PAGE No.
DATE / /

Probability & Statistics
Assignment - 2

Q1] let claim size of home insurance = x
let claim size of car insurance = y

$\therefore$ X, y belongs to normal distribution :-

$x \sim N(800, (100)^2)$
$y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$ [given]

$\therefore P((y_1+y_2+y_3) - (x_1+x_2+x_3+x_4) > 800)$
To find this, ~~we~~ we need to find -

$(y_1+y_2+y_3)-(x_1+x_2+x_3+x_4) \sim N(3(1200) - 4(800), 3(300^2) + 4(100)^2)$

$(y_1+y_2+y_3)-(x_1+x_2+x_3+x_4) \sim N(400, \cancel{290000}\ 310000)$

let $A = (y_1+y_2+y_3)-(x_1+x_2+x_3+x_4)$

$\therefore A \sim N(400, 310000)$

$P\left(A > \frac{800-400}{\sqrt{310000}}\right) \Rightarrow P(A > 0.7184212)$

$\therefore P(A > 0.7184212) = 1 - P(A < 0.7184212)$
$= 1 - 0.76424$

$\boxed{P(A > 0.7184212) = 0.23576}$

PAGE No.
DATE / /

Q2] $N \sim$ negative binomial (k, p)

$$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$$
$$= {}^{(n+3)-1}C_n (0.9)^3 (0.1)^n$$

from this we can see,

$k = 3$, $p = 0.9$

$\therefore$ $q = 0.1$

let $X \sim$ Gamma$(6, 2)$

let $y \Rightarrow$ total amount of claim.

$$M_y(t) = M_N(\log M_x(t))$$

(1) MGF of y -

$$M_y(t) = E(E(e^{yt}/N=n))$$
$$= E(e^{x_1 t + x_2 t + x_3 t + \dots + x_n t})$$
$$= E(e^{x_1 t}) + E(e^{x_2 t}) + E(e^{x_3 t}) + \dots E(e^{x_n t})$$
$$= \left(1 - \frac{t}{2}\right)^{-6n}$$

$$\therefore E(E(e^{yt}/N=n)) = E\left(\left(1-\frac{t}{2}\right)^{-6n}\right)$$
$$= E\left(e^{N \log\left(1-\frac{t}{2}\right)^{-6}}\right)$$
$$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$$

$$M_N(t) = \left[\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-(0.1)\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

(ii) $V(Y) = E(N)V(X) + E(X)^2 \cdot V(N)$

$$= \left(\frac{\cancel{3}^1}{\cancel{0.9}_{0.3}} \times \frac{\cancel{6}^2}{\cancel{4}^2}\right) + \left(\frac{36}{4}\right)\left(\frac{3(0.1)}{(0.9)^2}\right)$$

$$= \frac{1}{0.2} + \cancel{9}^1 \cdot \left(\frac{0.\cancel{3}}{0.\cancel{9} \times 0.\cancel{9}}\right)$$

$$= \frac{1}{0.2} + \frac{1}{0.3}$$

$$V(Y) = 8.33333$$

$$\therefore SD(Y) = \sqrt{V(Y)} = \sqrt{8.3333}$$

$$\boxed{SD(Y) = 2.8867513}$$

Q3.] (i) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

M.G.F is given by –

$$MGF = \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)}\, dx$$

$$M_X(t) = \lambda \int_{\alpha}^{\infty} e^{tx} \cdot e^{-\lambda x} \cdot e^{\lambda\alpha}\, dx$$

$$M_X(t) = \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{tx} \cdot e^{-\lambda x}\, dx$$

$$M_X(t) = \frac{e^{\alpha\lambda} \cdot \lambda}{t-\lambda} \left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty}$$

PAGE No.
DATE / /

$$\boxed{M_x(t) = e^{t\alpha} \frac{\lambda}{\lambda - t}}$$

(ii) $M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda - t}$

$E(x) = M_x'(t) = \lambda [e^{\alpha t}(\lambda-t)^{-2} + (\lambda-t)^{-1} e^{t\alpha} \cdot \alpha]$

$M_x'(t) = \lambda e^{\alpha t} (\lambda - t)^{-2} [\alpha(\lambda - t) + 1]$

when $t=0$,

$= \lambda (\lambda)^{-2} (\alpha . \lambda + 1)$

$E(x) = \frac{\alpha \lambda + 1}{\lambda}$

$E(x^2) = M_x''(t)$.

$= \lambda [\alpha e^{\alpha t}(\lambda - t)^{-2} + \alpha^2 (\lambda - t)^{-1} e^{\alpha t} + 2e^{\alpha t}(\lambda - t)^{-3} + (\lambda - t)^{-2} e^{\alpha t}]$

when $t=0$

$= \lambda(\alpha(\lambda)^{-2} + \alpha^2 (\lambda)^{-1} + 2(\lambda)^{-3} + \alpha(\lambda^{-2}))$

$E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$

$V(x) = E(x^2) - [E(x)]^2$

$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \frac{\alpha^2\lambda^2 + 1 + 2\alpha\lambda}{\lambda^2}$

$$\boxed{V(x) = \frac{1}{\lambda^2}}$$

Q4.] $G_x(t) = \left(\dfrac{p}{1-qt}\right)^k$

$G_V(t) = \sum t^r \cdot P(V=r)$

$P(V=4) = \dfrac{\Gamma(k+4)}{\Gamma 5\,\Gamma k} \cdot p^k q^k$

$P(V=4) = \dfrac{\Gamma(k+4)}{4!\,\Gamma k}\, p^k q^k$

$$\boxed{P(V=4) = \dfrac{(k+3)!}{4!\,(k-1)!} \cdot p^k (1-p)^k}$$

Q5.] $f(x,y) = ax(x+y)$, $0 \leq x \leq 5$ & $10 \leq y \leq 20$ $a = \frac{3}{6875}$

(1) $\cancel{P\left(y \geq \frac{1}{3}x + 10\right)}$ $\Rightarrow a \int_{10}^{20} \int_0^5 x^2 + xy \; dx\, dy = 1$

$a \int_{10}^{20} \left[\dfrac{x^3}{3}\right]_0^5 + \left[\dfrac{x^2}{2}\right]_0^5 y \; dy = 1$

$= \int_{10}^{20} \dfrac{125}{3} + \dfrac{25}{2} y \; dy = \dfrac{1}{a}$

$\dfrac{1}{a} = \dfrac{125}{3}[y]_{10}^{20} + \dfrac{25}{4}[y^2]_{10}^{20}$

$\therefore \boxed{a = 0.00046364}$

(ii) $P\left(y > \frac{1}{3}x + 10\right)$

$$f(y) = \frac{3}{6875}\int_0^5 x^2 + xy \, dx$$

$$= \frac{3}{6875}\left(\left[\frac{x^3}{3}\right]_0^5 + \left[\frac{x^2}{2}\right]_0^5 y\right)$$

$$= \frac{3}{6875}\left(\frac{125}{3} + \frac{25}{2}y\right)$$

$$\therefore P\left(y > \frac{1}{3}x + 10\right) = \frac{3}{6875}\int_{\frac{1}{3}x+10}^{20} \frac{125}{3} + \frac{25}{2}y \, dy$$

$$= \frac{3}{6875}\left[\frac{6875}{3} - \frac{25x^2}{36} - \frac{500x}{9}\right]$$

$$P\left(y > \tfrac{1}{3}x + 10\right) = 1 - \frac{x^2}{3300} - \frac{1500}{61875}x$$

$$\boxed{P\left(y > \tfrac{1}{3}x + 10\right) = 1 - \frac{x^2}{3300} - \frac{4x}{165}}$$

(iii)

PAGE No.
DATE / /

Q6.] $X \sim \text{Poisson}(\lambda)$, $y \sim \text{Poisson}(\mu)$

$M_x(t) = e^{\lambda(e^t-1)}$, $M_y(t) = e^{\mu(e^t-1)}$

$z = x - y$.

$M_z(t) = E(e^{tz})$

$= E(e^{t(x-y)}) \Rightarrow E(e^{tx} \cdot e^{-ty})$

$= \dfrac{E(e^{tx})}{E(e^{ty})} \Rightarrow \dfrac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$

$M_z(t) = e^{(\lambda-\mu)(e^t-1)}$

~~Which shows~~ let $\lambda = \mu = \alpha$

$\therefore M_z(t) = e^{\alpha(e^t-1)}$

which shows z belongs to poisson-

$z \sim \text{Poisson}(\alpha)$

Similarly,

for $z = x + y$.

$E(e^{tz}) = e^{\lambda(e^t-1)} \times e^{\mu(e^t-1)}$

$= e^{(\lambda+\mu)(e^t-1)}$

$\therefore$ let $\lambda + \mu = \beta$

$\therefore z \sim \text{Poisson}(\beta)$

		X			
		1	2	3	4
Q7. y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

(1) $V(X/Y=2) = E(x^2/Y=2) - [E(X/Y=2)]^2$

$$E(x^2/Y=2) = 1^2 \frac{P(x=1,y=2)}{P(y=2)} + 2^2 \frac{P(x=2,y=2)}{P(y=2)} + 3^2 \frac{P(x=3,y=2)}{P(y=2)} + 4^2 \frac{P(x=4,y=2)}{P(y=2)}$$

$$= \frac{1\times 0}{0.6} + \frac{4\times 0.3}{0.6} + \frac{9\times 0.1}{0.6} + \frac{16\times 0.2}{0.6}$$

$$E(x^2/Y=2) = 8.83333$$

$$E(x/Y=2) = 1 \frac{P(x=1,y=2)}{P(Y=2)} + 2 \frac{P(x=2,y=2)}{P(y=2)} + 3 \frac{P(x=3,y=2)}{P(y=2)} + 4 \frac{P(x=4,y=3)}{P(y=2)}$$

$$= 0 + \frac{2\times 0.3}{0.6} + \frac{3\times 0.1}{0.6} + \frac{4\times 0.2}{0.6}$$

$$= 1 + \frac{1}{2} + \frac{0.8}{0.6}$$

$$E(X/Y=2) = 0.807$$

$$[E(X/Y=2)]^2 = 0.651249$$

$$\therefore V(X/Y=2) = 8.8333 - 0.651249$$

$$\boxed{V(X/Y=2) = 8.182084}$$

DATE / /

Q8. $X \sim \text{Gamma}(3,2)$, $E(X) = \frac{3}{2}$, $V(X) = \frac{3}{4}$

$E(Y/X=x) = 3x+1$, $Var(Y/X=x) = 2x^2+5$

$E(Y) = E(E(Y/X=x)) \Rightarrow E(3x+1)$
$= 3E(X)+1 \Rightarrow \frac{9}{2}+1$

$\boxed{E(Y) = \frac{11}{2}}$

$V(Y) = E(V(X/Y)) + V(E(X/Y))$
$= E(2x^2+5) + V(3x+1)$
$= 2E(x^2)+5 + 9V(X) + 0$
$= 2[V(X)+E(X)^2] + 9V(X)$
$= 11V(X) + 2[E(X)]^2$
$= 11 \times \frac{3}{4} + 2 \times \frac{9}{4} + 5 \Rightarrow \frac{33}{4} + \frac{18}{4} + 5$

$\boxed{V(Y) = \frac{71}{4}}$

Q9. $E(X) = 3$, $V(X) = 4$, $E(Y) = 4$, $V(Y) = 1$

$Cov(X,Y) = -0.3\sqrt{4} = -0.6$

(1) $Z = X+Y$

$Cov(X, X+Y) = Cov(X/X) + Cov(X/Y)$
$= V(X) + (-0.6) \Rightarrow 4 - 0.6$

$\boxed{Cov(X, X+Y) = 3.4}$

(ii) $V(z) = \text{cov}(x+y, x+y)$

$= \text{cov}(x) + 2\,\text{cov}(x,y) + \text{cov}(y)$

$= V(x) + 2\,\text{cov}(x,y) + V(y)$

$= 4 - 1.2 + 1$

$\boxed{V(z) = 3.8}$

Q10.] $f(x,y) = k x^{-\alpha} e^{-y/\beta}$, $1<x<\infty$ & $1<y<\infty$

$\alpha, \beta > 1$

$$k \int_1^\infty \int_1^\infty x^{-\alpha} \cdot e^{-y\beta} \, dx\, dy = 1$$

$$\frac{k}{\alpha-1} \frac{\left[e^{-y/\beta}\right]_1^\infty}{-1/\beta} \quad -\frac{k\beta}{\alpha-1}\left(e^{-1/\beta}\right) = 1$$

$$\therefore \boxed{k = \frac{\alpha-1}{\beta e^{-1/\beta}}}$$