UNIT 2 → assignment

Q.1) $$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\,dw$$

let $\frac{2}{w} = t$

$\frac{-2}{w^2}\,dw = dt$

$dw = \frac{w^2\,dt}{-2}$

subsituting dw

$$\int_{100}^{1} -\frac{1}{2} e^t\,dt = \frac{1}{2}\int_{1}^{100} e^t\,dt$$

when, $w=2$, $t=1$

$w=\frac{1}{50}$, $t=100$

$$= \frac{1}{2}\left[e^t\right]_1^{100}$$

$$= \frac{1}{2}\left(e^{100} - e\right)$$

$$\boxed{\therefore \int_{1/50}^{2} \frac{e^{2/w}}{w^2}\,dw = \frac{e}{2}\left(e^{99}-1\right)}$$

Q.2) $$I = \int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

let $(t^4+2t) = u$

$4t^3+2\,dt = du$

$$dt = \frac{du}{(4t^3+2)} = \frac{1}{2}\frac{du}{(2t^3+1)}$$

$$\boxed{\int \frac{2t^3+1}{(t^4+2t)^3}\,dt = \int \frac{du}{2u^3} = -\frac{1}{4u^2} + C = \frac{-1}{4(t^4+2t)^2} + C}$$

Q.3) $f'(x) = x^4 + 3x - 9$

$$f(x) = \int f'(x)\,dx = \int x^4 + 3x - 9\,dx = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

$$\boxed{\therefore f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C.}$$

Q.4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx = \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \left[x^3\right]_{-2}^{1} + \left.6x\right]_{1}^{3}$$

$$= 1 + 8 + 6(2) = 21$$

$$\boxed{\therefore \int_{-2}^{3} f(x)\,dx = 21}$$

Q.5) $\int 3(8y-1)e^{4y^2-y}\,dy$

Let $4y^2-y=t$

$8y-1=\frac{dt}{dy}$

$\int 3(8y-1)e^{4y^2-y}\,dy=\int 3e^t\,dt=3e^t=3e^{4y^2-y}+c$

Method 2 → $\int f'(x)e^{f(x)}\,dx=e^{f(x)}+c$

$$\boxed{\int 3(8y-1)e^{4y^2-y}\,dy=3e^{4y^2-y}+c}$$

Q.6) $f''(x)=15\sqrt{x}+5x^3+6$

$f'(x)=\int f''(x)\,dx$

$=\int 15x^{1/2}+5x^3+6\,dx$

$=10x^{3/2}+\frac{5x^4}{4}+6x+k$

$f(x)=\int f'(x)\,dx=\int 10x^{3/2}+\frac{5}{4}x^4+6x+k\,dx$

$f(x)=4x^{5/2}+\frac{x^5}{4}+3x^2+kx+c$

$f(1)=-\frac{5}{4}=4+\frac{1}{4}+3+k+c$

$\therefore\ 4k+4c=-34\ \cdots(i)$

$f(4)=404=4(4)^2\sqrt{4}+\frac{(4)^5}{4}+3(4)^2+4k+c$

$404 = 128 + 256 + 48 + 4k + c$

$4k + c = -28 \cdots (2)$

from (i) and (2) we get,

$3c = -6 \quad \therefore c = -2$

putting value of c in (2) $\rightarrow$ $k = \frac{-26}{4} = \frac{-13}{2}$

$$\boxed{f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13x}{2} - 2}$$

Q.7) $z = f(x+iy) + g(x-iy)$

→ Partially differentiating w.r.t x, let it be a.

$a = f'(x+iy) + g'(x-iy) \cdots (i)$

→ Partially differentiating w.r.t y, let it be b.

$b = f'(x+iy)i + g'(x-iy)(-i) \cdots (ii)$

→ Partially differentiating (i) w.r.t x, let it be c.

$c = f''(x+iy) + g''(x-iy) \cdots (iii)$

→ Partially differentiating (ii) w.r.t y, let it be d.

$d = i^2 f''(x+iy) + i^2 g''(x-iy) \cdots (iv)$

$d = i^2 (f''(x+iy) + g''(x-iy))$

$d = (\sqrt{-1})^2 (c)$

$$\boxed{d = -c}$$

Q.8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$

integerating both side

$$\int r^{-2}\,dr = \int \theta^{-1}\,d\theta$$

when $r = 2, \theta = 1$ ~~and when~~ then

$$\Rightarrow \quad \frac{1}{-r} = \ln\theta + k$$

$$\therefore \; -k = \ln(1) + \frac{1}{2} = \frac{1}{2}$$

$$\therefore \; k = -1/2$$

then equation is

$$\frac{-1}{r} = \ln\theta - \frac{1}{2} \quad \Rightarrow \quad \boxed{\frac{1}{2} - \frac{1}{r} - \ln\theta = 0}$$

9) $\frac{dv}{dt} = 9.8 - 0.196V$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$\frac{1}{9.8} \int \frac{dv}{1 - 0.02v} = \int dt$$

$1 - 0.02v = u$

$-0.02dv = du$

$dv = \frac{du}{-0.02}$

$$\int dt = \frac{-1}{0.196} \int \frac{du}{u} = \frac{-1}{0.196} \log u$$

$$\frac{-\log u}{0.196} = t + k.$$

$$0.196t + 0.196k + \log u = 0$$

$$0.196t + 0.196k + \log(1 - 0.02v) = 0$$

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10) $t\frac{dy}{dt} + 2y = t^2 - t + 1$

$$\frac{dy}{dt} + \frac{2}{t}y = \frac{t^2 - t + 1}{t}$$

$IF \rightarrow e^{\int p\,dt} = e^{\int \frac{2}{t}dt} = expo(2\log t) = expo(\log t^2) = t^2$

$y * IF = \int Q * IF.$

$$yt^2 = \int \frac{t^2 - t + 1}{t} \times t^2 dt = \int t^3 - t^2 + t\,dt = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

$$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{c}{t^2}$$

when $t = 1$, $y = 1/2$ $\quad \cancel{\frac{1}{2}} = \frac{1}{4} - \frac{1}{3} + \cancel{\frac{1}{2}} + c \quad \boxed{c = 1/12}$

$$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{t^{-12}}{12}$$

Q.11) $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$$\int \frac{dy}{y^3} = \int \frac{x\,dx}{\sqrt{1+x^2}}$$ let $(1+x^2) = t$

$2x\,dx = dt$

$$-\frac{1}{2y^2} = \int \frac{1}{2} \times \frac{dt}{\sqrt{t}} = \frac{1}{2} \frac{\sqrt{t}}{1/2} = \sqrt{t} + c$$

$$-\frac{1}{y^2} = 2\sqrt{1+x^2} + c$$

when, $x=0$ put $y=-1$

$-1 = 2 + c$

$c = -3$

$$0 = 2\sqrt{1+x^2} - 3 + \frac{1}{y^2}$$

$$\boxed{0 = 2y^2\sqrt{1+x^2} - 3y^2 + 1}$$

$\sqrt{1+x^2} > 0$

$x \in (-\infty, \infty)$

thus, the validity of interval is $(-\infty, \infty)$

Q-12) $f(3) = (3)^3 - 10(3)^2 + 6 = -57$

$f(x) = x^3 - 10x^2 + 6$

$f'(x) = 3x^2 - 20x$

$f'(3) = 3(9) - 20(3) = -33$

$f''(x) = 6x - 20$

$f''(3) = -2$

$f'''(x) = 6$

$f'''(3) = 6$

$f''''(x) = 0$

$f''''(3) = 0$

TAYLOR SERIES→

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$$

$$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{6}(6) + \frac{(x-3)^4}{4!} \times 0$$

$$f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3 + \cdots 0$$

$$f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

Q.13) $f(0)=1$

$f'(x)=\mu(1+x)^{\mu-1}$ $\quad f'(0)=\mu$

$f''(x)=\mu(\mu-1)(1+x)^{\mu-2}$ $\quad f''(0)=\mu(\mu-1)$

$f'''(x)=\mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$ $\quad f'''(0)=\mu(\mu-1)(\mu-2)$

$f''''(x)=\mu(\mu-1)(\mu-2)(\mu-3)(1+x)^{\mu-4}$

$\cancel{f'(0)=0=f''(0)=f'''(0)=f''''(0)}$ $\quad f''''(0)=\mu(\mu-1)(\mu-2)(\mu-3)$

MACLAURIN SERIES →

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\frac{x^4}{4!}f''''(0)+\cdots\cdots$$

$$f(x)=1+\mu x+\frac{\mu(\mu-1)}{2!}x^2+\frac{\mu(\mu-1)(\mu-2)}{3!}x^3+\frac{\mu(\mu-1)(\mu-2)(\mu-3)}{4!}x^4+\cdots$$

Q.14) $f'(x)=\frac{1}{2\sqrt{1+x}}$, $\quad f(0)=1$, $f'(0)=1/2$

$f''(x)=\frac{-1}{4(1+x)^{3/2}}$, $\quad f''(0)=-1/4$

$f'''(x)=\frac{3}{8(1+x)^{5/2}}$, $\quad f'''(0)=3/8$

$f''''(x)=\frac{-15}{8\times 2}\times\frac{1}{(1+x)^{7/2}}$, $\quad f''''(0)=-15/16$

MACLAURIAN SERIES →

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\frac{x^4}{4!}f''''(0)+\cdots\cdots$$

$$f(x)=1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}+\cdots\cdots$$

Q·15) $f(x) = e^{-6x}$; $f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$; $f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$; $f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$; $f'''(-4) = -216e^{24}$

$f''''(x) = 1296e^{-6x}$; $f''''(-4) = 1296e^{24}$

TAYLOR SERIES →

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \frac{(x-a)^4}{4!}f''''(a) + \cdots$$

$$f(x) = f(-4) + (x+4)f'(-4) + \frac{(x+4)^2}{2!}f''(-4) + \frac{(x+4)^3}{3!}f'''(-4) + \frac{(x+4)^4}{4!}f''''(-4) \cdots$$

$$f(x) = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3 + 54e^{24}(x+4)^4 + \cdots$$

$$= e^{24}\left(1 - 6(x+4) + 18(x+4)^2 - 36(x+4)^3 + 54(x+4)^4 + \cdots\right)$$

Q·16) $f(x) = \ln(4x+3)$; $f(0) = \ln(3)$

~~f(0)~~ $f'(x) = \frac{1}{4x+3}$; $f'(0) = 1/3$

$f''(x) = \frac{-1}{(4x+3)^2}$; $f''(0) = -1/9$

$f'''(0) = 2/27$

$f'''(x) = \frac{+2}{(4x+3)^3}$; $f''''(0) = -\cancel{8}/81 = -2/27$

$f''''(x) = \frac{-6}{(4x+3)^4}$;

$$f(x) = f(0) + (x-0)f'(0) + \frac{(x-0)^2}{2!}f''(0) + \frac{(x-0)^3}{3!}f'''(0) + \frac{(x-0)^4}{4!}f''''(0) + \cdots$$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{x^2}{9} + \frac{x^3}{81} - \frac{x^4}{324} + \cdots$$