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F.Y. Section A
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Calculus Assignment

1) Given,

$$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = \lim_{h \to 0} \frac{2[9+h^2-6h]-18}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 - 12h + 18 - 18}{h}$$

$$= \lim_{h \to 0} \frac{2h(h-6)}{h}$$

$$= \lim_{h \to 0} 2h - 12$$

$$= 2(0) - 12$$

$$\Rightarrow \lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = -12$$

2) Given,

$$g(x) = \begin{cases} 2x & ; \; x < 6 \\ x-1 & ; \; x \geq 6 \end{cases}$$

a) At $x = 4$,

$$g(4) = 2 \times 4 = 8$$

$$\lim_{x \to 4} g(x) = \lim_{x \to 4} 2x = 2 \times 4 = 8$$

$\therefore$ Limit exists and since $g(4) = \lim_{x \to 4} g(x)$;

the function is continuous

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b) At $x=6$,

$g(6) = 6-1 = 5$

$$LHL = \lim_{x \to 6^-} 2x = 2(6) = 12$$

$$RHL = \lim_{x \to 6^+} x-1 = 6-1 = 5$$

$\therefore$ $LHL \neq RHL$.

$\therefore$ limit do not exist.

$\therefore$ At $x=6$, $g(x)$ is discontinuous

3) Given,

$$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x\to 9} - \frac{\left((\sqrt{x})^2 - (3)^2\right)}{\sqrt{x}-3}$$

$$= \lim_{x\to 9} - \frac{(\sqrt{x}+3)(\sqrt{x}-3)}{\sqrt{x}-3}$$

$$= \lim_{x\to 9} -(\sqrt{x}+3)$$

$$= -(\sqrt{9}+3)$$

$$\Rightarrow \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = -6$$

⑤

4) Given,

$$f(x) = \frac{4x+5}{9-3x}$$

a) At $x=-1$,

$$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$$

$$\lim_{x \to -1} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$$. ~~∅~~ limit exists

Here, $f(-1) = \lim_{x \to -1} f(x)$.

∴ limit ~~exists~~ $f(x)$ is continuous at $x=-1$

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b) At $x=0$,

$$\lim_{x \to 0} \frac{4x+5}{9-3x}$$ does not exist

b) At $x=0$,

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$$\lim_{x \to 0} \frac{4x+5}{9-3x} = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

Here $f(0) = \lim_{x \to 0} f(x)$.

$\therefore f(x)$ is continuous at $x=0$.

c) At $x=3$,

$$LHL = \lim_{x \to 3^-} \frac{4x+5}{9-3x} = \frac{4(3)+5}{9-3(3)} = \infty$$

$$RHL = \lim_{x \to 3^+} \frac{4x+5}{9-3x} = \frac{4(3)+5}{9-3(3)} = -\infty$$

$LHL \neq RHL$ $\therefore$ limit does not exist. Hence discontinuous

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5) Given,

$$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

Dividing numerator and denominator by e^{4x}.

$$= \frac{\lim_{x \to \infty} \left(6 - \frac{1}{e^{6x}}\right)}{\lim_{x \to \infty} \left(8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}\right)} \qquad \left(\because \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}\right)$$

$$= \frac{6 - \frac{1}{e^{6(\infty)}}}{8 - \frac{1}{e^{2(\infty)}} + \frac{3}{e^{5(\infty)}}}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4} \text{ or } 0.75$$

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6) Given,

$$g(y) = \begin{cases} y^2+5 & ; y < -2 \\ 1-3y & ; y \geq -2 \end{cases}$$

a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} 1-3y = 1-3(6) = 1-18 = -17$

b) $LHL = \lim_{y \to -2^-} g(y) = \lim_{h \to 0} (-2-h)^2+5 = (-2)^2+5 = 9$

$RHL = \lim_{y \to -2^+} g(y) = \lim_{h \to 0} 1-3(-2+h) = 1-3(-2) = 7$

$LHL \neq RHL$

$\therefore \lim_{x \to -2} g(y)$ does not exist

7) Given,

$$f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$$

Differentiating with respect to 't', we have,

$$f'(t) = (t^3 - 8t^2 + 12)\left(\frac{d}{dt}(4t^2 - t)\right) + (4t^2 - t)\left(\frac{d}{dt}(t^3 - 8t^2 + 12)\right)$$

$$\therefore \left((uv)' = vu' + uv' \right)$$

$$= (t^3 - 8t^2 + 12)(8t - 1) + (4t^2 - t)(3t^2 - 16t)$$

$$= 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12 + 12t^4 - 64t^3 - 3t^3 + 16t^2$$

$$= 8t^4 + 12t^4 - t^3 - 64t^3 - 64t^3 - 3t^3 + 8t^2 + 16t^2 + 96t - 12$$

$$f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

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8) Given,

$$R(w) = \frac{3w + w^4}{2w^2 + 1}$$

Differentiating with respect to w, we have,

$$R'(w) = \frac{(2w^2+1)\left(\frac{d}{dw}\, 3w + w^4\right) - (3w + w^4)\left(\frac{d}{dw}\, 2w^2+1\right)}{(2w^2+1)^2}$$

$$\left(\because \left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}\right)$$

$$= \frac{(2w^2+1)(3 + 4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$$

$$= \frac{4w^3+3}{(2w^2+1)} - \frac{(4w^5 + 12w^2)}{(2w^2+1)^2}$$

$$= \frac{(4w^3+3)(2w^2+1) - 4w^5 - 12w^2}{(2w^2+1)^2}$$

$$R'(w) = \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

9) Given,

$f(x) = 2e^x - 8^x$

Differentiating ~~w~~ with respect to x, we have,

$f'(x) = 2\frac{d}{dx}e^x - \frac{d}{dx}8^x$

$= 2e^x - \log(8) \cdot 8^x$

$\left(\because \frac{d}{dx}e^x = e^x, \frac{d}{dx}a^x = a^x \log a\right)$

$\therefore f'(x) = 2e^x - 8^x \cdot \log 8$

10) Given,

$$y = z^5 - e^z \log(z)$$

Diffentiating with respect to z, we have,

$$\frac{dy}{dz} = 5z^4 - \left[\log(z)\, e^z + e^z \times \frac{1}{z}\right]$$

$$\left(\because \frac{d}{dx} x^n = n x^{n-1};\ \frac{d}{dx} \log x = \frac{1}{x};\right.$$

$$\left.(uv)' = vu' + uv'\right)$$

$$\Rightarrow \frac{dy}{dz} = 5z^4 - \log(z) \cdot e^z - \frac{e^z}{z}$$

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11) a) Given,

$$f(x) = (6x^2 + 7x)^4$$

Differentiating with respect to x, we have,

$$f'(x) = 4(6x^2 + 7x)^3 \cdot (12x + 7)$$

b) Given,

$$f(t) = 5 + e^{4t + t^7}$$

Differentiating with respect to t, we have,

$$f'(t) = 0 + e^{4t + t^7} \times (4 + 7t^6)$$

$$= e^{4t+7}(4 + 7t^6)$$

12) a) Given,

$$z = \log(7-x^3)$$

Differentiating w.r.t. x, we have,

$$\frac{dz}{dx} = \frac{1}{7-x^3} \times (-3x^2) = \frac{-3x^2}{7-x^3}$$

Again, differentiating w.r.t. x, we have,

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$\frac{d^2z}{dz^2} = \frac{-(3x^4+42x)}{(7-x^3)^2}$$

b) Given,

$$g(v) = \frac{2}{(6+2v-v^2)^4} = 2(6+2v-v^2)^{-4}$$

Differentiating w.r.t. v, we have,

$$g'(v) = -8(6+2v-v^2)^{-5}(2-2v)$$

$$= -\cancel{8}(2-\cancel{2v})$$

$$= \frac{-16(1-v)}{(6+2v-v^2)^5}$$

Again differentiating w.r.t. v, we have,

$$g''(v) = -16\left[\frac{(6+2v-v^2)^5(-1) - (1-v)(5)(6+2v-v^2)^4(2-2v)}{(6+2v-v^2)^{10}}\right]$$

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$$= \frac{-16\,(6+2v-v^2)^4\left[v^2-2v-6-10(1-v)^2\right]}{(6+2v-v^2)^{10}}$$

$$= \frac{-16\left[v^2-2v-6-10v^2+20v-10\right]}{(6+2v-v^2)^6}$$

$$\phi''(v) = \frac{-16\left[-9v^2+18v-16\right]}{(6+2v-v^2)^6}$$

c) Given,

$$f(t) = \log(1+t^2)$$

Differentiating w.r.t. t, we have,

$$f'(t) = \frac{2t}{1+t^2}$$

Again differentiating w.r.t. t, we have,

$$f''(t) = \frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2} = \frac{2+2t^2-4t^2}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2}$$

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13) Given,

$f(x) = x^3 - 3x^2 - 9x + 12$

Differentiating wrt. x, we have,

$f'(x) = 3x^2 - 6x - 9$

To find critical points, let $f'(x) = 0$. Then,

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or -1

Now,

$f''(x) = 6x - 6$

$f''(3) = 6(3) - 6 = 18 - 6 = 12$ ($f''(3) > 0$. Hence minimum)

$f''(-1) = 6(-1) - 6 = -6 - 6 = -12 < 0$. Hence maximum

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Here, $f(x)$ is minimum at $x = 3$ and maximum at $x = \cancel{4} -1$.

$\therefore$ Maximum value $= (3)^3 - 3(3)^2 - 9(3) + 12 = 27 - 27 - 27 + 12 = -15$

Minimum value $= (-1)^3 - 3(-1)^2 - 9(-1) + 12 = -1 - 3 + 9 + 12 = 17$

14) Given,

$f(x) = 4x^3 - 18x^2 + 24x - 7$

Differentiating w.r.t.x, we have,

$f'(x) = 12x^2 - 36x + 24$

To find critical points, let $f'(x) = 0$. Then,

$12x^2 - 36x + 24 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\therefore x = 2, 1$

$f''(x) = 24x - 36$

$f''(2) = 24(2) - 36 = 48 - 36 = 12 > 0$. Hence minima

$f''(1) = 24(1) - 36 = 24 - 36 = -12 < 0$. Hence maxima

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Here $f(x)$ is maximum at $x = 1$ and minimum at $x = 2$.

$\therefore$ Maximum value $= 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 28 - 25$

$= 3$

Minimum value $= 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 4(8) - 18(4) + 48 - 7$

$= 32 - 72 + 48 - 7$

$= 80 - 79$

$= 1$

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15) Given,

$f(x) = x^2 (x+3) = y$ (say)

Here, ~~x y~~- intercepts are,

$x^2(x+3)=0$

$\Rightarrow \boxed{x = 0, -3}$

$f'(x) = (x+3)(2x) + x^2$

$= 2x^2 + 6x + x^2$

$= 3x^2 + 6x$

To find critical points, let $f'(x) = 0$. Then,

$3x^2 + 6x = 0$

$\Rightarrow x^2 + 2x = 0$

$\Rightarrow x(x+2) = 0$

$\Rightarrow x = 0, -2$

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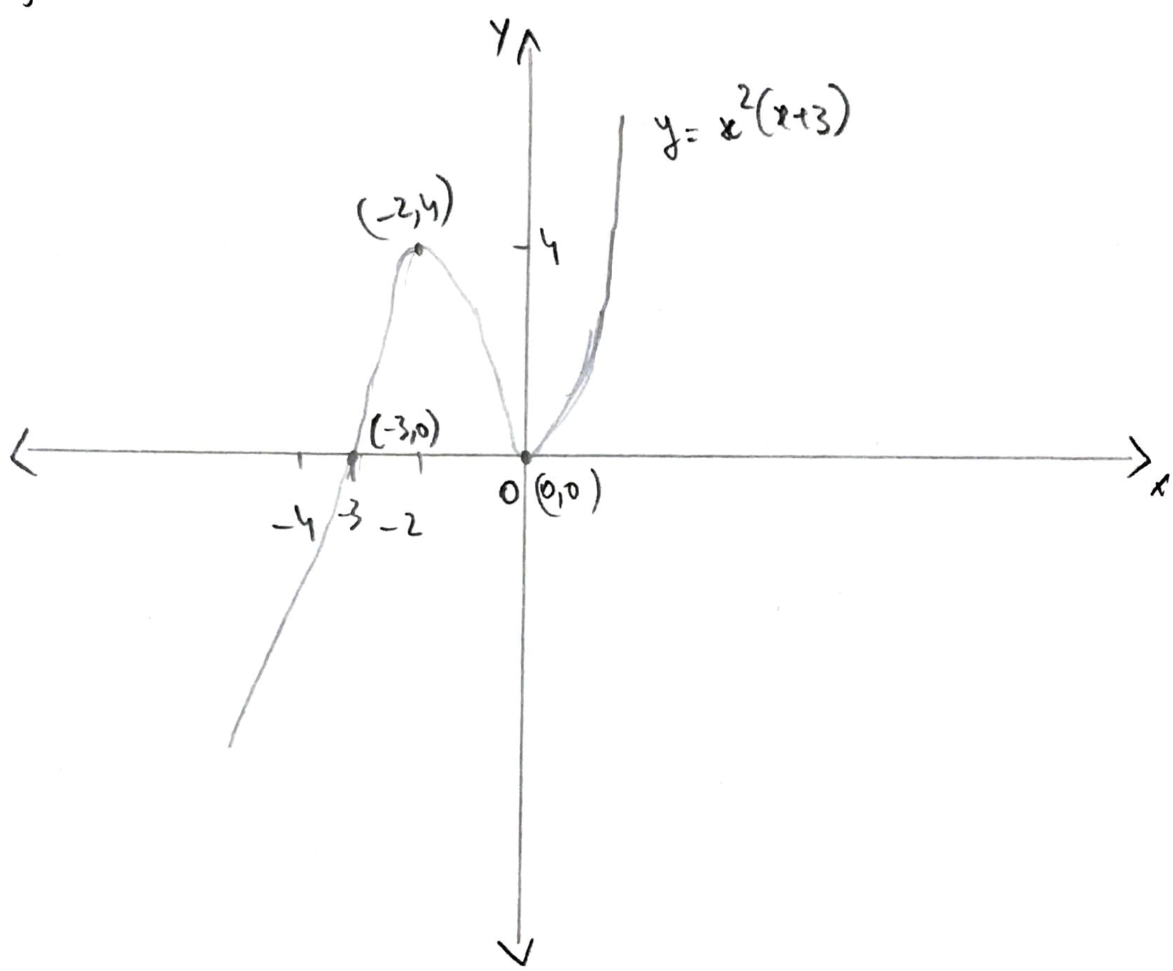
$f''(x) = 6x + 6$

$f''(0) = 6(0) + 6 = 6 > 0$, Hence minima

$f''(-2) = 6(-2) + 6 = -12 + 6 = -6$. Hence maxima



$\therefore$

$f(0) = 0^2(0+3) = 0$ and $f(-2) = (-2)^2(-2+3) = 4$

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16) Given,

$f(x) = 3x^2 - 6x$

Here x- intercepts are;

$$3x^2 - 6x = 0$$

$$\Rightarrow \boxed{x = 0, 2}$$

$f'(x) = 6x - 6$

To find critical points, let $f'(x) = 0$. Then,

$$6x - 6 = 0$$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

$f''(x) = 6$

$f''(1) = 6 > 0$. Hence minima

$\therefore f(x)$ is minimum at $x = 1$. i.e $(1, -3)$

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