

Ans-1 Actual length $= x_1 = 12.5 \text{ cm}$
 Actual area $= \pi x_1^2$
 $= \pi (12.5)^2 = 490.87365 \text{ cm}^2$

$x_2 = 12.65 \text{ cm}$

Approximate area $= \pi x_2^2$
 $= \pi (12.65)^2$
 $= 502.72551 \text{ cm}^2$

i) Absolute Error $=$ Approximate Area $-$ Actual Area
 $= 502.72551 - 490.87365$
 $= 11.84886 \text{ cm}^2$

ii) Relative Error $= \frac{\text{Absolute Error}}{\text{Actual Error}} = \frac{11.84886}{490.87365}$
 $= 0.0241383033$

iii) Percentage Error $=$ Relative Error $\times 100$
 $= 2.41383033\%$

Ans-2 i) $x_1 = 4$
 $y_1 = 0.60206$

$x_2 = 6$

$y_2 = 0.7781513$

$$x_2 - x_1 = y_2 - y_1 \quad (\because \text{Linear Interpolation})$$

$$6 - 4 = \cancel{y_2 - y_1} \quad 0.7781513 - 0.60206$$

$$\text{Change} = \frac{0.1760913}{2}$$

$$\text{Change} = 0.08804565$$

~~Figure~~ $\therefore x = 5$

$$y = 0.60206 + 0.08804565$$

$$= 0.69010565$$

ii) $x_1 = 4.5$

$$y_1 = 0.6532125$$

$$x_2 = 5.5$$

$$y_2 = 0.7403627$$

$$\therefore 5.5 - 4.5 = 0.7403627 - 0.6532125$$

$$\frac{6}{\therefore} 1 = 0.0871502$$

$$\text{Change} = 0.0871502$$

$$\therefore x = 5$$

$$y = 0.6532125 + 0.0871502$$

$$= 0.6967876$$

Ans-5 $A^2 = A$ (Given)

$$7A - (I + A)^3 = -7A + (I^3 + A^3 + 3A^2I + 3AI^2)$$

$$= -7A + (I^3 + A^2(A) + 3AI + 3AI^2)$$

$$\cancel{7A} - \cancel{A^2(A)} - \cancel{I^3} + \{3A(I)\}$$

$$= -7A + (I + A^2(A) + 3AI + 3AI^2)$$

$$= -7A + I + A^2 + 6A$$

$$= -A + I + A$$

$$= I$$

Ans-6 $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$|A| = 0 - (6-8) + (-1)4$$

$$= 2 - 4$$

$$= -2$$

$$\text{Adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$= -\frac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ 2 & 1/2 & -2 \end{bmatrix}$$

Ans-7

$$\vec{a} = 8\hat{i} - 9\hat{j}$$

$$\vec{b} = 7\hat{i} - 9\hat{j}$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= ab \cos \theta \\ &= (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j}) \\ &= 56 + 81 = 137\end{aligned}$$

$$a = \sqrt{64 + 81} = \sqrt{145}$$

$$b = \sqrt{49 + 81} = \sqrt{130}$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= ab \cos \theta \\ \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{ab} \\ &= \frac{137}{\sqrt{145} \cdot \sqrt{130}} \\ &= 0.998067232\end{aligned}$$

Ans-8

$$\vec{R} = \vec{A} + \vec{B}$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$\theta = 30^\circ \quad (\text{given})$$

$$\begin{aligned}R^2 &= (75)^2 + (25)^2 + 2(75)(25) \cos 30^\circ \\ R^2 &= 5625 + 625 + 3750 \cos 30^\circ \\ &= 9497.595284\end{aligned}$$

Ans-9

$$d = 3$$

$$AP = 101, 104, 107, 110, \dots, 998$$

$$a = 101$$

$$a_n = a + (n-1)d$$

$$998 = 101 + (n-1)3$$

$$897 = 3n - 3$$

$$900 = 3n$$

$$n = 300$$

$$S_n = \frac{n}{2} (a_n + a)$$

$$= \frac{300}{2} (998 + 101)$$

$$= 150 \times 1099$$

$$= 164850$$

Ans-10 $a_6 = 32$

$$a_8 = 128$$

$$\frac{a_{11}}{a_{15}} = \frac{128}{32}$$

$$r^2 = 4$$

$$r = 2$$

Ans-11 $(3x+4)^5 = (3x)^5 + 5(3x)^4 \cdot 4 + 10(3x)^3 \cdot 4^2 + 10(3x)^2 \cdot 4^3 + 5(3x) \cdot 4^4 + 4^5$

$$\therefore (3x+4)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

Ans-14 $(x^2 - x + 1)^4 = ((x^2 - 1)^2 + x)^4$

Using Pascal's triangle,

$$[(x-1)^2 + x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$

$$= (x-1)^8 + 4x(x-1)^6 + (x-1)^4 6x^2 + 4x^3(x-1)^2 + x^4$$

Ans-12 $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = -\lambda^3 + 13\lambda - 12$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

Comparing Coefficient

$$1) 1 - 13 + 12 = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$

$$\lambda = 3 \quad \text{or} \quad -4$$