ASSIGNMENT 1

1) $\lim_{h\to 0} \frac{2(h-3)^2 - 18}{h}$

$= \lim_{h\to 0} \frac{2(h^2 - 6h + 9) - 18}{h}$

$= \lim_{h\to 0} \frac{2h^2 - 12h + \cancel{18} - \cancel{18}}{h}$

$= \lim_{h\to 0} 2h - 12$

$= 2(0) - 12$

$= \boxed{-12}$

2)

Function is given $g(x) = \begin{cases} 2x, & x < 6 \\ x-1, & x \geq 6 \end{cases}$

(a) For $x = 4$,

$\rightarrow \lim_{x\to 4^+} g(x) = \lim_{x\to 4^-} g(x) = \lim_{x\to 4^\pm} 2x$

$= 2(4)$

$= 8$

(b) For $x = 6$,

$\rightarrow$ R.H.L $= \lim_{x\to 6^+} g(x) = \lim_{x\to 6^+} x - 1 = 6 - 1 = \boxed{5}$

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$\rightarrow$ $L.H.L = \lim_{x \to 6^-} g(x) = \lim_{x \to 6^-} 2x = \boxed{12}$

$\rightarrow$ $g(6) = (6)-1 = \boxed{5}$

$\rightarrow$ $g(x)$ is discontinuous at $x = 6$.

3)

Solⁿ

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

$$= \lim_{x \to 9} \frac{-(9-x)}{3-\sqrt{x}}$$

$$= \lim_{x \to 9} \frac{-(\cancel{3-\sqrt{x}})(3+\sqrt{x})}{\cancel{3-\sqrt{x}}}$$

$$= -(3+\sqrt{9})$$

$$= -3-3 \quad \text{or} \quad = -3+3$$
$$= -6 \qquad\qquad = 0$$

4)

Solⁿ

Here, $f(x) = \dfrac{4x+5}{9-3x}$

(a) For $x = -1$, $f(-1) = \dfrac{-4+5}{9+3} = \dfrac{1}{12}$

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(b) For $x=0$,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{4x+5}{9-3x} = \boxed{\frac{5}{9}}$$

(c) For $x=3$,

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{4x+5}{9-3x}$$

$$= \cancel{\lim_{x\to 3}} \frac{4(3)+5}{9-3(3)}$$

$$= \frac{17}{0}$$

= Not possible.

$\therefore$ The given function is not continuous at $x=3$.

5.)

Soln

$$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \lim_{x\to\infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$= \cancel{\lim} \frac{6 - \frac{1}{e^{\infty}}}{8 - \frac{1}{e^{\infty}} + \frac{3}{e^{\infty}}}$$

$$= \frac{6}{8}$$

$$= \boxed{\frac{3}{4}}$$

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6)

Sol^n $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

(a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} 1-3y$

$= 1-3(6)$

$= 18$

(b) For $y = -2$,

$\rightarrow$ R.H.L $= \lim_{y \to -2^+} g(y) = \lim_{y \to 2^+} g(1-3y)$

$= 1+6$

$= \boxed{7}$

$\rightarrow$ L.H.L $= \lim_{y \to -2^-} g(y) = \lim_{y \to -2^-} (y^2+5)$

$= 4+5$

$= 9$

$\rightarrow$ $g(-2) = 1-3(-2) = 7$

(7)

Sol^n $f(t) = (4t^2-t)(t^3-8t^2+12)$

$\Rightarrow f'(t) = (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t$

$= 16t^4 - 13\cancel{1}t^3 + 16t^2 + 96t - t^3 - 8t^2 - 12$

$= 16t^4 - 132t^3 + 8t^2 + 96t - 12$

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8.)

Soln $R(\omega) = \frac{3\omega + \omega^4}{2\omega^2+1}$

$\therefore R'(\omega) = \frac{(2\omega^2+1)(3+4\omega^3) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)^2}$

$= \frac{6\omega^2 + 3 + 8\omega^5 + 4\omega^3 - 12\omega^2 - 4\omega^5}{4\omega^4 + 4\omega^2 + 1}$

$= \frac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{4\omega^4 + 4\omega^2 + 1}$

9)

$f(x) = 2e^x - 8^x$

$\Rightarrow f'(x) = 2.e^x - 8^x \log 8$

$= 2.e^x - 8^x \log(2^3)$

$= 2.e^x - 8^x.3(0.3010)$

$= 2.e^x - 0.9030.8^x$

10).

$\rightarrow y = z^5 - e^z \ln(z)$

$\therefore \frac{dy}{dz} = 5z^4 - \left[e^z\left(\frac{1}{z}\right) + \ln(z)(e^z)\right]$

$= 5z^4 - \frac{e^z}{z} + \ln(z)e^z$

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11.)

(a)

$f(x) = (6x^2 + 7x)^4$

$\Rightarrow f'(x) = 4(6x^2+7x)^3 \cdot (12x+7) \cdot (12)$

$= 48(216x^6 + 343x^3 + 756x^5 + 882x^4)(12x+7)$

$= 48(2592x^7 + 4116x^4 + 9072x^6 + 10{,}584x^5$
$+ 1512x^6 + 2401x^3 + 5292x^5 + 6174x^4)$

$= 48(2592x^7 + 10584x^6 + 15876x^5 + 10{,}290x^4$
$+ 2401x^3)$

(b.) $f(t) = 5 + e^{4t+t^7}$

$\cancel{\therefore f'(t) = 4t+t^7 \; (5+e)^{4t}}$

$\therefore f'(t) = e^{4t+t^7} \cdot (4+7t^6)$

$\Rightarrow \boxed{= 4e^{4t+t^7} + 7t^6 \cdot e^{4t+t^7}}$

12) Second Derivative

a) $z = \ln(7 - x^3)$

$\Rightarrow \dfrac{dz}{dx} = \dfrac{1}{7-x^3}(-3x^2)$

$\Rightarrow \dfrac{d^2z}{dx^2} = \dfrac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$

$\Rightarrow = \dfrac{\cancel{2}\cdot 6x^4 - 42x - 9x^4}{49 - 14x^3 + x^6}$

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(b) $Q(v) = 2\,(6+2v-v^2)^{-4}$

$\Rightarrow Q'(v) = 2(-4)\,(6+2v-v^2)^{-5} \cdot (2-2v)$

$= \dfrac{-16(1-v)}{(6+2v-v^2)^5} = \dfrac{16(v-1)}{(6+2v-v^2)^5}$

$\Rightarrow Q''(v) = 16 \left[\dfrac{(6+2v-v^2)^5(1) - (v-1)5(6+2v-v^2)^4(2-2v)}{6+2\cdot} \right]$

(c) $f(t) = \ln(1+t^2)$

$\Rightarrow f'(t) = \dfrac{1}{1+t^2}(2t)$

$\Rightarrow f''(t) = \dfrac{(1+t^2)(2) - (2t)(2t)}{t^4+2t^2+1}$

$= \dfrac{2t^2+2-4t^2}{t^4+2t^2+1}$

$= \dfrac{2-2t^2}{t^4+2t^2+1}$

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13) $f(x) = x^3 - 3x^2 - 9x + 12$

$\Rightarrow f'(x) = 3x^2 - 6x - 9$
$= 3x^2 - 9x + 3x - 9$
$= 3x(x-3) + 3(x-3)$
$= (3x+3)(x-3)$

$\therefore$ zeros are -1 & 3.

$\Rightarrow f''(x) = 6x - 6$

$\rightarrow$ For $x = -1$,

$f''(x) = 6(-1) - 6 = -6 - 6 = -12$ Maxima

$\rightarrow$ For $x = 3$,

$f''(x) = 6(3) - 6 = 18 - 6 = 12$ Minima

14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$\rightarrow f'(x) = 12x^2 - 36x + 24$
$= 12[x^2 - 3x + 2]$
$= 12(x-2)(x-1)$

$\Rightarrow$ The roots are 1 & 2

$\rightarrow f''(x) = 24x - 36$

$\rightarrow$ For $x = 1$, $f''(x) = 24 - 36 = -12$ Maxima

$\rightarrow$ For $x = 2$, $f''(x) = 48 - 36 = 12$ Minima

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15. Sketch the curve:-

$f(x) = x^2(x+3)$

$\rightarrow f'(x) = x^2(1) + (x+3)(2x)$

$= x^2 + 2x^2 + 6x$

$= 3x^2 + 6x$

$= 3x(x+2)$

$\rightarrow$ The roots are, 0, −2

$\Rightarrow f''(x) = 6x + 6$

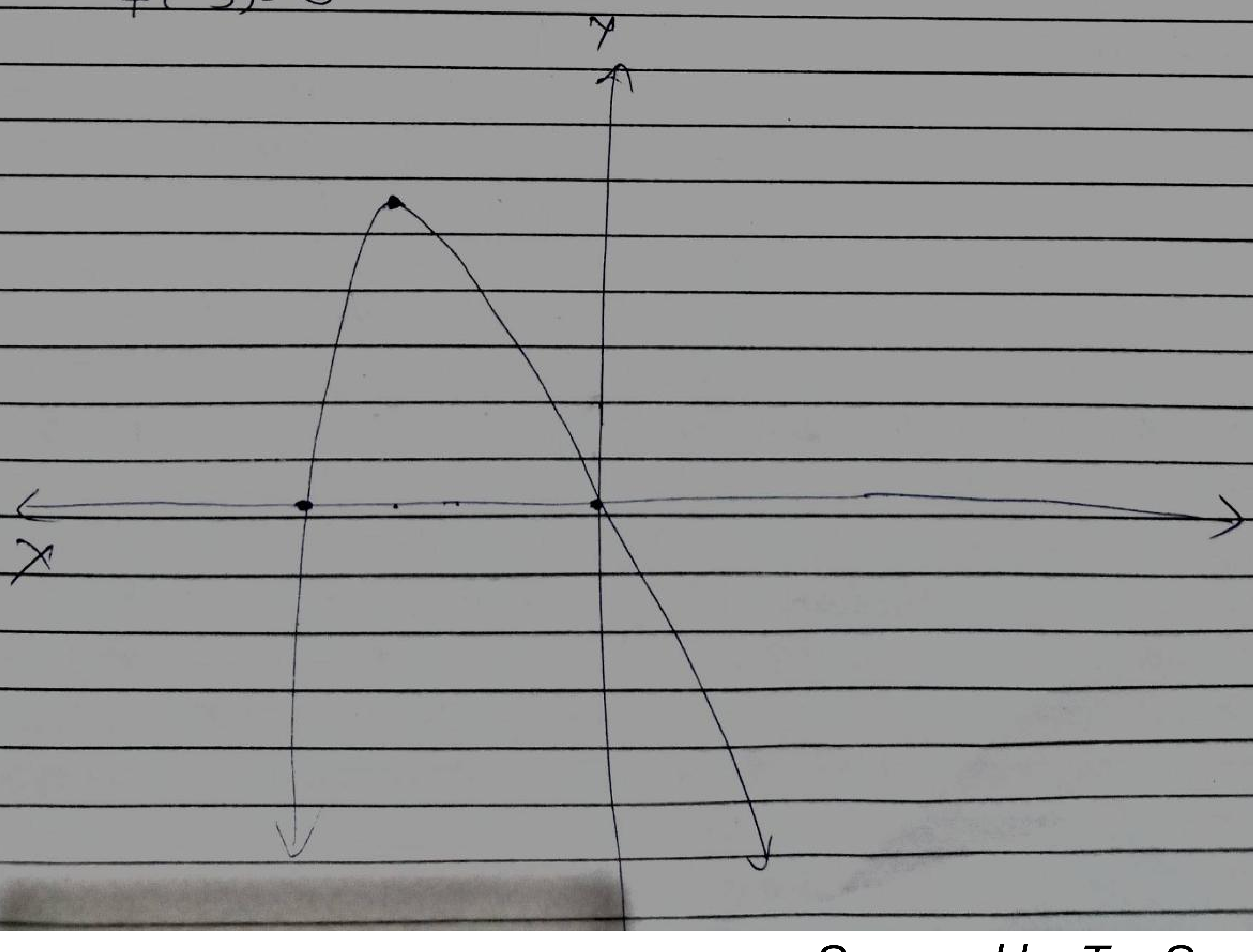
For $x = 0$, $f''(x) = 6$

For $x = -2$, $f''(x) = -12 + 6 = -6$

$\rightarrow$ For $x = -2$, For $x = 0$

$f(x) = 4(1) = 4$ $f(x) = 0$

$f(-3) = 0$



16. $f(x) = 3x^2 - 6x = 3x(x-2)$

$\rightarrow f'(x) = 6x - 6$
$= 6(x-1)$

$\Rightarrow$ Roots are $x = 1$

$\rightarrow f''(x) = 6$

$\therefore$ The graph is decreasing

$\Rightarrow f(1) = 3 - 6 = -3$

$\Rightarrow f(0) = 0$

$\Rightarrow f(-1) = 9$

$\Rightarrow f(2) = 0$
$\Rightarrow f(3) =$
$\Rightarrow f(4) = 24$