Calculus

Assignments - 1

(1.) $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = \lim_{h \to 0} \frac{2(h^2-12h+9)-18}{h}$

$= \lim_{h \to 0} \frac{2h^2-12h}{h} = \lim_{h \to 0} 2h-12$, $h \to 0$ $\therefore h \neq 0$

$= 2(0-6) = \underline{\underline{-12}}$

(2.) $g(x) = 2x \quad , x<6$
$\quad\quad = x-1 \quad , x \geq 6$

(a.) at $x=4$

~~$g(4) = g(4) = 8$~~

$\lim_{x \to 4} g(4) = \lim_{x \to 4} 2x = 2(4) = 8$

$\therefore$ $g(x)$ is continuous at $x=4$

(b.) at $x=6$

L.H.L. $= \lim_{x \to 6^-} 2x = 2(6) = 12$

R.H.L. $= \lim_{x \to 6^+} x-1 = 6-1 = 5$

$\therefore$ L.H.L. $\neq$ R.H.L.

$\Rightarrow \lim_{x \to 6^-} \neq \lim_{x \to 6^+}$

$\therefore$ $g(x)$ is discontinuous at $x=6$

(3.) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim_{x \to 9} \frac{-(3+\sqrt{x})\cancel{(9-x)}}{\cancel{(9-x)}}$, $x \to 9$, $\therefore x-9 \neq 0$

$= \lim_{x \to 9} -(3+\sqrt{x}) = -3 \pm 3$

$= \underline{\underline{-6 \text{ or } 0}}$

(4.) $f(x) = \frac{4x+5}{9-3x}$

(a.) at $x = -1$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \underline{\underline{\frac{1}{12}}}$

$\therefore$ $f(x)$ is continuous at $x = -1$

(b.) at $x = 0$

$f(0) = \frac{4(0)+5}{9-3(0)} = \underline{\underline{\frac{5}{9}}}$

$\therefore$ $f(x)$ is continuous at $x = 0$

(c.) at $x = 3$

$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \underline{\underline{\infty}}$

$\therefore$ $f(x)$ is discontinuous at $x = 3$.

5. $$\lim_{x\to\infty} \frac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+3e^{-x}}$$

$$= \lim_{x\to\infty} \frac{e^{x}(6e^{4}-e^{-2})}{e^{x}(8e^{4}-e^{2}+3e^{-1})}$$

$$= \frac{327.4535649}{430.4997825} = 0.760635843$$

6. $$g(y) = \begin{cases} y^2+5, & y<-2 \\ 1-3y, & y\geq -2 \end{cases}$$

(a.) $\lim_{y\to 6} g(y) = \lim_{y\to 6} 1-3y = 1-3(6) = -17$

(b.) $\lim_{y\to -2} g(y)$

L.H.L. $= \lim_{y\to -2^-} y^2+5 = (-2)^2+5 = 9$

R.H.L. $= \lim_{y\to -2^+} 1-3y = 1-3(-2) = 7$

~~7.~~ $\cancel{f(t) = (4t^2-t)(t^3-8t^2+12)}$

$\cancel{f'(t) = (t^3-8t^2+12)\left[\frac{4t^3}{3} - \frac{t^2}{2}\right] + (4t^2-t)\left[\frac{t^4}{4} - \frac{8t^3}{3} - 12t\right]}$

7. $f(t) = (4t^2-t)(t^3-8t^2+12)$

$f'(t) = (8t-1)(t^3-8t^2+12)+(4t^2-t)(3t^2-16t)$

$= 8t^4+96t-64t^3-t^3+8t^2-12+12t^4-64t^3$

$-3t^3+16t^2$

$= 20t^4-132t^3+24t^2+96t-12$

8. $R(w) = \dfrac{3w + w^4}{2w^2+1}$

$$R'(w) = \frac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$$

$$= \frac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$$

$$= \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

9. $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log 8$

10. $y = z^5 - e^z \ln(z)$

$y' = 5y^4 - e^z \ln(z) - \dfrac{e^z}{z}$

13. $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$(x-3)(x+1) = 0$

$\Rightarrow x = 3$ or -1

$f''(x) = 6x - 6$

$f''(-1) = -6 - 6 = -12$

$\Rightarrow$ Maxima at $x = -1$

$f''(3) = 18 - 6 = 12$

$\Rightarrow$ Minima at $x = 3$

$\Rightarrow$ Maximum value $= f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$
$= \underline{\underline{17}}$..

$\Rightarrow$ Minimum value $= f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$
$= \underline{\underline{-15}}$..

(14.) $f(x) = 4x^3 - 18x^2 + 24x - 7$
$f'(x) = 12x^2 - 36x + 24$
$0 = 12(x^2 - 3x + 2)$
$(x-1)(x-2) = 0$
$\therefore x = 1$ or 2
$f''(x) = 24x - 36$
$f''(1) = 24 - 36 = -12$
$\Rightarrow$ Maxima at $x = 1$
$f''(2) = 48 - 36 = 12$
$\Rightarrow$ Minimum at $x = 2$

$\Rightarrow$ Maximum Value $= f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$
$= \underline{\underline{3}}$..

$\Rightarrow$ Minimum value $= f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$
$= \underline{\underline{1}}$..

(15.) $f(x) = x^2(x+3) = x^3 + 3x^2$
$f'(x) = 3x^2 + 6x$
$0 = 3x(x+2)$
$x = -2$ or 0
$f''(x) = 6x + 6$
$f''(-2) = -6$
$f''(0) = 6$

16. $f(x) = 3x^2 - 6x$

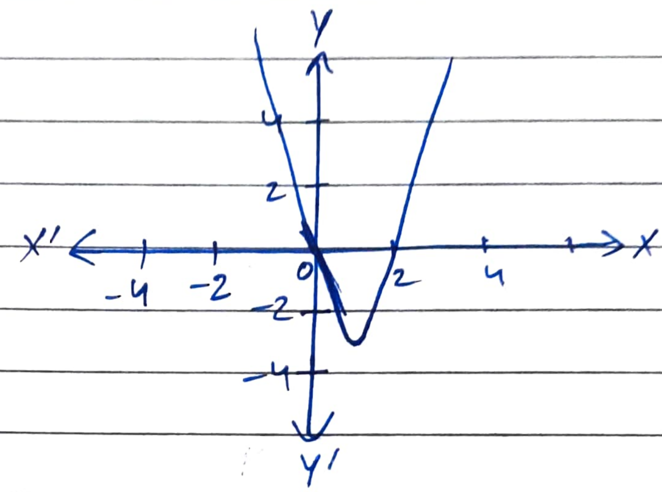
$f'(x) = 6x - 6$

$6x - 6 = 0$

$x - 1 = 0$

$\underline{\underline{x = 1}}$

$f''(x) = \underline{\underline{6}}$



12. (a.) $z = \ln(7 - x^3)$

$z' = \dfrac{-3x^2}{7 - x^3}$

$z'' = \dfrac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$

$= \dfrac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$

$= \dfrac{-42x - 3x^4}{(7-x^3)^2} = \dfrac{-3x(x^3+14)}{(7-x^3)^2}$

(b.) $Q(v) = \dfrac{2}{(6+2v-v^2)^4} = 2(6+2v-v^2)^{-4}$

$Q'(v) = 2 \times -4(6+2v-v^2)^{-5} \times (2-2v)$

$= \dfrac{-16(1-v)}{(6+2v-v^2)^5}$ or $-16(1-v)(6+2v-v^2)^{-5}$

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$$\phi''(v) = -16(6+2v-v^2)^{-5} + (-16)(-v+1)(2-2v)(-5)(6+2v-v^2)^{-6}$$

$$= -16(6+2v-v^2)^{-6}\left[1+2(1-v)^2(-5)\right](6+2v-v^2)$$

$$= -16(6+2v-v^2)^{-6}\left[6+2v-v^2-10(1+v^2-2v)\right]$$

$$= -16(6+2v-v^2)^{-6}\left[6+2v-v^2-10-10v^2+20v\right]$$

$$= \frac{-16(-11v^2+22v-4)}{(6+2v-v^2)^6} = \frac{16(11v^2-22v+4)}{(6+2v-v^2)^6}$$

(c.) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{2t}{1+t^2}$$

$$f''(t) = \frac{2(1+t^2) - 2t(2t)}{(1+t^2)^2}$$

$$= \frac{2(1+t^2-2t^2)}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2}$$

11. (a.) $f(x) = (6x^2+7x)^4$

$$f'(x) = 4(6x^2+7x)^3(12x+7)$$

$$= 4(12x+7)(6x^2+7x^3)$$

(b.) $f(x) = 5 + e^{4t+t^7}$

$$f'(x) = e^{4t+t^7} \times (4+7t^6)$$

$$= (4+7t^6)e^{4t+t^7}$$