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Probablity and Statistics.
- Assignment 1.

Q1. To find: mean, median, mode, std.dev., interquartile range.

i) mean $= \dfrac{4+7+9+9+11+15+18+18+18+25}{10}$

$\bar{x} = 13.4$

median, A.O = 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

median $= \dfrac{11+15}{2} = 13$

mode = 18

Standard Deviation =

x	$(x-\bar{x}) = d$	d^2
4	-9.4	88.36
7	-6.4	40.96
9	-4.4	19.36
9	-4.4	19.36
11	-2.4	5.76
15	1.6	2.56
18	4.6	21.16
18	4.6	21.16
18	4.6	21.16
25	11.6	134.56
n = 10	~~Σd = 0~~	$\Sigma d^2 = 374.4$

$\sigma = \sqrt{\dfrac{\Sigma d^2}{n-1}} = \sqrt{\dfrac{374.4}{9}} = \sqrt{41.6}$

$\sigma = \cancel{6.45}$

$\therefore \boxed{\sigma = 6.45 = \sqrt{41.6}}$

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Interquartile Range :

$= Q_3 - Q_1$, $\quad Q_3 = \frac{18+18}{2} = 18$

$Q_1 = \frac{7+9}{2} = 8$

$\Rightarrow IQR = 18-8 = 10$

Q1(ii) → after Q2.

Q2. It is assumed that claims arising on an i... policy can be modelled as a Poisson Process at the rate of 0.5 per year.

Soln. i) $\underset{\text{(claims)}}{X} \sim \text{Poisson}(\lambda = 0.5)$ $\quad$ rate $= \lambda = 0.5$ — (1)

$P(X=0) = \frac{e^{-0.5} \times 0.5^0}{0!} = \frac{e^{-0.5} \times 1}{1}$

(formula used : $\frac{e^{-\lambda} \cdot \lambda^x}{x!}$)

$= e^{-0.5} = \boxed{0.60653}$

ii) $3 \cdot [P(x \geq 1) \cdot P(X=0)^2] = 3 \cdot [(0.60653)^2 \cdot \{1 - P(x \leq 0)\}]$

$= 3\,[(0.36787) \cdot \{1 - 0.60653\}]$

$= \boxed{0.434257}$

iii) Exponential dist. :- $P(X \geq 2) = 1 - P(X \leq 2)$

here $P(X \leq 2) = (1 - e^{-\lambda x})$

$\therefore \cancel{1} - (\cancel{1} - e^{-\lambda x})$

$P(X \geq 2) = e^{-\lambda x}$

from (1) $= e^{-0.5(2)}$

$= e^{-1}$

$\boxed{P(x \geq 2) = 0.36787}$

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Q1. Part 2. Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$

Median $= \frac{(n+1)}{2}^{th}$ term $= 30.5^{th}$ term $= 2$

Mode $= 2$ (highest freq.)

Std. deviation $= \frac{\sqrt{\frac{1}{n-1} \Sigma fx^2 - \bar{x}^2 n}}{\sqrt{\frac{1}{n-1} (\Sigma f(fx - \bar{x})^2}} = \sqrt{\frac{58}{59}}$

$= \sqrt{0.98305} = \boxed{0.99148}$

Inter quartile Range: $Q_3 - Q_1$

$\frac{3}{4}(n+1)^{th} - \frac{1}{4}(n+1)^{th} = \frac{3(61)}{4}^{th} - \frac{(61)}{4}^{th}$

$= 45.75^{th} - 15.25^{th} = 3 - 1 = 2$

Q3. An analyst is interested in using gamma distribution with parameters $\alpha = 2$ and $\lambda = 1/2$. The range of x is defined as $0 < x < \infty$.

Soln →

$$X \sim \text{Gamma}\left(\alpha = 2, \lambda = \tfrac{1}{2}\right) \qquad 0 < x < \infty$$

i) • $E(x) = \dfrac{2}{1/2} = \boxed{4}$

$$V(x) = \frac{\alpha}{\lambda^2} = \frac{2}{(1/2)^2} = \boxed{8}$$

• $SD(x) = \sigma = \sqrt{V(x)} = \sqrt{8} = \boxed{2.828}$

ii) Since the variance is 8 i.e. very high the tiltedness is towards right making it positively skewed (as mean is 4.)

iii) $$F_X(x) = \int_0^x f_X(x) \cdot dx$$

$$= \int_0^x \frac{(1/2)^{(2)}}{\Gamma 2} \cdot x^{(2-1)} \cdot e^{-x/2}$$

$$= \frac{1}{4} \int_0^x x \cdot e^{-x/2} \cdot dx$$

$$= \frac{1}{4} \left[x \int e^{-x/2} - \int \left(\frac{d}{dx}(x) \cdot \int e^{-x/2} \cdot dx\right) \cdot dx \right]_0^x$$

$$= \frac{1}{4} \left[x \cdot \frac{e^{-x/2}}{-1/2} - \int \left(1 \cdot \frac{e^{-x/2}}{-1/2} \cdot dx\right) \right]_0$$

$$= \frac{1}{4} \left[x \cdot e^{-x/2}(-2) - \int e^{-x/2}(-2) \right]$$

$$= \frac{1}{4} \left[x \cdot e^{-x/2}(-2) - 4e^{-x/2} \right]$$

PTO

$$= \frac{1}{4} \times \left[\left(-2x e^{-x/2} - 4e^{-x/2} \right) - (-4) \right]$$

$$= \frac{1}{4} \cdot \left(\frac{-2x}{\sqrt{e^x}} - \frac{4}{\sqrt{e^x}} + 4 \right)$$

$$= 1 - \frac{0.5x}{\sqrt{e^x}} - \frac{1}{\sqrt{e^x}}$$

$$= \boxed{\frac{\sqrt{e^x} - 0.5x - 1}{\sqrt{e^x}}}$$

Q4.

10 M	→ 6 fully q.
8 F	7 fully q.

let x be the prob. tht. 2 q. females members w al least 1 female

$P(x) = ?$

$N = 0, 1, 2$ (females & males)

$$= \frac{{}^8C_1 + {}^8C_2 + {}^{10}C_1 + {}^{10}C_2}{{}^{18}C_2}$$

$\Rightarrow$ $P(M) = 10/18$ $P(F) = 8/18$

$P(F.Q/M) = 6/10$ $P(FQ/F) = 7/8$

$\therefore$ Prob. $= \frac{7}{\cancel{18}\,3} \times \frac{\cancel{6}}{17} = \boxed{\frac{7}{51}}$.

$P(2F) = \frac{\cancel{8}\,4}{\cancel{18}\,9} \times \frac{7}{17} = \boxed{\frac{28}{153}}$

$\therefore$ $\frac{\cancel{7}}{5\cancel{1}} \times \frac{\cancel{153}\,3}{\cancel{28}\,4} = \frac{3}{4} = \boxed{0.75}$.

Q5. The supply chain service in a country has 3 levels of service. Level 1 (next day del.)
Level 2 (2 day del.)
Level 3 (3-day del.)

40% of packages del. via. Level 1
50% via Level 2
10% via Level 3.

Calc. the prob. of that a late package was sent via Level 2 service.

Soln. →

$P(1) = 0.4$ + $P(L/L_1) = 0.2$

$P(2) = 0.5 \longrightarrow + P(L/L_2) = 0.4$

$P(3) = 0.1$ + $P(L/L_3) = 0.1$

to find : $P(L_2/L) = \dfrac{P(L/L_2).P(L_2)}{\sum_{i=1}^{3} P(L/L_i).P(L_i)}$

$= \dfrac{0.4 \times 0.5}{(0.8) + (0.2) + (0.01)} \rightarrow \dfrac{0.2}{(0.4\times0.2)+(0.5\times0.4)+(0.1\times0.1)}$

$= \dfrac{0.2}{0.29} = \boxed{0.689656}$

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Q6. If $X \sim U(1,2)$ and $Y = \frac{3}{6-X}$ determine PDF of Y

Soln. $\rightarrow$ PDF of $Y = F_Y(y) = P(Y \leq y) = P\left(\frac{3}{6-X} \leq y\right)$

$$\Rightarrow P\left(\frac{3}{6-X} \leq y\right)$$

Step 1 taking inverse of $\frac{3}{6-X} = Y$

$$3 = Y(6-X)$$

$$\frac{3}{Y} = 6 - X$$

$$x = 6 \overset{-}{\cancel{+}} \frac{3}{Y}$$

$$\boxed{x = \frac{6y \overset{-}{\cancel{+}} 3}{y}}$$

$$\therefore P\left(\frac{3}{6-X} \leq y\right) = P\left(x \geq \frac{6y \overset{-}{\cancel{+}} 3}{y}\right)$$

$$= P\left(x \geq \frac{6y - 3}{y}\right)$$

Step 2 differenciate to find PDF,

$$F_Y'(y) = f_X(y) = \frac{6y-3}{y} = 6 - \frac{3}{y}$$

$$\frac{dx}{dy} = 0 - \frac{3}{y^2}(-1)$$

$$\frac{dx}{dy} = \frac{3}{y^2}$$

$$\boxed{\text{PDF of } Y = \frac{3}{Y^2} = f_Y(y)}$$

Q7. i) Discrete R.V X.

Calc: $F(3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= \underline{\underline{0.55}}$.

ii) $E(x) = \sum_{i=x}^{n} x \cdot P(X=x)$

here : $\sum_{x=1}^{5} x \cdot P(X=x)$

$\rightarrow 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$\rightarrow \underline{\underline{3.25}}$

iii) $V(x) = E[x^2] - [E(x)]^2$

$\sum_{\text{all } x} x^2 \cdot P(X=x) - (3.25)^2$

$\rightarrow 1^2(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05) - (3.25)^2$

$\rightarrow 11.45 - 10.56$

$\rightarrow \underline{\underline{0.8875}}$

iv) Coeffecient of skewness.

$= \dfrac{3(\text{Mean} - \text{Median})}{\sigma}$

$= \dfrac{3(3.25 - 3)}{\sqrt{V(x)}}$

$= \dfrac{3(0.25)}{\sqrt{0.8875}}$

Coeff of skew. $= \boxed{0.7961}$

Q8. Insurance claims amounts are indepedent and exceed 1000 with probablity 0.2. In a sample of 15 claims, calculate the probablity that fewer than 3 of them exceed 1000.

formula $^nC_r \cdot p^r \cdot q^{n-r}$

Soln. →

$x \sim Bin(15, 0.2)$

to find $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= 15C_0 \cdot (0.2)^0 \cdot (0.8)^{15} + {}^{15}C_1 \cdot (0.2)^1 \cdot (0.8)^{14} + {}^{15}C_2 \cdot (0.2)^2 (0.8)^{13}$

$= 0.035184 + 0.13194 + 0.2308971$

$= \boxed{0.398022}$

$P(x<3) = 0.398022$

Q9. A man enters a raffle each week where the probablity of winning a price is 0.01. Calc. the prob. that he takes seven weeks up to & including the week where he wins the third price.

Soln. →

$P(\text{winning}) = 0.01$

3 successes out of 7 weeks.

$X \sim Binomial(3, 0.01)$

$= {}^nC_r \cdot p^r \cdot q^{n-r}$

$= {}^7C_3 \cdot (0.01)^3 (0.99)^4$

$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{4! \, 3! \times 3 \times 2} \cdot (0.01)^3 \cdot (0.99)^4$

$= 35 \times 9.6059601 \times 10^{-7}$

$= \boxed{3.36208605 \times 10^{-5}}$

Q10. The number of claims submitted to an insurance offices averages two per working week (of 5 days). Calc. the prob. that more than two claims arrive in a day.

Soln. given ⇒ 2/5 per week

$P(2/5 \text{ per day}) = ?$

$\therefore X \sim \text{Poisson}(\underset{\substack{\downarrow \\ \text{rate} = ?}}{\lambda}, 2/5)$

$P(x > 2) = 1 - P(x \leq 2)$

$P(x > 2) = 1 - 0.99207 = 0.00793$

Q11. If the prob. of having a male & female child is equal and a woman has 5 boys and 2 girls, calc. the prob. that her next three children are boys.

Soln. $P(B) = 0.5$

$P(G) = 0.5$

5B
2G

⇒ having a boy (next 3 times)

$= 0.5 \times 0.5 \times 0.5$

$= \underline{0.125}$

Q12. Claims follow a poisson process with rate 10 per day. Calc. the probablity the time bet. 2 claims is less than 0.1 days.

Soln. →

$X \sim$ Poisson (10 each day), λ = rate = 10

There is a gap bet. 2 claims ∴ Expo.

∴ $X \sim$ Expo. $(\lambda = 10)$

$$P(X<0.1) = 1 - e^{-\lambda x}$$
$$= 1 - e^{-10 \cdot (0.1)}$$
$$= 1 - e^{-1}$$
$$= 1 - 0.367879$$
$$\boxed{P(X<0.1) = 0.6321205588}$$

Q13. If X is following the parameters $a = 75$ & $b = 120$ with a uniform dist., calc. the prob. that X lies between 80 and 100.

Soln. → Since it is a uniform dist., the PDf is constant.

$$P(80 < x < 100) = P(100) - P(80)$$
$$= \frac{100-75}{120-75} - \frac{80-75}{120-75}$$
$$= \frac{25-15}{45} = \frac{10}{45} = \boxed{0.2222\ldots}$$

Q14. If $X \sim Gamma\ (5, 0.4)$, calc. $P(X<22\cdot$

*·

Soln.
→

*Q15. Cont. R.V 'x' has PDF :- $\frac{K}{(x+5)^4}$, $x > 0$

Calc. i) K
ii) F (10)
iii) E (X)

Soln.
→