

Ques ①

Let x_i be claim size of on home insurance

y_i " " " " car insurance

so, ATQ

$$X \sim N(800, 100^2) \quad ; \quad P(\cancel{x_1 + x_2 + x_3 + x_4})$$

$$Y \sim N(1200, 300^2) \quad ;$$

~~Ques 800~~ $P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$$P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 800)$$

$$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3 \times 1200 - 4 \times 800, 3 \times 300^2 + 4 \times 100^2)$$
$$\sim N(400, 310000)$$

~~Ques 800~~ $P(Z > \frac{800 - 400}{\sqrt{310000}})$

$$P(Z > 0.71842) \Rightarrow 1 - P(Z < 0.71842)$$

$$1 - P(Z < 0.72)$$

$$1 - 0.76424$$

$$\Rightarrow \boxed{0.23576}$$

Ques (2)

$N \sim \text{Neg. Binomial}(k, p)$

$$P(N=n) = \binom{n+2}{n} (0.9)^3 (0.1)^n$$

$$\Rightarrow \binom{(n+3)-1}{n-1} (0.9)^3 (0.1)^n$$

So, $k=3$, $p=0.9$

$$M_{Y,t} = M_N(\log M_{X,t})$$

$X \sim \text{Gamma}(6, 2)$

$Y \rightarrow$ Total claim amount

(i) MGF of Y

$$M_{Y,t} = E(e^{Yt} / N=n)$$

$$E(e^{Yt} / N=n) = E(e^{x_1 t + x_2 t + \dots + x_n t})$$

$$= E(e^{tx_1}) \cdot E(e^{tx_2}) \dots E(e^{tx_n})$$

$$\Rightarrow \left(1 - \frac{t}{2}\right)^{-6n}$$

$$\text{So, } E(e^{Yt} / N=n) = E\left(\left(1 - \frac{t}{2}\right)^{-6n}\right)$$

$$= E\left(e^{n \log \left(1 - \frac{t}{2}\right)^{-6}}\right)$$

$$= M_N(\log \left(1 - \frac{t}{2}\right)^{-6})$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$$

$$\Rightarrow \left(\frac{pe^{\log \left(1 - \frac{t}{2}\right)^{-6}}}{1 - qe^{\log \left(1 - \frac{t}{2}\right)^{-6}}}\right)^k$$

$$\Rightarrow \left(\frac{(0.9) \left(1 - \frac{t}{2}\right)^{-6}}{1 - 0.1 \left(1 - \frac{t}{2}\right)^{-6}}\right)^3$$

Q. $V(Y) = V(X)$

$$V(Y) = E(N) V(X) + (E(X))^2 \cdot V(N)$$

$$\Rightarrow \left[\frac{3}{0.9} + \frac{6}{4} \right] + \left(\frac{3.6}{2} \right)^2 \times \frac{(3)(0.1)}{(0.9)^2}$$

$V(X) =$

St. deviation $\rightarrow 2.887$.

Ques (3) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

MGF = $E(e^{tx}) \Rightarrow \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$

$$M_x t = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda x} dx \Rightarrow e^{\lambda \alpha} \lambda \int_{\alpha}^{\infty} e^{-(\lambda-t)x} dx$$

$$\frac{\lambda}{t-\lambda} \left[e^{-(\lambda-t)x} \right]_{\alpha}^{\infty} \Rightarrow \frac{e^{\lambda \alpha}}{t-\lambda} \left[-e^{-(\lambda-t)x} \right]$$

$$\frac{e^{\lambda \alpha}}{\lambda-t} \left[e^{-(\lambda-t)x} \right]_{\alpha}^{\infty} \Rightarrow e^{\lambda \alpha} \frac{1}{1-t}$$

(ii) $M_x t = e^{\lambda \alpha} \frac{1}{1-t} \Rightarrow E(X) = M_x'(t) \Rightarrow \lambda e^{\lambda \alpha} (1-t)$

$$\lambda \left[\lambda e^{\lambda \alpha} (1-t) + e^{\lambda \alpha} (-1) \right]$$

$$\Rightarrow \lambda \left[\lambda e^{\lambda \alpha} (1-t) - e^{\lambda \alpha} \right]$$

Put $t=0$ we get: $\lambda^2 \alpha - \lambda \Rightarrow \lambda(\lambda \alpha - 1)$

(ii)

$$M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda - t} \Rightarrow \lambda e^{t\alpha} (\lambda - t)^{-1}$$

$$M_x'(t) = \lambda \left[e^{t\alpha} (\lambda - t)^{-2} (\alpha) + (\lambda - t)^{-1} e^{t\alpha} (\alpha) \right]$$

$$\Rightarrow \lambda \left[(\lambda - t)^{-1} e^{t\alpha} (\alpha) + e^{t\alpha} (\lambda - t)^{-2} \right]$$

$$\Rightarrow \lambda e^{t\alpha} (\lambda - t)^{-2} [\alpha (\lambda - t) + 1]$$

t=0

$$\Rightarrow \lambda (\lambda)^{-2} [\alpha (\lambda) + 1]$$

$$\alpha + \frac{1}{\lambda}$$

$$\Rightarrow \lambda \frac{1}{\lambda} [\lambda \alpha + 1] \Rightarrow \frac{\lambda \alpha + 1}{1}$$

$$E(x) = M_x''(t) \Rightarrow \lambda \left[\alpha e^{t\alpha} (\lambda - t)^{-2} + \alpha^2 (\lambda - t)^{-1} e^{t\alpha} + \alpha e^{t\alpha} (\alpha + 2) (\lambda - t)^{-3} + (\lambda - t)^{-2} e^{t\alpha} \right]$$

$$\lambda \left[\alpha e^{t\alpha} \right] \text{ put } t=0$$

$$\lambda \left[\alpha (\lambda)^{-2} + \alpha^2 (\lambda)^{-1} + (2) (\lambda)^{-3} + \alpha (\lambda)^{-2} \right]$$

$$\lambda \left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3} \right]$$

$$E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(x) = E(x^2) - [E(x)]^2 \Rightarrow \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\frac{\alpha + 1}{\lambda} \right]^2$$

$$2\alpha \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left(\alpha^2 + \frac{1}{\lambda^2} + \frac{2\alpha}{\lambda} \right)$$

$$\frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \alpha^2 - \frac{1}{\lambda^2} - \frac{2\alpha}{\lambda} \Rightarrow \boxed{\frac{1}{\lambda^2}}$$

Ques (4) $q_v(t) = \left(\frac{p}{1-q}\right)^k$

$q_v(t) = \sum t^v \times P(V=v)$

~~P(B)~~ $P(V=4) \Rightarrow \frac{\sqrt{k+4}}{\sqrt{5} \sqrt{k}} p^k q^k$

$= \frac{\sqrt{k+4}}{4 \sqrt{k}} p^k q^k$

$\Rightarrow \frac{(k+3)!}{(k-1)! 4!} p^k (1-p)^k$

Ques (5) $f(x,y) = ax(x+y)$ $0 < x < 5, 10 < y < 20$

(i) $P\left(y > \frac{1}{3}x + 10\right)$

$a \int_{10}^{20} \int_0^5 (x^2 + xy) dx dy = 1$

$a \left(\frac{2500}{3} + 2500 - \frac{1250}{3} - 625 \right)$

$a \left(\frac{1250}{3} + 1875 \right) = 1$

$a \left(\frac{6875}{3} \right) = 1$

$a = 0.000436$

$a = \frac{3}{6875}$

$\int_{10}^{20} \left(\frac{x^3}{3} + \frac{x^2 y}{2} \right) dy = \frac{1}{a}$

$\int_{10}^{20} \left(\frac{125}{3} + \frac{25}{2}y \right) dy = \frac{1}{a} \Rightarrow \left(\frac{125}{3}y + \frac{25}{4}y^2 \right) \Big|_{10}^{20} = \frac{1}{a}$

$833.33 + 2500 - (416.667 + 625) = \frac{1}{a}$

$a = 0.00043636$

(ii)

$$E\left(\frac{Y}{X}\right)$$

$$E\left(\frac{Y}{X}\right)$$

$$f_{Y/X} = \frac{f(x,y)}{f(x)} \Rightarrow$$

$$f(x) = \frac{3}{6875} \int_{10}^{20} (x^2 + xy) dy \Rightarrow \frac{3}{6875} \left[x^2 y + \frac{xy^2}{2} \right]_{10}^{20}$$

$$\Rightarrow \frac{3}{6875} [20x^2 + x(900) - 10x^2 - 50x]$$

$$= \frac{3}{6875} [10x^2 + 150x] \Rightarrow \frac{30x}{6875} [x+15]$$

$$f_{Y/X} = \frac{\frac{3}{6875} \frac{x(x+y)}{30x} \times 6875(x+15)}{\frac{3}{6875} \frac{x(x+y)}{30x} \times 6875(x+15)} = \frac{x(x+y)}{30x(x+15)}$$

$$f_{Y/X} \Rightarrow \frac{(x+y)(x+15)}{10} \quad f_{Y/X} = \frac{(x+y)}{10(x+15)}$$

$$So, E\left(\frac{Y}{X}\right) = \int_{10}^{20} y \cdot f_{Y/X} dy$$

$$\frac{30x(x+15)}{6875} \left(\frac{x+15}{10} \right) \int_{10}^{20} (x+y)(y) dy = \frac{x+15}{10} \left(xy + \frac{y^2}{2} \right)_{10}^{20}$$

$$\frac{x+15}{10} [20x + 400 - 10x - 100]$$

$$\frac{x+15}{10} [10x + 300] \Rightarrow (x+15)(x+30)$$

$$so, E(Y|X) = \int_{10}^{20} y \cdot \frac{(x+y)}{10(x+15)} dy \Rightarrow \frac{1}{10(x+15)} \int_{10}^{20} (xy + y^2) dy$$

$$\Rightarrow \frac{1}{10(x+15)} \left[\frac{xy^2}{2} + \frac{y^3}{3} \right]_{10}^{20} \Rightarrow \frac{1}{10(x+15)} \left[\frac{200x + 8000}{3} - \frac{50x}{3} \right]$$

$$\frac{1}{10(x+15)} \left[150x + \frac{7000}{3} \right]$$

$$\frac{1}{x+15} \left[\frac{150x + 700}{3} \right]$$

$$\frac{20 \times 20 \times 20}{3}$$

$$\frac{8000}{3}$$

$$(ii) P(Y > \frac{1}{3}x + 10)$$

$$f(y) = \frac{3}{6875} \int_0^5 (x^2 + xy) dx \Rightarrow \frac{3}{6875} \left[\frac{x^3}{3} + \frac{x^2 y}{2} \right]_0^5$$

$$f(y) \Rightarrow \frac{3}{6875} \left[\frac{125}{3} + \frac{25y}{2} \right]$$

$$\Rightarrow \frac{20}{6875}$$

$$so, \frac{3}{6875} \int_{\frac{1}{3}x+10}^{20} \left(\frac{125}{3} + \frac{25y}{2} \right) dy \Rightarrow \frac{3}{6875} \left[\frac{125y}{3} + \frac{25y^2}{4} \right]_{\frac{1}{3}x+10}^{20}$$

$$\Rightarrow \frac{3}{6875} \left[\frac{125(20)}{3} + \frac{25(400)}{4} - \frac{125}{3} \left(\frac{1}{3}x + 10 \right) - \frac{25}{4} \left(\frac{1}{3}x + 10 \right)^2 \right]$$

$$\frac{3}{6875} \left[\frac{2500}{3} + 2500 - \frac{125x}{9} - 1250 - \frac{25}{4} \left(\frac{1}{9}x^2 + \frac{10}{3}x + 100 \right) \right]$$

$$\frac{3}{6875} \left[\frac{2500}{3} + 2500 - \frac{125x}{9} - 1250 - \frac{25x^2}{36} - 625 - \frac{25x}{3} \right] \Rightarrow \frac{3}{6875} \left[\frac{1250}{3} + 1150 - \frac{125x}{9} - \frac{25x^2}{36} - \frac{25x}{3} \right]$$

Sol (6)

$$X \sim \text{Poi}(1)$$

$$Y \sim \text{Poi}(4)$$

$$M_X(t) = e^{1(e^t-1)}$$

$$M_Y(t) = e^{4(e^t-1)}$$

So, $Z = X + Y$

$$M_Z(t) = E(e^{tz}) = E(e^{t(X+Y)}) = E(e^{tX} \cdot e^{tY})$$

$$= E(e^{tX}) \times E(e^{tY}) \Rightarrow \frac{E(e^{tX})}{E(e^{tY})}$$

$$\Rightarrow \frac{M_X(t) \cdot M_Y(-t)}{e^{1(e^t-1)} \cdot e^{4(e^{-t}-1)}} \Rightarrow \frac{e^{1(e^t-1)}}{e^{4(e^t-1)}}$$

$$\Rightarrow e^{1e^t-1+4e^{-t}-4} \Rightarrow e^{(1-4)(e^t-1)}$$

$$\Rightarrow e^{-(1+4)} \Rightarrow e^{-5}$$

So, By uniqueness MBF of $Z = e^{(1-4)(e^t-1)}$
 $\sim \text{Poi}(1-4)$

Now: let $Z = X + Y$

$$E(e^{tz}) = e^{1(e^t-1)} \times e^{4(e^t-1)}$$

$$\Rightarrow e^{(1+4)(e^t-1)}$$

So, By uniqueness $Z \sim \text{Poi}(1+4)$

Ans (7)

		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2
					2

$$(i) V(X|Y=2) = E(X^2|Y=2) - [E(X|Y=2)]^2$$

$$\text{Now: } E(X^2|Y=2) \Rightarrow \sum_x x^2 \frac{P(X=x, Y=2)}{P(Y=2)}$$

when $x=1$ when $x=2$ when $x=3$ when $x=4$

$$\frac{(1)^2 P(X=1, Y=2)}{P(Y=2)} + \frac{(2)^2 P(X=2, Y=2)}{P(Y=2)} + \frac{(3)^2 P(X=3, Y=2)}{P(Y=2)} + \frac{(4)^2 P(X=4, Y=2)}{P(Y=2)}$$

$$\Rightarrow \frac{(1)(0)}{0.6} + \frac{(4)(0.3)}{0.6} + \frac{(9)(0.1)}{0.6} + \frac{16(0.2)}{0.6}$$

$$\Rightarrow 8.8333$$

$$\text{Now: } E(X|Y=2) \Rightarrow \sum_x x \frac{P(X=x, Y=2)}{P(Y=2)}$$

$$\frac{(1)(0)}{0.6} + \frac{(2)(0.3)}{0.6} + \frac{(3)(0.1)}{0.6} + \frac{4(0.2)}{0.6}$$

$$\Rightarrow \frac{0.6 + 0.3 + 0.8}{0.6} \Rightarrow 2.8333$$

$$\Rightarrow \boxed{0.807}$$

Ques

ii)

$$f(u, v) = \frac{48}{67} (2uv - u^2)$$

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$$0 < u < 1; \frac{u}{2} < v < 2$$

$$E(u/v=v) = ?$$

$$\frac{f(u, v)}{f(v)} \Rightarrow \int \frac{f(u, v)}{f(v)} du$$

$$f(v) \Rightarrow \frac{48}{67} \int_0^1 (2uv - u^2) du \Rightarrow \frac{48}{67} \left[\frac{2u^2v}{2} - \frac{u^3}{3} \right]_0^1$$

$$\frac{48}{67} \left[v - \frac{1}{3} \right] \Rightarrow \frac{16}{67} [3v - 1]$$

$$f(v) = \frac{16}{67} [3v - 1]$$

$$\frac{f(u, v)}{f(v)} = \frac{3 \cdot \frac{48}{67} (2uv - u^2) \times 67}{67 \times 16 (3v - 1)}$$

$$\Rightarrow \frac{3}{3v-1} (2uv - u^2) \quad 2u^2v - u^3$$

Now: $E(u/v=v) = \frac{3}{3v-1} \int_0^1 u (2uv - u^2) du$

$$\frac{3}{3v-1} \left[\frac{2u^3v}{3} - \frac{u^4}{4} \right]_0^1 \Rightarrow \frac{3}{3v-1} \left[\frac{2v^2}{3} - \frac{1}{4} \right]$$

$$\frac{3}{3v-1} \left[\frac{8v^2-3}{12} \right] \Rightarrow \frac{8v^2-3}{4(3v-1)}$$

Q8..

$X \sim \text{Gamma}(3, 2)$

$$E(X) = \frac{3}{2}$$

$$V(X) = \frac{3}{4}$$

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$$E(Y|X=x) = 3x + 1$$

$$\text{Var}(Y|X=x) = 2x^2 + 5$$

$$E(Y) = E(E(Y|X=x)) \Rightarrow E(3x+1)$$

$$= 3E(X) + 1$$

$$\Rightarrow 3\left(\frac{3}{2}\right) + 1 \Rightarrow \frac{9}{2} + 1 = \frac{9+2}{2}$$

$$= \frac{11}{2}$$

$$\bullet V(Y) = E(V(X|Y)) + V(E(X|Y))$$

$$\Rightarrow E(2x^2 + 5) + V(3x+1)$$

$$= 2E(x^2) + 5 + 9V(x)$$

$$\bullet V(X) = E(x^2) - [E(X)]^2$$

$$\frac{3}{4} + \frac{9}{4} = E(x^2)$$

$$\frac{12}{4} = E(x^2) \Rightarrow \boxed{E(x^2) = 3}$$

$$V(Y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$$

$$= 11 + \frac{27}{4} = \frac{44+27}{4}$$

$$= \boxed{\frac{71}{4}}$$

Ques 9) $E(x) = 3$, $V(x) = 4$.

$$E(y) = 4$$
 , $V(y) = 1$

Correlation coeff. $\text{Corr}(x, y) = -0.3\sqrt{4} \Rightarrow \text{Cov}(x, y)$
 $= -0.6$

$$Z = x + y .$$

(i) $\text{Cov}(x, x+y) = \text{Cov}(x, x) + \text{Cov}(x, y)$

$$= V(x) + \text{Cov}(x, y)$$

$$\Rightarrow 4 + (-0.6)$$

$$= 3.4$$

(ii) $\text{Var}(Z) = \text{Cov}(x+y, x+y)$

$$= \text{Cov}(x, x) + 2\text{Cov}(x, y) + \text{Cov}(y, y)$$

$$= V(x) + 2\text{Cov}(x, y) + V(y)$$

$$\Rightarrow 4 + 2(-0.6) + 1$$

$$\Rightarrow 5 - 1.2$$

$$\Rightarrow 3.8$$

Q10

$$f(x, y) = k x^{-\alpha} e^{-y/\beta}$$

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$$k \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta} dx dy = 1$$

$1 < x < \infty$, $1 < y < \infty$
 $\alpha > 1$, $\beta > 0$.

$$k \int_1^{\infty} e^{-y/\beta} \left[\frac{x^{-(\alpha-1)}}{-(\alpha-1)} \right]_1^{\infty} dy = 1$$

$$\frac{k}{1-\alpha} \int_1^{\infty} e^{-y/\beta} [-1]_1^{\infty} dy \Rightarrow \frac{k}{\alpha-1} \int_1^{\infty} e^{-y/\beta} dy = 1$$

$$\frac{k}{\alpha-1} \left[\frac{e^{-y/\beta}}{(-1/\beta)} \right]_1^{\infty} = \frac{k}{\alpha-1} (\beta) (1 - e^{-1/\beta}) = 1$$

$$k = \frac{\alpha-1}{\beta (1 - e^{-1/\beta})}$$