

NMA Homework (10th Feb 2021)

* HW-1: Lagrange's Interpolation.

Q1) $y = f(x)$

x	$f(x)$
7	3
8	1
9	1
10	9

Find $f(9.5) = ?$

$\therefore x_k = 9.5$

$\therefore$ Using Lagrange's Interpolation

$$\begin{aligned}
 \rightarrow \therefore f(9.5) &= \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3) f(x_0) + (x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \\
 &\quad + \frac{(x_k - x_0)(x_k - x_2)(x_k - x_3) f(x_1) + (x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \\
 &\quad + \frac{(x_k - x_0)(x_k - x_1)(x_k - x_3) f(x_2) + (x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \\
 &\quad + \frac{(x_k - x_0)(x_k - x_1)(x_k - x_2) f(x_3) + (x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \\
 &= \frac{(9.5 - 8)(9.5 - 9)(9.5 - 10) \times 3}{(7 - 8)(7 - 9)(7 - 10)} + \frac{(9.5 - 7)(9.5 - 9)(9.5 - 10) \times 1}{(8 - 7)(8 - 9)(8 - 10)} \\
 &\quad + \frac{(9.5 - 7)(9.5 - 8)(9.5 - 10) \times 1}{(9 - 7)(9 - 8)(9 - 10)} + \frac{(9.5 - 7)(9.5 - 8)(9.5 - 9) \times 9}{(10 - 7)(10 - 8)(10 - 9)} \\
 &= 0.1875 + (-0.3125) + (0.9375) + 2.8125 \\
 &= \underline{\underline{3.625}}
 \end{aligned}$$

$\therefore f(9.5)$ Using Lagrange's Interpolation is 3.625

Q2)

x	$f(x)$
3	168
7	120
9	72
10	63

To find:- $f(6) = ?$

→ By Lagrange's Interpolation formula,

$$\begin{aligned}
 f(6) &= \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 + \\
 &\quad \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 + \\
 &\quad \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 + \\
 &\quad \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63 \\
 &= 12 + 180 + (-72) + 27 \\
 &= \underline{\underline{147}}
 \end{aligned}$$

$\therefore f(6) = 147$

Q3)

x	$f(x)$
1	4
2	5
7	5
8	4

To find:- $f(6) = ?$

→ By Lagrange's Interpolation formula,

$$f(6) = \frac{(6-2)(6-7)(6-8) \times 4}{(1-2)(1-7)(1-8)} +$$

$$\frac{(6-1)(6-7)(6-8) \times 5}{(2-1)(2-7)(2-8)} +$$

$$\frac{(6-1)(6-2)(6-8) \times 5}{(7-1)(7-2)(7-8)} +$$

$$\frac{(6-1)(6-2)(6-7) \times 4}{(8-1)(8-2)(8-7)}$$

$$= -0.7619 + 1.6666 + 6.6666 + (-1.9047)$$

$$= 5.6667$$

$$\therefore \underline{\underline{f(6) = 5.6667}}$$

Q4)

x	$f(x)$
1	2
2	4
3	8
4	16
7	128

To find :- $f(5) = ?$

→ By Lagrange's Interpolation,

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7) \times 2}{(1-2)(1-3)(1-4)(1-7)} +$$

$$\frac{(5-1)(5-3)(5-4)(5-7) \times 4}{(2-1)(2-3)(2-4)(2-7)} +$$

$$\frac{(5-1)(5-2)(5-4)(5-7) \times 8}{(3-1)(3-2)(3-4)(3-7)} +$$

$$\frac{(5-1)(5-2)(5-3)(5-7) \times 16}{(4-1)(4-2)(4-3)(4-7)}$$

$$\begin{aligned}
 & \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128 \\
 & = -0.5714 + 6.4 + (-24) + 42.6667 + \\
 & \quad 8.5333 \\
 & = \underline{\underline{33.0286}}
 \end{aligned}$$

$$\therefore \underline{\underline{f(5) = 33.0286}}$$

Q5)

x	$f(x)$
10	0.25
20	0.2
14	0.15
16	0.10

To find :- $f(15) = ?$

→ By Lagrange's Interpolation,

$$\begin{aligned}
 f(15) &= \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25 + \\
 & \quad \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2 + \\
 & \quad \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15 + \\
 & \quad \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10 \\
 &= (-0.005208) + (-0.004166) + \\
 & \quad (0.078125) + 0.052083 \\
 &= \underline{\underline{0.120834}}
 \end{aligned}$$

$$\therefore \underline{\underline{f(15) = 0.1208}}$$

* HW-2 :- Regula Falsi Method.

[Root of the equations = ?]

1) $x^3 + x - 1 = 0$ lying between 0.5 & 1.

→ When $x = 0.5$; LHS = -0.375

$x = 1$; LHS = 1

∴ By Regula Falsi Method,

$$\bar{x} = 1(0.5) - (-0.375)$$

$$= 0.875$$

and LHS = 0.544

When $x = 0.5$; LHS = -0.375

$x = 0.875$; LHS = 0.544

$$\therefore \bar{x} = \frac{0.544(0.5) - 0.875(-0.375)}{0.544 + 0.375}$$

$$= 0.6535$$

and LHS = -0.0673

When $x = 0.653$; LHS = -0.067

$x = 0.875$; LHS = 0.544

$$\therefore \bar{x} = \frac{0.544(0.653) - 0.875(-0.067)}{0.544 + 0.067}$$

$$= 0.677$$

and LHS = -0.01189

close enough to
Zero.

∴ Root of the equation is 0.677.

2) $x^3 - 2x + 0.5 = 0$ lying betⁿ 0 & 1.

→ When $x = 0$; LHS = 0.5

$x = 1$; LHS = -0.5

∴ By Regula Falsi Method,

$$\bar{x} = \frac{0 - 0.5}{-0.5 - 0.5} = \frac{-0.5}{-1} = 0.5$$

∴ LHS = -0.375

When $x = 0$; LHS = 0.5

$x = 0.5$; LHS = -0.375

∴ $\bar{x} = \frac{0 - 0.25}{-0.875} = 0.285$ ∴ LHS = -0.048

When $x = 0$; LHS = 0.5

$x = 0.285$; LHS = -0.048

∴ $\bar{x} = \frac{0 - 0.1425}{-0.548} = 0.260$ ∴ LHS = -0.002

close enough
to zero

∴ Root of the equation is 0.260.

3) $x^3 + 2x - 20 = 0$ lying between 2 and 2.5.

→ When $x = 2$; LHS = -8

$x = 2.5$; LHS = 0.625.

∴ By Regula Falsi Method,

$$\bar{x} = \frac{1.25 + 20}{8.625} = 2.4637$$

$$\therefore \text{LHS} = -0.1170$$

When $x = 2.4637$; LHS = -0.1170

$x = 2.5$; LHS = 0.625.

$$\therefore \bar{x} = \frac{1.5398 + 0.2925}{0.742} = 2.4694$$

$$\therefore \text{LHS} = -0.00281$$

i.e. close enough to zero.

∴ Root of the equation is 2.4694.

4) $3x - \cos x - 1 = 0$ lying between 0 & 1.

→ When $x = 0$; LHS = -2

$x = 1$; LHS = 1.0001

∴ By Regula Falsi Method,

$$\bar{x} = \frac{0 + 2}{3.0001} = 0.6666$$

$$\therefore \text{LHS} = -0.000033$$

which is very close
to zero.

∴ Root of the equation is 0.6666.

5) $x^3 - 3x - 5$ lying between 2 & 2.5.

→ When $x = 2$; LHS = -3

$x = 2.5$; LHS = 3.125.

∴ By Regula Falsi Method,

$$\bar{x} = \frac{6.25 + 7.5}{6.125} = 2.2448$$

$$\therefore \text{LHS} = -0.4225$$

When $x = 2.2448$; LHS = -0.4225

$x = 2.5$; LHS = 3.125

$$\therefore \bar{x} = \frac{7.015 + 1.056}{3.5475} = 2.2751$$

$$\therefore \text{LHS} = -0.0489$$

∴ Root of the equation is 2.2751.

6) $x \log_{10} x - 1.2 = 0$ lying betⁿ 2 & 3.

→ When $x=2$; LHS = -0.5979

$x=3$; LHS = 0.2313

∴ By Regula Falsi Method,

$$\bar{x} = \frac{0.4627 + 1.7937}{0.8292} = 2.7211$$

∴ LHS = -0.0169

When $x = 2.7211$; LHS = -0.0169

$x = 3$; LHS = 0.2313 .

$$\therefore \bar{x} = \frac{0.6293 + 0.0507}{0.2482} = 2.7397$$

∴ LHS = -0.00080

∴ Root of the equation is 2.7397.

7) $x^3 - 4x + 1 = 0$

→ When $x=0$; LHS = 1

$x=1$; LHS = -2

∴ By Regula Falsi method,

$$\bar{x} = \frac{0-1}{-3} = 0.3334 \quad \therefore \text{LHS} = -0.2962$$

When $x=0$; LHS = 1

$x = 0.3334$; LHS = -0.2962 .

$$\therefore \bar{x} = \frac{0-0.3334}{-1.2962} = 0.2571 \quad \therefore \text{LHS} = -0.0117$$

∴ Root of Equation is 0.2571.

$$8) x^3 + 2x^2 - 8 = 0$$

$$\rightarrow \text{When } x=0 ; \text{LHS} = -8$$

$$x=1 ; \text{LHS} = -5$$

$$x=2 ; \text{LHS} = 8$$

$\therefore$ By Regula Falsi Method,

$$\bar{x} = \frac{8+10}{13} = 1.3846 ; \text{LHS} = -1.5111$$

$$\text{When } x = 1.3846 ; \text{LHS} = -1.5111$$

$$x = 2 ; \text{LHS} = 8$$

$$\therefore \bar{x} = \frac{11.0768 + 8.0222}{9.5111} = 1.4823$$

$$\therefore \text{LHS} = -0.3477$$

$$\text{When } x = 1.4823 ; \text{LHS} = -0.3477$$

$$x = 2 ; \text{LHS} = 8$$

$$\therefore \bar{x} = \frac{11.8584 + 0.6954}{8.3477} = 1.5038$$

$$\therefore \text{LHS} = -0.0756$$

~~$$\text{When } x = 1.4823 ; \text{LHS} = -0.3477$$~~

~~$$x = \quad ; \text{LHS} = \quad$$~~

$\therefore$ Root of the equation is 1.5038

$$9) x^3 - 9x + 1 = 0$$

$$\rightarrow \text{When } x = 0 ; \text{ LHS} = 1$$

$$x = 1 ; \text{ LHS} = -7$$

$\therefore$ By Regula Falsi Method,

$$\bar{x} = \frac{0 - 1}{-8} = 0.125 \quad \therefore \text{LHS} = -0.123$$

$$\text{When } x = 0 ; \text{ LHS} = 1$$

$$x = 0.125 ; \text{ LHS} = -0.123$$

$$\therefore \bar{x} = \frac{0 - 0.125}{-1.123} = 0.1113 \quad \therefore \text{LHS} = -0.0004$$

$\therefore$ Root of the equation is 0.1113.

$$10) x^3 - x - 4 = 0$$

$$\rightarrow \text{When } x = 0 ; \text{ LHS} = -4$$

$$x = 1 ; \text{ LHS} = -4$$

$$x = 2 ; \text{ LHS} = 2$$

$\therefore$ By Regula Falsi Method,

$$\bar{x} = \frac{2 + 8}{6} = 1.6667 ; \text{ LHS} = -1.0370$$

$$\text{When } x = 1.6667 ; \text{ LHS} = -1.0370$$

$$x = 2 ; \text{ LHS} = 2$$

$$\therefore \bar{x} = \frac{3.3334 + 2.074}{3.0370} = 1.7805 \quad \therefore \text{LHS} = -0.1359$$

$$\text{When } x = 1.7805 ; \text{ LHS} = -0.1359$$

$$x = 2 ; \text{ LHS} = 2$$

$$\therefore \bar{x} = \frac{3.561 + 0.2718}{2.1359} = 1.7944$$

$$\therefore \text{LHS} = -0.0160$$

$\therefore$ Root of the Equation is 1.7944.

