

Assignment-2.

Ans 1)

$$\text{Area} = \pi r^2$$

$$\text{Actual} = \pi (12.5)^2 = 156.25\pi = 490.8738$$

$$\text{Error value} = \pi (12.65)^2 = 160.0225\pi = 502.7255$$

i) Absolute error = $502.7255 - 490.8738$
 $= 11.8517$

ii) Relative error = $\frac{11.8517}{490.8738} \times 100 = 2.4144$

iii) Percentage error = $\frac{0.15}{12.5} \times 100 = 1.2\%$

$$\text{Error in calculation} = 1.2 \times 2 = 2.4\%$$

Ans 2)

x	$f(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

a) Interpolate $\log(5)$ using $x=4$ & $x=6$

4.0		0.60206
6.0		0.7781513

$$\Rightarrow h = 2$$

$$u = \frac{x - x_0}{h} = \frac{5 - 4}{2} = 0.5$$

$$\begin{aligned} f(x) &= 1.f(x_0) + u \Delta f(x_0) \\ &= 0.60206 + \frac{1}{2} (0.7781513 - 0.60206) \\ &= 0.690105 \end{aligned}$$

b)

4.5		0.6532125
5.5		0.7403627

$$\begin{aligned} f(x) &= 1.f(x_0) + u \Delta f(x_0) \\ &= 0.6532125 + \frac{1}{2} (0.7403627 - 0.6532125) \end{aligned}$$

$$= 0.696787$$

$$x^4 - x - 10 = 0$$

$$\begin{array}{r} x \\ 1.5 \\ 2 \end{array} \quad \begin{array}{r} y \\ -6.4375 \\ 4 \end{array}$$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1.5 + 2}{2} = 3.5 = 1.75$$

$$y_3 = (1.75)^4 - 1.75 - 10 = -2.3710$$

$$\begin{array}{r} x \\ 1.75 \\ 2 \end{array} \quad \begin{array}{r} y \\ -2.3710 \\ 4 \end{array}$$

$$x_4 = \frac{1.75 + 2}{2} = 3.75 = 1.875$$

$$y_4 = (1.875)^4 - 1.875 - 10 = 0.48461$$

$$\begin{array}{r} x \\ 1.75 \\ 1.875 \end{array} \quad \begin{array}{r} y \\ -2.3710 \\ 0.48461 \end{array}$$

$$x_5 = \frac{1.75 + 1.875}{2} = 3.625 = 1.8125$$

$$y_5 = (1.8125)^4 - 1.8125 - 10$$

$$= -1.0202$$

x	y
1.8125	-1.0202
1.8750	0.48461

$$x_6 = \frac{1.8125 + 1.8750}{2} = 1.84375$$

$$y_6 = (1.84375)^4 - (1.84375) - 10$$

$$= -0.2877$$

x	y
1.84375	-0.2877
1.8750	0.48461

$$x_7 = \frac{1.84375 + 1.8750}{2} = 1.8593$$

$$y_7 = 0.09152$$

Ans 4)

$$f(x) = x^3 - x - 1$$

$$f'(x) = 3x^2 - 1$$

$$x_1 = 1$$

$$f(x) = -1$$

$$f'(x) = 2$$

$$\begin{aligned}x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\&= 1 - \frac{(-1)}{2}\end{aligned}$$

$$= \frac{1+1}{2} = \frac{2}{2} = 1.5$$

$$f(x_2) = 0.875$$

$$f'(x_2) = 5.75$$

$$x_3 = 1.5 - \frac{0.875}{5.75}$$

$$= 0.84782$$

$$f(x_3) = -1.2884$$

$$f'(x_3) = 1.15684$$

$$x_4 = 0.84782 - \frac{(-1.2884)}{1.1564}$$

$$= 1.9187$$

$$f(x_4) = 4.1448$$

$$f'(x_4) = 10.044$$

$$x_5 = 1.9187 - \frac{4.1448}{10.044} = 1.5060$$

$$f(x_5) = 0.9096$$

$$f'(x_5) = 5.8041$$

$$x_6 = 1.5060 - \frac{0.9096}{5.8041}$$

$$= 1.3492$$

$$f(x_6) = 0.10680$$

$$f'(x_6) = 4.46102$$

$$x_7 = 1.3492 - \frac{0.10680}{4.46102}$$

$$= 1.3252$$

Ans 5) $A^2 = A$

$$7A - (1+A)^3$$

$$= 7A - (1^3 + A^3 + 3A^2 \cdot 1 + 3 \cdot 1^2 \cdot A)$$

$$\Rightarrow 7A - 1^3 - A^3 - 3A^2 - 3A$$

$$\Rightarrow 7A^2 - 1 - A^3 - 3A^2 - 3A^2$$

$$\Rightarrow -A^3 + A^2 - I$$

$$\Rightarrow -(A^2 \cdot A) + A^2 - I$$

$$\Rightarrow -A^2 + A^2 - I$$

$$= -I$$

$$[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix} A^2 = A]$$

$$[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix} A^2 = A]$$

Ans 6)

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\begin{aligned} |A| &= 1(0-6) - (-1)(-4-0) + 2(4-0) \\ &= -6 - 4 + 8 \\ &= -10 + 8 \\ &= -2 \end{aligned}$$

$$\text{adj } A = \begin{bmatrix} -6 & 4 & 4 \\ 4 & -1 & -1 \\ 4 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

Q7)

$$A = 8\hat{i} - 9\hat{j}$$

$$B = 7\hat{i} - 9\hat{j}$$

$$\vec{A} \cdot \vec{B} = 8(7) + (-9)(-9)$$

$$= 56 + 81$$

$$= 137$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$|\vec{a}| = \sqrt{8^2 + 9^2} = \sqrt{81 + 64} = \sqrt{145}$$

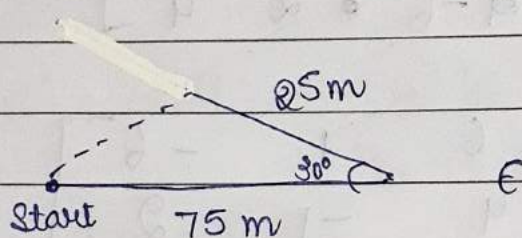
$$|\vec{b}| = \sqrt{7^2 + 9^2} = \sqrt{49 + 81} = \sqrt{130}$$

$$137 = \sqrt{145} \cdot \sqrt{130} \cos \theta$$

$$\cos \theta = \frac{137}{\sqrt{145} \cdot \sqrt{130}}$$

$$\theta = \cos^{-1} \left(\frac{137}{\sqrt{145} \cdot \sqrt{130}} \right)$$

Ans 8)



$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$R^2 = (75)^2 + (25)^2 + 2(75)(25) \cos 30^\circ$$

$$R^2 = 9497.595264$$

$$R = 97.4556$$

Ans 9) AP = 101, 104, 107, ..., 998.

$$\text{Sum of AP} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2(101) + (n-1)3]$$

$$a_n = a + (n-1)d$$

$$998 = 101 + (n-1)3$$

$$897 = 3n - 3$$

$$900 = 3n$$

$$300 = n$$

$$\text{Sum} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{300}{2} [2(101) + (299)3]$$

$$= 164850$$

Ans 10)

$$ar^5 = 32$$

$$ar^7 = 128$$

$$r^2 = \frac{128}{32} = 4$$

$$r^2 = 4$$

$$r = 2$$

$$\text{Ans 11)} (4 + 3x)^5$$

$$\Rightarrow {}^5C_0 4^5 + {}^5C_1 4^4 \cdot (3x)^1 + {}^5C_2 4^3 \cdot (3x)^2 + {}^5C_3 4^2 \cdot (3x)^3 + {}^5C_4 4^1 \cdot (3x)^4 + {}^5C_5 (3x)^5$$

$$\Rightarrow 1024 + (5)(256)(3x) + (10)(64)(9x^2) + (10)(16)(27x^3) + (5)(4)(81x^4) + 243x^5$$

$$\Rightarrow 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$$

Ans 12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$|A| = 2(-15) + 3(6) - 0 = 18 - 30 = -12$$

$$\Rightarrow A^3 - (a + e + i)A^2 + (m_{11} + m_{22} + m_{33})A - |A| = 0$$

$$\Rightarrow A^3 - (0)A^2 + (-15 + 6 - 4)A - (-12) = 0$$

$$= A^3 - 14A + 12 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -14 & 12 \\ & & 1 & -1 & -15 \\ x & 1 & -1 & -15 & \end{array}$$

Ans 13)

$$f(x) = x^3 - 7x^2 + 8x - 3$$

$$f'(x) = 3x^2 - 14x + 8$$

$$x_0 = 5$$

$$f(x_0) = -13$$

$$f'(x_0) = 13$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{(-13)}{13} = 5 + 1 = 6$$

$$x_1 = 6$$

$$f(x_1) = 9$$

$$f'(x_1) = 32$$

$$x_2 = 6 - \frac{9}{32} = 5.71875.$$

Ans 14) $(1 - x + x^2)^4$

$$= [1 + \cancel{x} + \cancel{x^2} - \cancel{x}]^4$$

$$= (1 - x + x^2)^4$$

$$= {}^4C_0 (1-x)^4 + {}^4C_1 x^2 (1-x)^3 + {}^4C_2 (x^2)^2 (1-x)^2 + {}^4C_3 (1-x)(x^2)^3 + {}^4C_4 (x^2)^4$$

$$= (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$$

Ans 15) $e^{-x} = 3 \log(x)$

$$e^{-x} - 3 \log(x) = 0$$

x	y
1	0.3678
2	-0.7677

$$x_1 = \frac{-(-0.7677) + 2(0.3678)}{2}$$

$$x_1 = \frac{1 + 2}{2} = 1.5$$

$$y_1 = e^{-1.5} - 3 \log(1.5) = -0.30514$$

x	y
1	0.3678
1.5	-0.305143

$$x_2 = \frac{x_0 + x_1}{2} = \frac{1 + 1.5}{2} = 1.25$$

$$y_2 = e^{-1.25} - 3 \log(1.25) = -0.004225$$

x	y
1	0.3678
1.25	-0.004225

$$x_3 = \frac{1 + 1.25}{2} = 1.125$$

$$y_3 = 0.17119$$

x	y
1.125	0.17119
1.25	-0.004225

$$x_4 = 1.1875$$

$$y_4 = 0.08108$$