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Financial mathematics

① $APR = \dfrac{\text{average annual interest}}{1/2 \times \text{original loan}}$

$= \dfrac{2 \times \text{average annualization}}{\text{origination}}$

original loan amount = Rs 20,000

$= 20000 = 12 \times 427.90 a^{(12)}_{\overline{5|}}$

$= 3.8499$

Flate rate $= \dfrac{\text{total interest}}{\text{loan} \times \text{total years}}$

$= \dfrac{5 \times 12 \times 427.90 - 20000}{20000 \times 5}$

$= \underline{5.67\%}$

$APR \approx 2 \times 5.67 \approx 11\%$

$i = 11\%, \quad a^{(12)}_{\overline{5|}} = 3.878$

$i = 10\,i, \quad a^{(12)}_{\overline{5|}} = 3.961$

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I is $\frac{P - 10\%}{3.8499 - 3.96154}$

$= \frac{11\% - 10\%}{3.87872 - 3.96514}$

$i = 10.8\%$

$12 \times 427.9\, a_{\overline{5}|}^{(12)} = 20000.2$

$i = 10.9\%$,

$12 \times 427.9\, a_{\overline{5}|}^{(12)} = 19958.2$

$i = 10.8\%$ is more close.

$\therefore$ APR $= 10.8\%$

ii) APR $= 10.8\%$

After 1year capital left to pay $= 12 \times$

$a_{\overline{4}|@\,10.8}^{(12)} \times 427.90 = 16775.98$

let t denote term of new loan thus

$16775.98 = 12 \times 274.49\, a_{t}^{(12)}$

$= 167795.98 = 3293.88 \frac{(1 - 1.108^{t})}{12 \times (1.108^{1/12} - 1)}$

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$1.108^{-t} = 0.4754$

$t = 7.25$ years.

(ii) Total interest $= 4 \times 12 \times 427.90 - 16775.68$

$= 3763.22$

~~PAO~~ New loan has term of 7.25 years hence.

Total interest $= 7.25 \times 12 \times 274.49 -$

16775.68

$= 7104.65$

- They'll have to pay (7104.65 − 3763.22)

3341.23

Q2 (i) Discounted payback period is the

a. time for which the present value of the rise up to time exceeds the present value the cost up to time t.

b. Payback period: It is the exact time for which the monetary value of the return exceeds the monetary value of the costs.

(ii) Unlike the NPV, neither the PPP nor the PP give indication.

PP can give misleading results as it doesn't consider the time value of money.

iii) a). The rate of interest that satisfies the equation of value:

$$-170000 - 20000v - 10000v^2 + 20000v + 20000v^2 + 200000v^2 = 0$$

$$= 10v^2 + 200v^3 - 170$$
$$= 10v^2 + 200v^3 - 170 = 0$$

$0.9 \rightarrow -16.1$
$1 \rightarrow 40$
$0.93 \rightarrow -0.479$
$0.94 \rightarrow 4.9538$
$0.9309 \rightarrow 0.0046$

$\therefore v = 0.9309$

$$\frac{1}{1+i} = 0.9309$$

$$i = 7.4\%$$

b) Net present value =

Project A NPV $= -170000 + 10000v^2 + 200000v^3 = 6823.82$

Project B NPV $= -200000 + 14000a_{\overline{6}|} + 200000v^6 = 9834.645$

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~~c)~~

(3) Value on 1st Nov 1985 is

$\Rightarrow$ Annuity 1 $= 200 \times v^{(1/4)} \times \ddot{a}_{22}$ at 8%

$= 200 \times v^{1/4} \times a_{\overline{22}|} \times \frac{i}{d}$

$= 200 \times 0.980944 \times 10.2007 \times \frac{0.08}{0.074074}$

$= 2161.36630.$

$v^{1/4} = 0.980944$

$a_{\overline{22}|} = 10.2007$

$d = 0.074074$

$= 2161.366301$

$\rightarrow$ Annuity 2 $= 320(i+1)^{1/12} \times a^{(4)}_{\overline{16.25}|}$ at 8%

$= 320 \times 1.006434 \times 9.184257$

$= 2957.870081$

$\rightarrow$ Annuity 3 $= 180 \times a_{\overline{18.75}|}$

$= 180 \times \frac{1 - (0.925926)^{18.75}}{0.077208}$

$= 180 \times 9.89257870$

$= 1780.66416$

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Revised annuity =

$$\times (1+i)^{1/4} \times a^{(2)}_{\overline{21.5|}} (1+i)^{1/4}.$$

$$A = \frac{6899.90}{1.019427 \times 10.30886}$$

$$= \underline{\underline{656.56}}$$

④ $(I\ddot{a})_n$ for $i > 0$

$$(I\ddot{a})_{\overline{n|}} = 1 + 2v + 3v^2 + 4v^3 - - \cdot nv^{n-1}$$

$$(I\ddot{a})_{\overline{n|}} = (1+i)(Ia)_{\overline{n|}}$$
$$= \frac{(1+i)\ \ddot{a}_{\overline{n|}} - nv^n}{i}$$

$$= \ddot{a}_{\overline{n|}} - nv^n / i \Big/ 1+i$$

$$(I\ddot{a})_{\overline{n|}} = \ddot{a}_{\overline{n|}} - nv^n \Big/ d$$

⑤ Property fund $= \frac{16.4}{12.4} - 1$

$$= \underline{0.3226}$$

Equity fund $= \frac{15.5}{12.1} - 1$

$$= \underline{\underline{0.2810}}$$

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ii)

a) Assuming that investor bought 100 units per quarter.

$$1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1580(1+i)^{1/4} = \underline{4 \times 1640}$$

- $1+i = x$.

$= 1240x + 131x^{3/4} + 148x^{1/2} + 158x^{1/4} - 656 = 0$.

$\rightarrow x \approx 1.2932$.

$i = 29.32\%$

b) $$400 \times \ddot{s}_{1}^{(4)} = 100 \times 16.4 \left(\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{15.8} \right)$$

$$= \ddot{s}_{1}^{(4)} = 100 \times 4.72051784 / 400$$

$$= \ddot{s}_{1}^{(4)} = \underline{1.1801}$$

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$\therefore$ $i = 2.956\%$

$i \to$ $1.2 \to -0.057$
$1.3 \to 0.0012$

$i = 0.2979$

(6) i) Initial ~~annuale~~ annual repayment :

$160000 = X a_{\overline{10}|}$ @ 8%

$X = \dfrac{160000}{a_{\overline{10}|}} = \dfrac{160000}{6.7_{\overline{10}|}} = 23844.65$

ii) Loan outstanding after 4th payment :

$23844.65 a_{\overline{6}|}$ @ 8% $= 23844.65 \times 4.6229 = 110{,}231.442$

Revised annual instalment :

$y a_{\overline{6}|} = 110231.44$ @ 10%

$y = 110231.442 / a_{\overline{6}|}$

$= 110231.447 / 4.353$

$= 25309.72$

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(ii) The loan outstanding after 7th repayment is

$25309.72 \, a_{\overline{3}|} @ 10\%$

$= 25309.72 \times 2.4869 = 62942.74.$

→ Revised annual instalment → Z

$Z a_{\overline{3}|} = 62942.74 @ 9\%$

$Z = 62942.74 / a_{\overline{3}|}$

$= 62942.74 / 2.5313$

$= 24865.77$

→ $160000 + 1465.07 a_{\overline{4}|} - 443.95 * a_{\overline{7}|} - 24865.77 \, a_{\overline{10}|} = 0.$

$= 160000 + 1465.07 \left(\frac{1 - (1/1+i)^4}{i} \right) -$

$443.95 \left(\frac{i - (1/1+i)^7}{i} \right) -$

$24865.77.$