

Homework - Error, Difference & Interpolation

Lagrange's Interpolation

1.

x	7	8	9	10
$y = f(x)$	3	1	1	9

To find value of y at $x = 9.5$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$= \frac{(1.5)(0.5)(-0.5)}{(-1)(+2)(-3)} \times 3 + \frac{(2.5)(0.5)(-0.5)}{(-1)(+2)}$$

$$+ \frac{(2.5)(1.5)(-0.5)}{(2)(1)(+1)} + \frac{(2.5)(0.5)(1.5)}{(3)(2)(1)} \times 9$$

$$= 0.1875 - 0.3125 + 0.9375 + 2.8125$$

$$f(9.5) = \underline{\underline{3.625}}$$

2.

x	3	7	9	10
y	168	120	72	63

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168$$

$$+ \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72$$

$$+ \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$= \frac{(+1)(-2)(-4) \times 168}{(+4)(+6)(+7)} + \frac{(3)(-3)(-4) \times 120}{(4)(-2)(+3)}$$

$$+ \frac{(3)(-1)(-4) \times 72}{(6)(2)(-1)} + \frac{(3)(-1)(+3) \times 63}{(7)(3)(1)}$$

$$= 12 + 180 - 72 + 27$$

$$f(6) = 147$$

3.

x	1	2	7	8
$f(x)$	4	5	5	4

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$= \frac{(4)(-1)(-2)(4)}{(-1)(+6)(-7)} + \frac{(5)(+1)(-2)(5)}{(+5)(+6)}$$

$$+ \frac{(5)(4)(-2)(5)}{(6)(5)(+1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$= -0.7619 + 0.8333 + 6.667 - 1.90476$$

$$f(6) = \underline{\underline{4.83364}}$$

4.

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$= \frac{(3)(2)(-2)(2)}{(-1)(-2)(-3)(-6)} + \frac{(4)(2)(+2)(4)}{(-1)(+2)(-5)}$$

$$+ \frac{(4)(3)(-2)(8)}{(2)(-1)(-4)} + \frac{(4)(3)(2)(+2)(16)}{(3)(2)(+3)}$$

$$+ \frac{(4)(3)(2)(128)}{(6)(5)(4)(3)}$$

$$= -0.6667 + 6.4 - 24 + 42.6667 + 8.533$$

Ans $f(5) = 32.933$

5.

x	10	20	14	16
$f(x)$	0.25	0.2	0.15	0.10

$$f(15) = \frac{(15)(1)(+1)}{(-10)(-4)(+6)} \times 0.25$$

$$+ \frac{(15)(1)(-1)}{(10)(20)(4)} \times 0.2$$

$$+ \frac{(5)(+5)(1)}{(4)(-6)(-2)} \times 0.15$$

$$+ \frac{(5)(+5)(1)}{(10)(-4)(2)} \times 0.1$$

$$= -0.005208 + 0.004167 + 0.078125 + 0.05208$$

Ans $f(15) = 0.12083$

Regula Falsi Method

1. Root of $x^3 + x - 1 = 0$; b/w 0.5 & 1

x	0.5	1
$f(x)$	-0.375	1

$$x_0 = 0.5 \quad x_1 = 1$$

$$y_0 = -0.375 \quad y_1 = 1$$

$$x_2 = \frac{(0.5)(1) - (1)(-0.375)}{1 - (-0.375)}$$

$$x_2 = 0.63636$$

$$x_1 = 1$$

$$y_2 = -0.10594$$

$$y_1 = 1$$

$$x_3 = \frac{0.63636 - (-0.10594)}{1 - (-0.10594)}$$

$$x_3 = 0.67119$$

$$x_1 = 1$$

$$y_3 = -0.02644$$

$$y_1 = 1$$

$$x_4 = \frac{0.67119 + 0.02644}{1 + 0.02644}$$

$$x_4 = 0.67966$$

$$x_1 = 1$$

$$y_4 = -0.006379$$

$$y_1 = 1$$

$$x_5 = \frac{0.67966 + 0.006379}{1 + 0.006379}$$

$$x_5 = 0.68169, \quad y_5 = -0.0015278$$

$$n_c = \frac{0.68169 + 0.0015278}{1 + 0.0015278}$$

$$n_6 = 0.68217$$

$$y_6 = -0.000378$$

Ans | $n = 0.68217$

2. $n^3 - 2n + 0.5 = 0$, b/w 0 & 1

n	0	1
$f(n)$	0.5	-0.5

~~$$n_0 = 0$$~~
~~$$y_0 = 0.5$$~~

~~$$n_1 = 1$$~~
~~$$y_1 = -0.5$$~~

n	0	0.1	0.2	0.3	1
$f(n)$	0.5	0.801	0.108	-0.073	-0.5

$$n_0 = 0.3 \quad n_1 = 0.2$$

$$y_0 = 0.801 - 0.073 \quad y_1 = 0.108$$

~~$$n_2 = (0.2)(0.801)$$~~

$$n_2 = \frac{(0.2)(-0.073) - (0.3)(0.108)}{0.108 + 0.073}$$

$$n_2 = 0.25967$$

$$n_1 = 0.2$$

$$y_2 = -0.001828$$

$$y_1 = 0.108$$

$$n_3 = \frac{(0.25967)(0.108) + (0.2)(0.001828)}{0.108 + 0.001828}$$

$$x_3 = 0.25867$$

$$y_3 = -0.0000411$$

Ans Root is 0.25867

3. $x^3 + 2x - 20 = 0$ b/w 2 & 2.5

x	2	2.1	2.2	2.3	2.4	2.5
y	-8	0.018 -6.539	-4.952	-3.233	-1.376	0.625

$$x_0 = 2.4$$

$$y_0 = -1.376$$

$$x_1 = 2.5$$

$$y_1 = 0.625$$

$$x_2 = \frac{(2.4)(0.625) + (2.5)(-1.376)}{0.625 + 1.376}$$

$$x_2 = 2.46877$$

$$y_2 = -0.015827$$

$$x_1 = 2.5$$

$$y_1 = 0.625$$

$$x_3 = \frac{(2.46877)(0.625) + (2.5)(-0.015827)}{(2.46877)(0.625) + (2.5)(-0.015827)}$$

$$x_3 = 2.469537$$

$$y_3 = -0.00017479$$

Ans Root is 2.469537

4. $3n - \cos n - 1 = 0$, b/w 0 & 1

n	0	1
y	-2	1.0001523

$$n_0 = 0 \quad n_1 = 1$$

$$y_0 = -2 \quad y_1 = 1.0001523$$

$$n_2 = \frac{2}{2 + 1.0001523}$$

$$n_2 = 0.66663$$

$$y_2 = -0.00004231$$

Ans Root is 0.66663

5. $n^3 - 3n - 5$, b/w 2 & 2.5

n	2	2.1	2.2	2.3
y	-3	-2.039	-0.952	0.267

$$n_0 = 2.2 \quad n_1 = 2.3$$

$$y_0 = -0.952 \quad y_1 = 0.267$$

$$n_2 = \frac{(2.2)(0.267) + (2.3)(-0.952)}{(0.952) + (0.267)}$$

$$n_2 = 2.27809 \quad n_1 = 2.3$$

$$y_2 = -0.0116798 \quad y_1 = 0.267$$

$$n_3 = \frac{(2.27809)(0.267) - (2.3)(-0.0116798)}{(0.267) - (-0.0116798)}$$

$$x_3 = 2.279$$

$$y_3 = -0.000236361$$

Ans Root is ~~2.279~~ 2.279

G. $x \log_{10} x - 1.2 = 0$, b/w 2 & 3

x	2	3
y	-0.59794	0.23136

$$x_0 = 2$$

$$x_1 = 3$$

$$y_0 = -0.59794$$

$$y_1 = 0.23136$$

$$x_2 = \frac{2(0.23136) - 3(-0.59794)}{0.23136 - (-0.59794)}$$

$$x_2 = 2.721018$$

$$x_1 = 3$$

$$y_2 = -0.0170879$$

$$y_1 = 0.23136$$

$$x_3 = \frac{(2.721018)(0.23136) - 3(-0.0170879)}{(-0.0170879) + (0.23136)}$$

$$x_3 = 2.740206$$

$$y_3 = -0.00038381$$

Ans Root is 2.740206

7.

$$x^3 - 4x + 1 = 0$$

$$\begin{array}{c|c|c|c} x & 0 & 1 & \\ \hline 4 & 1 & -2 & \end{array}$$

$$x_0 = 0$$

$$y_0 = -2$$

$$x_1 = 0$$

$$y_1 = 1$$

~~$$x_2 = 0.0020 - 1.1$$~~

$$x_2 = \frac{1(1) - (-2)(0)}{1 - (-2)}$$

$$x_2 = 0.3333$$

$$y_2 = -0.296174$$

$$x_1 = 0$$

$$y_1 = 1$$

$$x_3 = \frac{0.3333}{1.296174}$$

$$x_3 = 0.25714$$

$$y_3 = -0.011557$$

$$x_1 = 0$$

$$y_1 = 1$$

~~$$x_4 = \frac{0.25714}{1.011557}$$~~

$$x_4 = 0.2542022$$

$$y_4 = -0.000382569$$

Ans

Root is 0.2542022

$$8. \quad x^3 + 2x^2 - 8 = 0$$

x	0	1	2
y	-8	-5	8

$$x_0 = 1$$

$$x_1 = 2$$

$$y_0 = -5$$

$$y_1 = 8$$

$$x_2 = \frac{1(8) - (2)(-5)}{8 - (-5)}$$

$$x_2 = 1.384615$$

$$x_1 = 2$$

$$y_2 = -1.511155913$$

$$y_1 = 8$$

$$x_3 = \frac{(1.384615)(8) - (2)(-1.511155913)}{8 - (-1.511155913)}$$

$$x_3 = 1.48247$$

$$x_1 = 2$$

$$y_3 = -0.346515$$

$$y_1 = 8$$

$$x_4 = \frac{(1.48247)(8) - (2)(-0.346515)}{8 - (-0.346515)}$$

$$x_4 = 1.503956$$

$$x_1 = 2$$

$$y_4 = -0.074459$$

$$y_1 = 8$$

$$x_5 = 1.50853$$

$$x_1 = 2$$

$$y_5 = -0.0157689$$

$$y_1 = 8$$

$$x_6 = 1.509496$$

$$y_6 = 0.003339$$

Ans Root is 1.509496

9. $x^3 - 9x + 1 = 0$

x	0	0.1	0.2
y	1	0.101	-0.792

$$x_0 = 0.2 \quad x_1 = 0.1$$

$$y_0 = -0.792 \quad y_1 = 0.101$$

$$x_2 = \frac{(0.2)(0.101) - (0.1)(-0.792)}{0.101 - (-0.792)}$$

$$x_2 = 0.1131019$$

$$y_2 = -0.0004125$$

Ans Root is 0.1131019

10. $x^3 - x - 4 = 0$

x	0	1	2
y	-4	-4	2

$$x_0 = 1 \quad x_1 = 2$$

$$y_0 = -4 \quad y_1 = 2$$

$$x_2 = \frac{(1)(2) - (2)(-4)}{2 - (-4)}$$

$$x_2 = 1.6667 \quad x_1 = 2$$

$$y_2 = -1.03679 \quad y_1 = 2$$

$$x_3 = \frac{(1.6667)(2) - 2(-1.03679)}{2 - (-1.03679)}$$

$$x_3 = 1.78049$$

$$y_3 = -0.136079$$

$$n_1 = 2$$

$$y_1 = 2$$

$$x_4 = 1.79447$$

$$y_4 = -0.0160567$$

$$n_1 = 2$$

$$y_1 = 2$$

$$x_5 = 1.796107$$

$$y_5 = -0.00186517$$

$$n_1 = 2$$

$$y_1 = 2$$

$$x_6 = 1.796247$$

$$y_6 = -0.000216164$$

Ans Root is 1.796247