

ASSIGNMENT-1

Chapter: Error, Dimensions and Interpolation

Homework-1: Lagrange's Interpolation

Ans 1)

x	$f(x)$
7	3
8	1
9	1
10	9

To find $f(9.5)$

$$\begin{aligned}
 f(9.5) &= \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3 \\
 &+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1 \\
 &+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1 \\
 &+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9
 \end{aligned}$$

$$\Rightarrow 0.4375 + (-2.5) + 8.4375 + 2.125$$

$$\Rightarrow 0.1875 + (-0.3125) + 0.9375 + (2.8125)$$

$$= 3.625$$

Ans 2)

x	$f(x)$
3	168
7	120
9	72
10	63

Find at $x = 6$

$$f(6) \Rightarrow \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168$$

$$+ \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72$$

$$+ \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$\Rightarrow 12 + 180 + (-72) + 27 = 147$$

Ans 3)	n	$f(n)$
	1	4
	2	5
	7	5
	8	4

To find $f(6)$

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4$$

$$+ \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$= -0.7619 + 1.6667 + 6.6667 + (-1.9047)$$

$$= 5.668 \text{ or } 4.6664 \text{ (if rounded up values).}$$

Ans 4)

f(5) w	f(x)
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1	2
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2	4
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3	8
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4	16
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7	128
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To find f(5)

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

$$+ \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$= (-0.6667) + 6.4 + (-24) + (42.6667) + 8.5334$$

$$= 32.9$$

Ans 5)

x	$f(x)$	x	$f(x)$
10	0.25	10	0.25
14	0.15	20	0.20
16	0.10	14	0.15
20	0.2	16	0.10

$$\begin{aligned}
 f(15) &= \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25 \\
 &+ \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.20 \\
 &+ \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15 \\
 &+ \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow (-0.0052) + (-0.0041) + (0.078125) + (0.0052) \\
 &= 0.120825 \quad \text{or} \quad 0.1208
 \end{aligned}$$

Ans 6) Home-work-2: Regula Falsi method

Ans 1) $x^3 + x - 1 = 0$

x	y
0.5	-0.375
1	1

$$\bar{x} = \frac{0.5(1) - 1(-0.375)}{1 - (-0.375)} = 0.6363$$

$$\bar{y} = -0.10607$$

x	y
0.6363	-0.10607
1	1

$$\bar{x} = \frac{0.6363(1) - (1)(-0.10607)}{1 - (-0.10607)}$$

$$\bar{x} = 0.6711$$

$$\bar{y} = -0.0266$$

x	y
0.6711	-0.0266
1	1

$$\bar{x} = \frac{0.6711(1) - 1(-0.0266)}{1 - (-0.0266)} = 0.6796$$

$$\bar{y} = -0.0065$$

x	y
0.6796	-0.0065
1	1

$$\bar{x} = \frac{0.6796(1) - (1)(-0.0065)}{1 - (-0.0065)} = 0.6816$$

$$\bar{y} = -0.0017$$

$$\bar{x} = 0.6816 \quad \bar{y} = -0.0017$$

Ans 2) $x^3 - 2x + 0.5 = 0$

x	y
0	0.5
1	-0.5

$$\begin{aligned} \bar{x} &= \frac{0(-0.5) - (1)(0.5)}{-0.5 - (-0.5)} \\ &= \frac{-0.5}{-1} = 0.5 \end{aligned}$$

$$\bar{y} = -0.375$$

x	y
0.5	-0.375
1	

x	y
0	0.5
0.5	-0.375

$$\bar{x} = \frac{0(-0.375) - (0.5)(0.5)}{-0.375 - 0.5} = 0.2857$$

$$\bar{y} = -0.0480$$

Ans 3) $x^3 + 2x - 20 = 0$

x	y
2	-8
2.5	0.625

$$\bar{x} = \frac{2(0.625) - (2.5)(-8)}{0.625 - (-8)} = 2.4637$$

$$\bar{y} = -0.6046$$

x	y
2.4637	-0.6046
2.5	0.625

$$\bar{x} = \frac{2.4637(0.625) - (2.5)(-0.6046)}{0.625 - (-0.6046)} = 2.4815$$

$$\bar{y} = 0.2436$$

x	y
2.4637	-0.6046
2.4815	0.2436

$$\bar{x} = \frac{2.4637(0.2436) - (2.4815)(-0.6046)}{0.2436 - (-0.6046)}$$

$$= 2.47639$$

$$\bar{y} = 0.13926$$

x	y
2.4637	-0.6046
2.47639	0.13926

$$\bar{x} = 2.4740 \Rightarrow \left(\frac{2.4637(0.13926) - 2.47639(-0.6046)}{2.47639(0.13926) + 0.6046} \right)$$

$$\bar{y} = 0.0905$$

Ans.)

$$3x - \cos x - 1 = 0$$

x	y
0	-2
1	1.4596

$$\bar{x} = \frac{0(1.4596) - (1)(-2)}{1.4596 - (-2)} = 0.5781$$

$$\bar{y} = -0.1032$$

x	y
0.5781	-0.1032
1	1.4596

$$\bar{x} = \frac{(0.5781)(1.4596) - (1)(-0.1032)}{1.4596 - (-0.1032)}$$

$$= 0.6059$$

$$\bar{y} = -0.004$$

Ans 5) $x^3 - 3x - 5 = 0$

x	y
2	-3
2.5	3.125

$$\bar{x} = \frac{2(3.125) - (-3)(2.5)}{3 - (-3.125 - (-3))}$$

$$= 2.2448$$

$$\bar{y} = -0.4225$$

x	y
2.2448	-0.4225
2.5	3.125

$$\Rightarrow \frac{2.2448(3.125) - (2.5)(-0.4225)}{3.125 - (-0.4225)}$$

$$\Rightarrow 2.2751$$

$$\bar{y} = -0.049$$

x	y
2.2751	-0.049
2.5	3.125

$$\bar{x} = \frac{2.2751(3.125) - (2.5)(-0.049)}{3.125 - (-0.049)}$$

$$\bar{x} = 2.2785$$

$$\bar{y} = -0.0065$$

Ans 6) $x \log_{10} x - 1.2 = 0$

x	y
2	-0.5979
3	0.2313

$$\bar{x} = \frac{2(0.2313) - (3)(-0.5979)}{0.2313 - (-0.5979)}$$

$$= 2.7210$$

$$\bar{y} = -0.0171$$

x	y
2.7210	-0.0171
3	0.2313

$$\bar{x} = \frac{2.7210(0.2313) - (3)(-0.0171)}{0.2313 - (-0.0171)}$$

$$\bar{x} = 2.7402$$

$$\bar{y} = -0.0003$$

Ans 7) $x^3 - 4x + 1 = 0$

x	y
1	-2
2	1

$$\bar{x} = \frac{1(1) - 2(-2)}{1 - (-2)} \Rightarrow \frac{1+4}{3} = \frac{5}{3} = 1.6667$$

$$= -1.0368$$

$$\bar{y} = -1.0368$$

x	y
1.6667	-1.0368
2	1

$$\bar{x} = \frac{1.6667(1) - (2)(-1.0368)}{1 - (-1.0368)} = 1.8363$$

$$\bar{y} = -0.1532$$

x	y
1.8363	-0.1532
2	1

$$\begin{aligned}\bar{x} &= \frac{1.8363(1) - (2)(-0.1532)}{1 - (-0.1532)} \\ &= 0.9951 \quad 1.8580\end{aligned}$$

$$\bar{y} = -0.017$$

x	y
1.8580	-0.017
2	1

$$\bar{x} = \frac{1.8580(1) - (2)(-0.017)}{1 - (-0.017)}$$

$$\bar{x} = 1.8525 \quad 1.8603$$

$$\bar{y} = -0.00322$$

Ans 8) $x^3 + 2x^2 - 8 = 0$

x	y
1	-5
2	8

$$x = \frac{8 - (-10)}{8 - (-5)} = \frac{18}{13} = 1.2$$

$$y = -3.392$$

x	y
1.2	-3.392
2	8

$$\begin{aligned}\bar{x} &= \frac{1.2(8) - (2)(-3.392)}{8 - (-3.392)} \\ &= 1.4382\end{aligned}$$

$$\bar{y} = -0.8883$$

x	y
1.4382	-0.8883
2	8

$$\begin{aligned}\bar{x} &= \frac{1.4382(8) - (2)(-0.8883)}{8 - (-0.8883)} \\ &= 1.4943\end{aligned}$$

$$\bar{y} = -0.1974$$

x	y
1.4943	-0.1974
2	8

$$\bar{x} = \frac{1.4943(8) - (-0.1974)(2)}{8 - (-0.1974)}$$

$$\bar{x} = 1.506$$

$$\bar{y} = -0.04$$

Ans 9) $x^3 - 9x + 1 = 0$

x	y
2	-9
3	1

$$\bar{x} = \frac{2(1) - 3(-9)}{1 - (-9)} = 2.9$$

$$\bar{y} = -0.711$$

x	y
2.9	-0.711
3	1

$$\bar{x} = \frac{2.9(1) - (3)(-0.711)}{1 - (-0.711)}$$

$$= 2.941$$

$$\bar{y} = -0.030$$

x	y
2.941	-0.03
3	1

$$\begin{aligned}\bar{x} &= \frac{2.941(1) - (-0.03)(3)}{1 - (-0.03)} \\ &= \frac{2.941 + (0.03)(3)}{1.03}\end{aligned}$$

$$\bar{x} = 2.9427$$

$$\bar{y} = -0.002$$

Ans 10) $x^3 - x - 4 = 0$

x	y
1	-4
2	2

$$\bar{x} = \frac{2(1) - (-4)(2)}{2 - (-4)} = 1.6667$$

$$\bar{y} = -1.0367$$

x	y
1.6667	-1.0367
2	2

$$\bar{x} = \frac{1.6667(2) - (2)(-1.0367)}{2 - (-1.0367)}$$

$$\bar{x} = 1.7804$$

$$\bar{y} = -0.1368$$

x	y
1.7804	-0.1368
2	2

$$\bar{x} = \frac{1.7804(2) - (2)(-0.1368)}{2 - (-0.1368)}$$

$$\bar{x} = 1.7944$$

$$\bar{y} = -0.021$$

x	y
1.7944	-0.021
2	2

$$\bar{x} = \frac{1.7944(2) - (-0.021)(2)}{2 - (-0.021)}$$

$$\bar{x} = 1.7965$$

$$\bar{y} = 0.0015$$