$

$$= 2.3333$$

4 Apply Lagrange's interpolation formula to find $f(5)$ given that $f(1)=2$, $f(2)=4$, $f(3)=8$, $f(4)=16$ and $f(7)=128$.

x $f(x)$ find $f(5)$

$$f(5) = \frac{(5-1)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} \\ + \frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)} \\ + \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}$$

$$= \frac{(3)(2)(1)(-2)(2)}{(1)(-2)(-3)(-6)} + \frac{(4)(2)(1)(-2)(4)}{(1)(1)(-2)(+5)}$$

$$+ \frac{(4)(3)(1)(-2)(8)}{(2)(1)(-1)(-4)} + \frac{(4)(3)(2)(+2)(16)}{(3)(2)(1)(+3)} + \frac{(128)(4)(3)(2)(1)}{(6)(5)(4)(3)}$$

$$= \frac{1}{3} + \frac{32}{5} + (-6) + \frac{64}{3} + \frac{128}{15}$$

$$= \underline{\underline{30.6}}$$

5) Apply Lagrange's formula to find $f(15)$ using the following values of function $f(x)$

x	10	12	14	16
$f(x)$	0.25	0.2	0.15	0.10

Find $f(15)$

$$f(15) = \frac{(15-12)(15-14)(15-16)(0.25)}{(10-12)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)(0.2)}{(12-10)(12-14)(12-16)} \\ + \frac{(15-10)(15-12)(15-16)(0.15)}{(14-10)(14-12)(14-16)} + \frac{(15-10)(15-12)(15-14)(0.10)}{(16-10)(16-12)(16-14)}$$

$$= \frac{(3)(1)(+1)}{(2)(+4)(+6)} \cdot 0.25 + \frac{(5)(1)(+1)}{(2)(+2)(-4)} \cdot 0.1 + \frac{(5)(3)(+1)}{4(2)(+2)} \cdot 0.15$$

$$+ \frac{(5)(3)(1)}{(6)(4)(2)} \cdot 0.1$$

$$= \frac{0.25}{16} + \left(\frac{-0.5}{8} \right) + \left(\frac{2.25}{16} \right) + \left(\frac{1.5}{48} \right)$$

$$= \underline{\underline{0.125}}$$

Homework-2 Regula falsi method.

a) $x^3 + x - 1 = 0$ lying between 0.5 and 1.

x	y
0.5	-0.375
1	1

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0.5(1) - (1)(-0.375)}{1 - (-0.375)}$$

$$= \frac{0.5 + 0.375}{1 + 0.375}$$

$$= \frac{0.875}{1.375} = 0.6364$$

$$\bar{y} = x^3 + x - 1$$

$$= (0.6364)^3 + 0.6364 - 1$$

$$= -0.1059$$

x	y
0.6364	-0.1059
1	1

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0.6364(1) - (1)(-0.1059)}{1 - (-0.1059)}$$

$$= \frac{0.6364 + 0.1059}{1 + 0.1059}$$

$\therefore 0.6816$ is the root of
 $x^3 + x - 1 = 0$.

$$\bar{y} = x^3 + x - 1$$

$$= (0.7009)^3 + 0.7009 - 1$$

$$= 0.0452$$

x	y
0.6364	-0.1059
0.7009	0.0452

$$\bar{x} = \frac{0.6364(0.0452) - (-0.1059)(0.7009)}{0.0452 - (-0.1059)}$$

$$= \frac{0.02876528 + 0.07422531}{0.0452 + 0.1059}$$

$$= \frac{0.10299059}{0.1511} = 0.6816$$

2. $x^3 - 2x + 0.5 = 0$ lying between 0.4 and 1.

x	y	$\bar{x} = x_1 y_2 - x_2 y_1$ $y_2 - y_1$
0	0.5	
1	-0.5	$\bar{x} = \frac{0(-0.5) - 1(0.5)}{-0.5 - 0.5}$

$$\bar{y} = \frac{x^3 - 2x + 0.5}{-1} = \frac{-0.5}{-1}$$

$$= \frac{(0.5)^3 - 2(0.5) + 0.5}{-1}$$

$$= -0.375$$

x	y	$\bar{x} = x_1 y_2 - x_2 y_1$ $y_2 - y_1$
0	0.5	
0.5	-0.375	$\bar{x} = \frac{0(-0.375) - (0.5)(0.5)}{-0.375 - 0.5}$

$$\bar{y} = \frac{x^3 - 2x + 0.5}{-0.875} = \frac{+0.25}{-0.875} = -0.2857$$

$$= \frac{(0.2857)^3 - 2(0.2857) + 0.5}{-0.875}$$

$$= -0.0480$$

x	y	$\bar{x} = x_1 y_2 - x_2 y_1$ $y_2 - y_1$
0	0.5	
0.2857	-0.0480	$\bar{x} = \frac{0(-0.0480) - (0.5)(0.2857)}{-0.0480 - 0.5}$

$$\bar{y} = \frac{x^3 - 2x + 0.5}{-0.5480} = \frac{+0.14285}{-0.5480} = -0.2607$$

$$= \frac{(0.2607)^3 - 2(0.2607) + 0.5}{-0.5480}$$

$$= -0.00368$$

$\therefore 0.2607$ is a root of $x^3 - 2x + 0.5 = 0$.

3 $x^3 + 2x - 20 = 0$ lying between 2 and 2.5.

x	y
2	-8
2.5	0.625

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{2(0.625) - (-8)(2.5)}{0.625 - (-8)}$$

$$= \frac{1.25 + 20}{8.625} = \frac{21.25}{8.625} = 2.4638.$$

$$y = (2.4638)^3 + 2(2.4638) - 20$$

$$= -0.1164$$

x	y
2.4638	-0.1164
2.5	0.625

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{2.4638(0.625) - 2.5(-0.1164)}{0.625 - (-0.1164)}$$

$$= \frac{1.539875 + 0.291}{0.7414} = 1.830875$$

$$y = (2.4695)^3 + 2(2.4695) - 20$$

$$= -0.000926497625 = 2.4694834$$

$$= 2.4695.$$

$\therefore x = 2.4695$ is a root of $x^3 + 2x - 20 = 0$.

4 $3x - \cos x - 2 = 0$ lying between 0 & 1.

x	y
0	-2
1	1.0002

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0(1.0002) - (-2)(1)}{1.0002 - (-2)}$$

$$= \frac{2}{3.0002} = 0.6666$$

$$y = 3(0.6666) - \cos(0.6666) - 1$$

$$= -0.00006564$$

$\therefore x \approx 0.6666$

5 $x^3 - 3x - 5$ lying between 2 and 2.5.

x	y
2	-3
2.5	3.125

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{2(3.125) - (-3)(2.5)}{3.125 - (-3)}$$

$$= \frac{6.25 + 7.5}{6.125} = \frac{13.75}{6.125} = 2.2449$$

$$\bar{y} = \frac{x^3 - 3x - 5}{x^3 - 3x - 5}$$

$$= \frac{(2.2449)^3 - 3(2.2449) - 5}{(2.2449)^3 - 3(2.2449) - 5}$$

$$= 1.8235 - 0.42184$$

x	y
2	-3
2.2449	1.8235

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{2(1.8235) - (-3)(2.2449)}{1.8235 - (-3)}$$

$$= \frac{3.647 + 6.7347}{4.8235} = \frac{10.3817}{4.8235} = 2.1523$$

$$\bar{y} = \frac{x^3 - 3(x) - 5}{x^3 - 3(x) - 5}$$

$$= \frac{(2.1523)^3 - 3(2.1523) - 5}{(2.1523)^3 - 3(2.1523) - 5}$$

$$= -1.4866$$

x	y
2.1523	-1.4866
2.2449	1.8235

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{2.1523(1.8235) - (-1.4866)(2.2449)}{1.8235 - (-1.4866)}$$

$$= \frac{3.92471905 + 3.33726839}{3.3101} = 2.1939$$

$$\bar{y} = \frac{x^3 - 3(x) - 5}{x^3 - 3(x) - 5}$$

$$= \frac{(2.1939)^3 - 3(2.1939) - 5}{(2.1939)^3 - 3(2.1939) - 5}$$

$$= -1.02202$$

x	y
2.1939	-1.0220
2.2449	1.8235

$$\bar{x} = \frac{(2.1939)(1.8235) - (-1.0220)(2.2449)}{1.8235 - (-1.0220)}$$

$$= \frac{4.00057665 + 2.2942878}{2.8455} = 2.2122$$

$$\bar{y} = \frac{(2.2122)^3 - 3(2.2122) - 5}{(2.2122)^3 - 3(2.2122) - 5}$$

$$= -0.8105$$

x	y
2.1939	-0.8105
2.2449	1.8235

$$\bar{x} = \frac{(2.1939)(1.8235) - (-0.8105)(2.2449)}{1.8235 - (-0.8105)}$$

$$= \frac{5.8200681}{2.634} = 2.2096$$

Wrong

$$\bar{y} = (2.2096)^3 - 3(2.2096) - 5$$

$$= -0.8408$$

$$\bar{x} = (2.2096)(1.8235) - (-0.8408)(2.2449)$$

$$1.8235 - (-0.8408)$$

$$\begin{array}{c|c} x & y \\ \hline 2.2096 & -0.8408 \\ 2.2449 & 1.8235 \end{array}$$

$$= \frac{5.91671752}{2.6643} = 2.2207$$

$$2.2449$$

$$1.8235$$

$$= \frac{5.91671752}{2.6643} = 2.2207$$

$$2.6643$$

$$x^3 - 3x - 5$$

$$\bar{y} = (2.2207)^3 - 3(2.2207) - 5$$

$$= -0.7107$$

$$\begin{array}{c|c} x & y \\ \hline 2.2449 & -0.4214 \\ 2.5 & 3.125 \end{array}$$

$$2.2449$$

$$-0.4214$$

$$2.5$$

$$3.125$$

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$y_2 - y_1$$

$$= \frac{2.2449(3.125) - (-0.4214)(2.5)}{3.125 + 0.4214}$$

$$3.125 + 0.4214$$

$$= \frac{8.0688125}{3.5464}$$

$$3.5464$$

$$\bar{y} = x^3 - 3x - 5$$

$$= (2.2752)^3 - 3(2.2752) - 5$$

$$= -0.0479$$

$$= 2.2752$$

$$\begin{array}{c|c} x & y \\ \hline 2.2752 & -0.0479 \\ 2.5 & 3.125 \end{array}$$

$$2.2752$$

$$-0.0479$$

$$2.5$$

$$3.125$$

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$y_2 - y_1$$

$$= \frac{2.2752(3.125) - (-0.0479 \times 2.5)}{3.125 + 0.0479}$$

$$3.125 + 0.0479$$

$$\bar{y} = x^3 - 3x - 5$$

$$= (2.2786)^3 - 3(2.2786) - 5$$

$$= -0.0052$$

$$= \frac{7.22975}{3.1729} = 2.2786$$

$$3.1729$$

$$\boxed{x \approx 2.2786}$$

$$x \log_{10} x - 1.2 = 0 \text{ lying between 2 \& 3}$$

x	y	$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$
2	-0.5979	$\frac{2(0.2314) - (-0.5979)(3)}{0.2314 - (-0.5979)}$
3	0.2314	$= \frac{2.2565}{0.8293} = 2.7210$

$$\begin{aligned} \bar{y} &= x \log_{10} x - 1.2 \\ &= 2.7210 \log_{10} (2.7210) - 1.2 \\ &= -0.0171 \end{aligned}$$

x	y	$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$
2.7210	-0.0171	$\frac{2.7210(0.2314) - (-0.0171)(3)}{0.2314 - (-0.0171)}$
3	0.2314	$= \frac{0.6809394}{0.2485} = 2.7402$

$$\begin{aligned} \bar{y} &= x \log_{10} x - 1.2 \\ &= 2.7402 \log_{10} 2.7402 - 1.2 \\ &= -0.000389 \end{aligned}$$

$$\therefore x \approx 2.7402$$

$$x^3 - 4x + 1 = 0 \quad \bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0(-2) - 1(1)}{-2 - 1} = \frac{-1}{-3} = \frac{1}{3} = 0.3333$$

x	y
0	1
1	-2

$$\begin{aligned} \bar{y} &= x^3 - 4x + 1 \\ &= (0.3333)^3 - 4(0.3333) + 1 \\ &= -0.2962 \end{aligned}$$

x	y
0	1
0.3333	-0.2962

$$\begin{aligned} \bar{x} &= \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0(-0.2962) - 1(0.3333)}{-0.2962 - 1} \\ &= \frac{-0.3333}{-1.2962} = 0.2571 \end{aligned}$$

$$\begin{aligned} \bar{y} &= x^3 - 4x + 1 \\ &= (0.2571)^3 - 4(0.2571) + 1 \\ &= -0.0112 \end{aligned}$$

x	y
0	1
0.2571	-0.0114

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0(1) - 1(0.2571)}{-0.0114 - 1} = \frac{-0.2571}{-1.0114} = 0.2542$$

$$\begin{aligned}\bar{y} &= x^3 - 4x + 1 \\ &= (0.2542)^3 - 4(0.2542) + 1 \\ &= -0.00037\end{aligned}$$

$$\therefore x \approx 0.2542$$

$$8 \quad x^3 + 2x^2 - 8 = 0$$

$$8 \quad x^3 + 2x^2 - 8 = 0$$

x	y
0	-8
1	-5
2	8

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1(8) - (-5)(2)}{8 - (-5)} = \frac{8 + 10}{13} = \frac{18}{13} = 1.3846$$

$$\bar{y} = x^3 + 2x^2 - 8$$

$$= (1.3846)^3 + 2(1.3846)^2 - 8$$

$$= -1.5113$$

x	y
1.3846	-1.5113
2	8

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{(1.3846)8 - (2)(-1.5113)}{8 - (-1.5113)}$$

$$= \frac{14.0994}{9.5113} = 1.4824$$

$$\begin{aligned}\bar{y} &= x^3 + 2x^2 - 8 \\ &= (1.4824)^3 + 2(1.4824)^2 - 8 \\ &= -0.3474\end{aligned}$$

x	y
1.4824	-0.3474
2	8

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.4824(8) - (2)(-0.3474)}{8 - (-0.3474)}$$

$$= \frac{12.554}{8.3474} = 1.5039$$

$$\bar{y} = x^3 + 2x^2 - 8$$

$$(1.5039)^3 + 2(1.5039)^2 - 8 = -0.0752$$

x	y
1.5039	-0.0752
2	8

$$\bar{x} = \frac{1.5039(8) - (-0.0752 \times 2)}{8 + 0.0752}$$

$$= \frac{12.1816}{8.0752} = 1.5085$$

$$\bar{y} = (1.5085)^3 + 2(1.5085)^2 - 8$$

$$= -0.0162$$

x	y
1.5085	-0.0162
2	8

$$\bar{x} = \frac{1.5085(8) - (-0.0162 \times 2)}{8 + 0.0162}$$

$$= \frac{12.1004}{8.0162} = 1.5095$$

$$\bar{y} = (1.5095)^3 + 2(1.5095)^2 - 8$$

$$= -0.0032$$

$$\therefore x \approx 1.5095$$

9 $x^3 - 9x + 1 = 0$

x	y
0	1
1	-7

$$\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1} = \frac{0(-7) - (1)(1)}{-7 - 1} = \frac{-1}{-8} = \frac{1}{8} = 0.125$$

$$\bar{y} = x^3 - 9x + 1$$

$$= (0.125)^3 - 9(0.125) + 1$$

$$= -0.1230$$

x	y
0	1
0.125	-0.1230

$$\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1} = \frac{0(-0.1230) - (1)(0.125)}{-0.1230 - 1}$$

$$= \frac{-0.125}{-1.1230} = 0.1113$$

$$\bar{y} = x^3 - 9x + 1$$

$$= (0.1113)^3 - 9(0.1113) + 1$$

$$= -0.0003$$

$$\therefore x \approx 0.1113$$

$$10 \quad x^3 - x - 4 = 0$$

x	y
0	-4
1	-4
2	2

$$\bar{x} = \frac{1(2) - 4(2)}{2+4} = \frac{2+8}{6} = \frac{10}{6} = 1.6667$$

$$\bar{y} = x^3 - x - 4$$

$$= (1.6667)^3 - (1.6667) - 4$$

$$= -1.0368$$

x	y
2	-1.0368
1.6667	-1.0368
2	2

$$\bar{x} = \frac{1.6667(2) - (-1.0368 \times 2)}{2+1.0368} = \frac{5.407}{3.0368}$$

$$= 1.7805$$

$$\bar{y} = x^3 - x - 4$$

$$= (1.7805)^3 - 1.7805 - 4$$

$$= -0.1360$$

x	y
1.7805	-0.1360
2	-0.1360
2	2

$$\bar{x} = \frac{1.7805(2) - (-0.1360 \times 2)}{2+0.1360} = \frac{3.833}{2.1360}$$

$$= 1.7945$$

$$\bar{y} = x^3 - x - 4$$

$$= (1.7945)^3 - 1.7945 - 4$$

$$= -0.0158$$

x	y
1.7945	-0.0158
2	-0.0158
2	2

$$\bar{x} = \frac{1.7945(2) - (0.0158 \times 2)}{2+0.0158} = \frac{3.6206}{2.0158} = 1.7961$$

$$y = x^3 - x - 4$$

$$= (1.7961)^3 - 1.7961 - 4$$

$$= -0.0019$$

$$\therefore x \approx 1.7961$$