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ASSIGNMENT - 2

1 iii) $\Delta c = \frac{dC}{dS_t}$

i) $C_t = S_0 \phi(d_1) - Xe^{-rt} \phi(d_2)$

$d_1 = 1.8855$

$d_2 = 1.7087$

$\therefore C_0 = £1.42$

ii) $\phi(d_1) = 0.86213$

iii) Put call parity : $S_0 + P = C + Xe^{-rt}$

Differentiating wrt. S_t

$1 + \Delta p = \Delta c$

$\therefore \Delta p = (-0.1379)$

2) i) Delta measures the change in price of an option wrt price of underlying

Vega measures the change in price of an option wrt the volatility of the underlying.

ii) PCP : $S_0 + p = C + Xe^{-rt}$

Differentiating wrt σ

$V_p = V_c$

iii) $d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{t}$

$d_1 = 0.7062$, $d_2 = 0.4562$

$C_t = S_0 \phi(d_1) - Xe^{-rt} \phi(d_2)$

$= £9.65$

Using PCP ;

$p = 2.2140$

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iv) Delta-hedged portfolio is a portfolio which is unaffected by changes in the price of the underlying. Basically, delta is 0.
Vega-hedged portfolio is a portfolio that is unaffected by changes in the volatility of the underlying. Basically, vega is 0.

v) ?

Q3 i) $e^{-r(T-t)} \cdot E_Q[X_T | F_t]$

ii) $d_1 = 0.34409$, $d_2 = 0.16732$

Using Blackscholes formula

$C_t = £4.66$

iii) The value of an American call is always equal to European call

iv) PCP : $50 + p = 4.66 + 49e^{-0.05 \times 0.5}$

$p = £2.45$

v) European and American's call price would decrease as dividend reduces the price of the underlying
The value of the ~~American~~ put would increase as the dividend reduces the price of the underlying

Q4 i) Suppose Z_t is a SBM under P. and P & Q are equivalent measures.

$\tilde{Z}_t = Z_t + \gamma_t$

ii) Discounted value of a security price process is a martingale under the risk neutral measure.

Q5 i) $\Delta_C = \frac{dC_t}{dS_t} = \phi(d_1)$

ii) ?

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Q6 i) $\Delta_p = \phi(d_1) - 1$

$V_0 = \Psi - 24830 S_0$

$\Delta_V = -24830$

$\Delta_p = \frac{\Delta_V}{100000} = -0.24823$

(ii) $\therefore \phi(d_1) = 0.7517$

$d_1 = 0.68$

Substituting in formula

$\sigma = 7.1\%$

(iii) a) $P_t = Ke^{-rt}\phi(-d_2) - S_0\phi(-d_1)$

Substituting in the values:

$P_t = £0.0696$

b) $V_0 = \Psi - 24830 \times 6.40$

$\Psi = £165872$

7 i) Delta : Measures change in prices of option wrt change in price of underlying

Gamma : Measures change in delta of option wrt change in price of underlying

Vega : Measures change in price of option wrt change in volatility of underlying.

ii) $\Delta = \phi(d_1) = 0.80106$

iii) $C_0 = \Psi + \phi S_0$

$\Delta_C = \phi = 080106$

$17.91 = \Psi + 0.801 \times 60$

$\Psi = (-30.15)$

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iv) $V = \frac{\text{change in option}}{\text{change in underlying}}$

$\frac{C_2 - C_1}{\Delta\sigma} = 29$

$C_2 = \$18.49$

8) i) Answered already

ii) $C_t = S_0 \Phi(d_1) - Xe^{-rt}\Phi(d_2)$

$d_1 = \frac{-0.09 - (0.03 + \sigma^2)}{\sigma} = \frac{\sigma^2 - 0.12}{\sigma}$

$\Phi(d_1) = 0.42074$

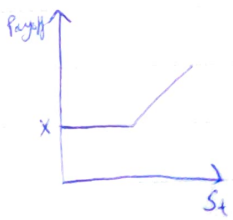
$\therefore d_1 = -0.2 = \frac{\sigma^2 - 0.12}{\sigma}$

$\sigma^2 + 0.2\sigma - 0.12 = 0$

$\sigma = \frac{-0.2 + 0.72111}{2} = 0.26056$

9) Need to ask Akash sir

10)



Portfolio A : Underlying + put Portfolio B : Cash + call

Same payoff $\therefore p + S_t = C + Xe^{-rt}$

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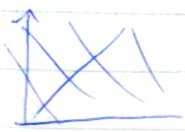
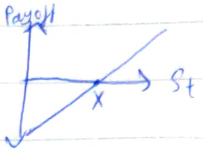
ii) $10.09 = S_0 \Phi(d_1) - X e^{-rt} \Phi(d_2)$

$10.09 = 110\,\Phi(d_1) - 117.62\,\Phi(d_2)$

(iii)





(iv) ?

(v) Using the Black-scholes formula

$p_t = \$18.35$

Q11 i) $\Delta_p = 100{,}000\,\Delta_c - 18673\,\Delta_s$

$0 = 100{,}000\,\Delta_c - 18673$

$\Delta_{sc} = 0.18673$

(ii) $\Phi(d_1) = \Delta_c$

$\therefore d_1 = -0.89$

~~$d_2 = -0.89 - \sigma\sqrt{t}$~~

Assuming ~~$r = 2\%$~~ 1%.

~~$C_t = 117.98\,\Phi(d_1) - 150 e^{-0.02}\,\Phi(d_2)$~~

$$-0.89 = \frac{\ln(117.98/150) + (0.02 + \frac{1}{2}\sigma^2)1}{\sigma\sqrt{1}}$$

$-0.89\sigma = \ln(0.78653) + 0.02 + \frac{1}{2}\sigma^2$

$\sigma^2 + 1.78\sigma - 0.4402 = 0$

$\sigma = \frac{-1.78 + 2.2202}{2} = 22\%$

(iii) Using the Black Scholes formula

$p = £31.45$

(iv) ?

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12) Assumptions

1) No taxes or transaction costs
2) Complete divisibility of holdings
3) Unlimited buying and selling
4) Underlying follows a continuous path
5) Risk free rate and volatility is constant
6) Underlying follows GBM
7) Investors are rational and risk-averse
8) $dS_t = \mu S_t dt + \sigma S_t dB_t$ where $B_t \rightarrow$ SBM

(ii) Using Blackscholes formula

$p = 1.61$

(iii) $r \propto \frac{1}{p}$