Probability & Statistics - II

Assignment - 1

1) P.Q $\rightarrow \dfrac{\hat{\theta} - \theta}{\sqrt{\hat{\theta}/n}} \sim N(0,1)$,

$$\hat{\theta} = \frac{\sum X_i}{n} = \frac{200}{1} = 200.$$

C.I = 95%.

$$P\left(-1.96 < \frac{\hat{\theta} - \theta}{\sqrt{\hat{\lambda}/n}} < 1.96\right)$$

$$P\left(-1.96\sqrt{\frac{\hat{\theta}}{n}} < \hat{\theta} - \theta < 1.96\sqrt{\frac{\hat{\theta}}{n}}\right) = 0.9$$

$$P\left(\hat{\lambda} - 1.96\sqrt{\frac{\hat{\theta}}{n}} < \theta < \hat{\theta} + 1.96\sqrt{\frac{\hat{\lambda}}{n}}\right) = 0.9$$

95% CI for θ = (172.28, 227.72)

2) i) $\bar{X} = \frac{\sum X_i}{n}$. ~~$X_i \sim N(\mu, \sigma^2)$~~ $E(X_i) = \mu$, $V(X_i) = \sigma^2$

$E(\bar{X}) = E\left(\frac{\sum X_i}{n}\right) = \frac{1}{n} E(X_1 + X_2 + \dots X_n)$

$= \frac{1}{n}\left[E(X_1) + E(X_2) + \dots E(X_n)\right]$

$= \frac{1}{n}\left[\mu + \mu + \dots \mu\right]$

$= \frac{n\mu}{n} = \mu$

$V(\bar{X}) = V\left(\frac{\sum X_i}{n}\right) = \frac{1}{n^2} V(X_1 + X_2 + \dots X_n)$

$= \frac{1}{n^2}\left[V(X_1) + \dots V(X_n)\right] =$

$= \frac{1}{n^2}\left[\sigma^2 + \sigma^2 + \dots \sigma^2\right]$

$= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$

ii) $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$E(S^2) = \frac{1}{n-1} E\left(\sum X_i^2 + \bar{X}^2 - 2\sum X_i \bar{X}\right)$

$= \frac{1}{n-1}\left[E(\sum X_i^2) + \sum E(\bar{X})^2 - 2\bar{X}.E(\right.$

$= \frac{1}{n-1}\left[nE(X_i^2) + nE(\bar{X})^2 - 2\cancel{n}\, E(\bar{X})^2\right.$

$\Rightarrow \frac{1}{n-1}\left[nE(X_i^2) - E(\bar{X})^2\right]$

$E(X_i^2) = \left(V(X_i) + [E(X_i)]^2\right)$

$= \sigma^2 + \mu^2$

$E(\bar{X}^2) = V(\bar{X}) + (E(\bar{X}))^2$

$= \frac{\sigma^2}{n} + \mu^2$

$\cancel{= \frac{1}{n-1}\left[n\sigma^2 + n\left[\frac{\sigma^2}{n} + \mu^2\right] +\right.}$

$= \frac{1}{n-1}\left[n\sigma^2 + n\mu^2 \overset{-n\mu^2 - \sigma^2}{+ \cancel{\sigma^2 + n\mu^2 - 2n\mu^2}}\right]$

$= \frac{1}{n-1}\left[\sigma^2(n-1)\right] = \sigma^2$

$\therefore E(S^2) = \sigma^2$

iii) $X_i \sim N(\mu, \sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V(S^2) = V\left(\frac{(n-1)S^2}{\sigma^2}\right) \times \frac{\sigma^4}{(n-1)^2} \quad \cancel{\sim \chi^2_{n-1}}$$

$$= V(\chi^2_{n-1}) \times \frac{\sigma^4}{(n-1)^2}$$

$$= \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{n-1}$$

iv) $P[-0.5E(S^2) < S^2 < 0.5E(S^2)]$

$E(S^2) = \sigma^2 = 100, \quad n = 10$

$P(-50 < S^2 < 50)$

$$P\left(\frac{9 \times -50}{100} < \frac{(n-1)S^2}{\sigma^2} < \frac{9 \times 50}{100}\right)$$

$$P(-4.5 < \chi^2_9 < 4.5)$$

$$= P(\chi^2_9 < 4.5) - P(\chi^2_9 \leq -4.5)$$

$$= 0.1245$$

3) $X \sim Bin(4, p)$

$$L = [P(X=0)]^{86} \cdot [P(X=1)]^{75} \cdot [P(X=2)]^{16} \cdot$$
$$[P(X=3)]^{2} \cdot [P(X=4)]^{1}$$

$$\cancel{{}^4C_0\, p^4 \cdot (1-p)^0} =$$

$$L = ((1-p)^4)^{86} \cdot ((1-p)^3 \cdot (p)^1)^{75} \cdot ((1-p)^2 \cdot (p)^2)^{16} \cdot$$
$$((1-p) \cdot (p)^3)^2 \cdot (p)^4 \cdot Const$$

$$L = (1-p)^{603} \cdot (p)^{117}$$

$$l = 603\log(p) + 117\log(p)$$

$$\frac{dl}{dp} = \frac{603}{1-p} + \frac{117}{p}(-1) = 0$$

$$\cancel{603p - 603p + 117p = 0}$$

$$\cancel{603 - 720p = 0}$$

$$\cancel{\hat{p} = \frac{603}{720}}$$

$$\hat{p} =$$

$$603p - 117 + 117p = 0$$

$$720p - 117 = 0$$

$$\hat{p} = 0.1625$$

ii) H_0 : Data for no. of claims ~~follow~~ confirms to be Binomial Dist.

H_1 : " " " doesn't "

$\hat{p} = 0.1625$, $P(X=x) = {}^nC_x\, p^x (1-p)^{n-x}$

$P(X=0) = {}^4C_0 . (0.1625)^0 . (0.8375)^4 = 0.4920$

$P(X=1) = {}^4C_1 . (0.1625)^1 . (0.8375)^3 = 0.3818$

$P(X=2) = {}^4C_2\, (0.1625)^2 . (0.8375)^2 = 0.1111$

$P(X=3) = {}^4C_3 . (0.1625)^3 . (0.8375)^1 = 0.0144$

$P(X=4) = {}^4C_4 . (0.1625)^4 . (0.8375)^0 = 0.0007$

No. of claims →	0	1	2	3	4
O_i →	86	75	16	2	1

$= 180$

$E_i \rightarrow$ 180×0.492 | 180×0.3818 | 180×0.1111 | 180×0.0144 | 180×0.0007

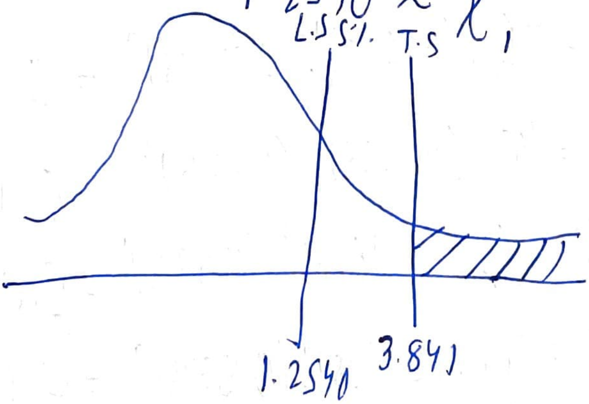
$E_i \rightarrow$ 88.56 | 68.72 | 19.99 | 2.59 | 0.13

	0	1	2+
O_i	86	75	19
E_i	88.56	68.72	22.71

$$\sum_{i=1}^{n} \left(\frac{(O_i - E_i)^2}{E_i} \right) = 1.2540 \sim \chi^2_{5-1-1-2}$$

$$1.2540 \sim \chi^2_1$$

L.S 5% T.S



$\therefore$ Since, T.S lies in A.R, we have insufficient evidence to reject H_0 at 5% L.S.

$\therefore$ It concludes that data confirms to follow binomial distn.

4) $$f(x) = \frac{cB^3}{(x+B)^4} \; ; \; x > 0 \, , \, B > 0$$

$$\int_0^{\infty} f(x) \cdot dx = 1$$

$$cB^3 \int_0^{\infty} \frac{1}{(x+B)^4} = 1$$

$$cB^3 \left[\frac{(x+B)^{-3}}{-3} \right]_0^{\infty} = 1$$

$$\frac{cB^3}{-3} \left[0 - B^{-3} \right] = 1$$

$$\boxed{c = 3}$$

$$E(X) = \int_0^{\infty} x \cdot f(x) \cdot dx$$

$$= 3 B^3 \int_0^{\infty} \frac{x}{(x+B)^{+4}} \cdot dx$$

$$= 3B^3 \int_0^{\infty} \frac{x+B}{(x+B)^{+4}} - \frac{B}{(x+B)^{+4}} \cdot dx$$

$$= 3B^3 \left[\frac{(x+B)^{-2}}{-2} - \frac{B(x+B)^{-3}}{-3} \right]_0^{\infty}$$

$$= 3B^3 \left[0 - 0 - \left[\left(\frac{B^{-2}}{-2} \right) - \left(\frac{B^{-2}}{-3} \right) \right] \right]$$

$$= 3B^3 \left[\frac{B^{-2}}{2} - \frac{B^{-2}}{3} \right]$$

$$= \frac{B^3}{2} \left[B^{-2} \right] = \frac{B}{2}$$

$$E(X^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx$$

$$= 3B^3 \int_0^\infty \frac{x^2}{(x+B)^4} \cdot dx$$

Using Pareto Dist^n $\longrightarrow$ $\dfrac{3B^2}{5} \Rightarrow V(X)$ where $\alpha = 3$, $\lambda = B$

ii) $B = X_1, X_2 \ldots X_n$

$$E(X) = \overline{X}$$

$$\frac{B}{2} = \overline{X}$$

$$\boxed{\widehat{B} = 2\overline{X}_n}$$

$$E(\widehat{B}) = E(2\overline{X}_n) = 2E\left(\frac{\Sigma X_i}{n}\right)$$

$$= \frac{2}{n} \Sigma E(X_i) = \frac{2}{n} \times n \times \frac{B}{2} = B$$

$\therefore$ Unbiased

$$\cancel{M.S.E = V(\widehat{B}) = V(2\overline{X}_n) = 4V\left(\frac{\Sigma X_i}{n}\right)}$$

$$\cancel{= \frac{4}{n^2} \Sigma V(X_i) = \frac{4}{n^2} \times n \times \frac{3B^2}{5} = \frac{12B^2}{5n}}$$

~~As n inc. MSE dec, ∴ it is consistent~~

ii) $Var(\hat{B}) = Var(2\bar{X}_n) = 4\,Var[\bar{X}_n] = 4\,Var[X]/n$

$$= 4 \times \frac{3B^2}{4} / n = \frac{3B^2}{n}$$

Hence $M.S.E[\hat{B}] = \frac{3B^2}{n} + 0 = \frac{3B^2}{n}$

the estimator is consistent since MSE tends to zero as $n \to \infty$.

iii) $MSE[b\bar{X}_n] = Var[b\bar{X}_n] + (Bias[b\bar{X}_n])^2$

$$= b^2 \times \frac{3}{4} \times \frac{B^2}{n} + (E(b\bar{X}_n) - B)^2$$

$$= b^2 \times \frac{3}{4} \times \frac{B^2}{n} + \left(B\left(\frac{b}{2} - 1\right)\right)^2$$

$$= \frac{B^2}{n}\left[\frac{3b^2}{4} + n\left(\frac{b}{2} - 1\right)^2\right]$$

Diff. the MSE respect to b:

$$\frac{d(MSE)}{db} = \frac{B^2}{n}\left(\frac{3}{2}b + n\left(\frac{b}{2} - 1\right)\right)$$

Setting this equal to 0 gives $b = \frac{2n}{n+3} = \frac{2}{1+3/n}$

Diff MSE for 2nd time wrt b:

$$\frac{d^2}{db^2}(MSE) = \frac{B^2}{n}\left(\frac{3}{2} + \frac{n}{2}\right) > 0$$

∴ Min^m value for MSE at $b = \frac{2}{1+3/n}$

iv) It is seen in ii) that $\hat{B} = 2\bar{X}_n$ is an unbiased estimator. The estimator $b\bar{X}_n$ in iii) when $b = \frac{2}{1+\frac{3}{n}}$ is -vely biased estimator for $b < 2$, $n \geq 1$.

As $n \to \infty$, b tends to 2. so estimator in iii) is also consistent as in ii)

5) i) $E(X) = \bar{X}$

$$\frac{a+b}{2} = \bar{X}$$

$$a = 2\bar{X} - b$$

$V(X) = S^2$

$$\frac{(b-a)^2}{12} = S^2$$

$$b - a = 2\sqrt{3}\,S$$

$$a = b - 2\sqrt{3}\,S$$

$$2\bar{X} - b = b - 2\sqrt{3}\,S$$

$$2b = 2\bar{X} + 2\sqrt{3}\,S$$

$$\boxed{\hat{b} = \bar{X} + \sqrt{3}\,S}$$

$$a = 2\bar{X} - \bar{X} - \sqrt{3}\,S$$

$$\boxed{\hat{a} = \bar{X} - \sqrt{3}\,S}$$

iii) Sample → 1, 2, 3, 4, 50 , $\bar{x} = 12$, $s = 21.2$

$\hat{a} = 12 - \sqrt{3}(21.27) = -24.84$

$\hat{b} = 12 + \sqrt{3}(21.27) = 48.84$

For $U(a,b)$, the prob. of a sample pt. being less than 'a' or greater than 'b' is zero, and we have a sample value 50 that is greater than our estimate 'b'.

It highlights a potential weakness of MOM

iv) $X \sim U(0,b)$

$$f(x) = \frac{1}{b-a} = \frac{1}{b}$$

$$L = f(x_1) \cdot f(x_2) \cdots f(x_n)$$

$$= \frac{1}{b} \cdot \frac{1}{b} \cdots \frac{1}{b} = \frac{1}{b^n}$$

$$l = -n \log b$$

$$\frac{dl}{db} = \frac{-n}{b} = 0, \quad b \to \infty$$

$$\frac{d^2 l}{db^2} = \frac{+n}{b^2} > 0, \quad \text{min}^m \text{ at } b \to \infty$$

So, using common sense, $L(\theta)$ is max^m when θ is minimum. $b > \text{Max } x_i$. $\boxed{\hat{b} = \text{Max } x_i}$

6) i) $P(\text{type I error}) = P(\text{Reject } H_0 / H_0 \text{ is T})$

X be no. of hits in 1^{st} step 12 missiles

Y be no. of hits in 2^{nd} step 12 missiles

$P(\text{Type I error}) = P(X \geq 3 / p = 0.1)$
$+ P(X = 0 / p = 0.1) . P(Y \geq 5 / p = 0.1) +$
$P(X = 1 / p = 0.1) . P(Y \geq 4 / p = 0.1) +$
$P(X = 2 / p = 0.1) . P(Y \geq 3 / p = 0.1)$

$= (1 - 0.8891) + (0.2824)(1 - 0.9957)$
$+ (0.6590 - 0.2824)(1 - 0.9944) +$
$(0.8891 - 0.6590)(1 - 0.8891)$

$= 0.14727 \approx 15\%$

ii) $P(X \geq 3 / p = 0.3) + P(X = 0 / p = 0.3) . P(Y \geq 5 / p = 0.3)$
$+ P(X = 1 / p = 0.3) . P(Y \geq 4 / p = 0.3) + P(X = 2 / p = 0.3) . P(Y \geq 3 / p = 0.3)$

$= 0.91253 \approx 91\%$

iii) $P(\text{type II error})$ is the prob. of accepting H_0 when H_0 is F.

$= 1 - 0.91253 = 0.08747 \approx 9\% \, (p = 0.3)$

iv) 10 space agencies fire 120 missiles and record 40 hits.

$X \sim Bin(120, 1/3)$ $\hat{p} = \frac{40}{120} = \frac{1}{3}$

$$\frac{\frac{X}{n} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

$$P\left(-1.2816 < \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} < 1.2816\right)$$

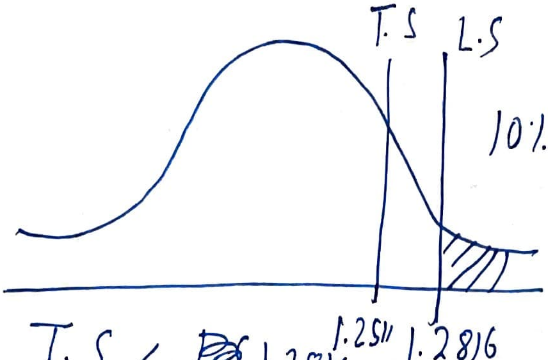
At CI 90% $p \in (0.3333, 0.2782)$

v) $p > 0.278$ at 10% LS

$H_0: p = 0.278$
$H_1: p > 0.278$

$$\frac{X - np_0}{\sqrt{np_0 q_0}} \sim N(0,1)$$

$$\frac{39.5 - 120 \times 0.278}{\sqrt{120 \times 0.278 \times 0.722}} = 1.2511$$



As T.S < ~~L.S~~ 1.2816

We have insufficient evidence to reject H_0 at L.S 10%.

vi) Lower Bound of C.I implies that there is only a 10% chances 'p' ≤ 0.2782, whereas from the hypothesis test, 'p' could be less than = to 0.2780 with prob. more than 10% (approx 10.5% corresponding to 1.2511)

The minor disconnect b/w C.I & H.T at some level is due to –

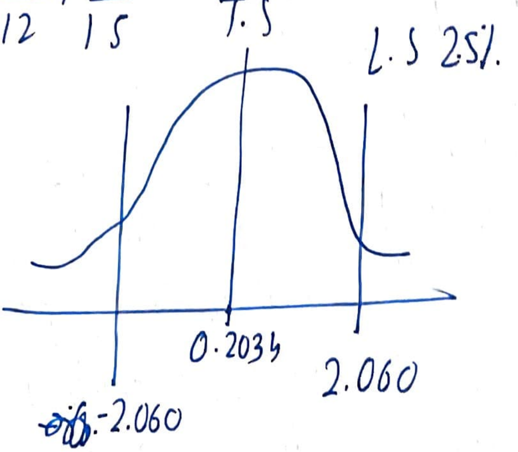
1) Applying continuity correction in H.T
2) Use of sample proportion $\hat{p}$ to estimate pop^n variance in calculating C.I.

vii) $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$

$$P.Q \rightarrow \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$S_p^2 = \frac{1}{25}\left(11 \times 2916 + 14 \times 4900\right) = 4027.04$$

$$\frac{65-70-0}{63.459\sqrt{\frac{1}{12}+\frac{1}{15}}} = 0.2034$$



Since T.S lies in A.R, we have insuff. evidence to reject H_0 at 5% L.S

7) $\sum X_{Pub} = 270$, $\bar{X}_{Pub} = 30$

$\sum X_{Pri} = 315.5$, $\bar{X}_{Pri} = 35.06$

$\sum X^2_{Pub} = 8614.5$, $S_1^2 = \frac{8614.5 - 9 \times 30^2}{8}$

$= 64.31$

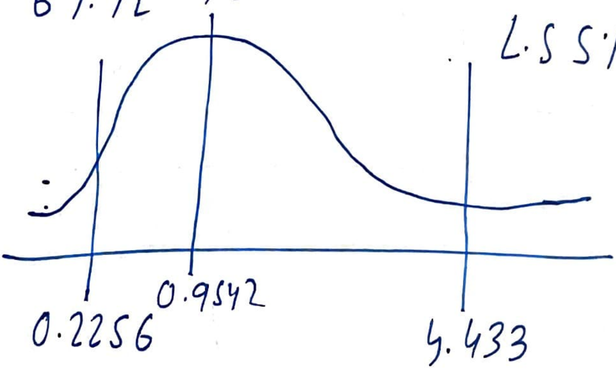
$$S_2^2 = \frac{11599.25 - 9 \times 35.06^2}{8} = 67.42$$

ii) $H_0: \sigma_1^2 = \sigma_2^2$, $H_1: \sigma_1^2 \neq \sigma_2^2$

$$T.S = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,\, n_2-1}$$

$$= \frac{64.31}{67.42} = 0.9542 \sim F_{8,8}$$

L.S 5%



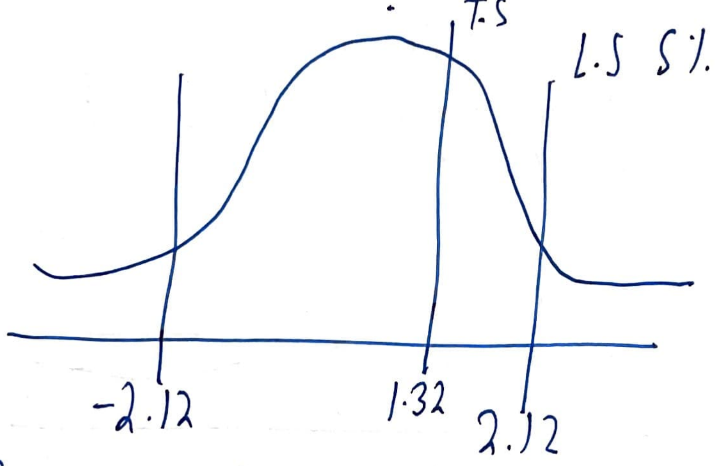
Since T.S lies in A.R, we have insuff. evidence to reject H_0 at L.S 5%.
∴ Pop^n Var. are =.

iii) $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 < \mu_2$

$$T.S = \frac{(\bar{X}_2 - \bar{X}_1) - (\mu_2 - \mu_1)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}$$

$$S_p^2 = \frac{64.31 \times 8 + 67.42 \times 8}{16} = 65.86$$

$$T.S \rightarrow \frac{(35.06-30)-0}{\sqrt{65.86}\sqrt{\frac{1}{9}+\frac{1}{9}}} = 1.32$$



Since, T.S lies in A.R, we have insuff. evidence to reject H_0 at 5% L.S.

$\therefore$ Cost of claims in private hospitals is similar to that of public hospitals.

8) i) $\hat{\theta}$ is said to be unbiased when $E(\hat{\theta}) = \theta$

ii) Measure of bias is given by $E(\hat{\theta}) - \theta$

iii) M.S.E of this estimator $\hat{\theta} = (E(\hat{\theta}) - \theta)^2$

iv) $\tilde{\theta}$ is efficient as an estimator with lower M.S.E is said to be more efficient than one with higher M.S.E.

v) An estimator is termed as consistent if MSE converges to zero as sample size tends to ∞.

vi) θ can be estimated using:

a) Method of Moments: The pop^n moments are equated to the sample moments to estimate the parameters.

b) Maximum Likelihood Method: A max^m likelihood f^n $L(\theta) = \prod_{i=1}^{n} f(x_i ; \theta)$ is generated. A MLE of parameter is given by sol^n to $\frac{dL(\theta)}{d\theta} = 0$

9) i) Variable t_k is defined as $t_k = \frac{N(0,1)}{\sqrt{\chi^2_k / k}}$

where $k \rightarrow$ degree of freedom

and 2 R.V $N(0,1)$ & χ^2_k are indep.

ii) Mean & Var. of t_k for $k>2$ are 0, and $\frac{k}{k-2}$ resp.

iii) $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$, $t_9 = 3.25$

C.I for μ is thus $\left(50-3.25\sqrt{\frac{48.667}{10}}, 50+3.2\ldots\right.$

i.e $(42.83, 57.17)$

b) $t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}}\right) \sim N(0, \sqrt{9/7})$

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N(0, \sqrt{9}/\sqrt{7})$$

$$\frac{\bar{X}-\mu}{S/\sqrt{7n/9}} \sim N(0,1)$$

C.I for $\mu \in \left(50-2.58\sqrt{\frac{48.667}{10}\times\frac{9}{7}},\right.$
$\left.50+2.58\sqrt{\frac{48.667}{10}\times\frac{9}{7}}\right)$

i.e $(43.54, 56.45)$

10) i) P(type I error) = P(Rej H_0 | H_0 is T)

$X \sim Pois(n\mu)$ i.e $Poi(3000)$

H_0, $X \sim Poi(3000)$

P(Reject H_0 | H_0 is T) = $P(X > 3100$ when $X \sim Poi(3000))$

$X \sim N(3000, 3000)$

$$P(X > 3100) = P\left(Z > \frac{3100 - 300}{\sqrt{3000}}\right)$$

$$1 - P(Z < 1.825) = 3.39\%$$

ii) P(type II error) = P(Accepting H_0 / H_0 is F)

iii) Power of test - Prob. of Rejecting / H_0 is F

P(reject H_0 / H_0 is F) = $P(X > 3100$ when $X \sim Poi(n\mu) \sim N(n\mu, n\mu)$

$$P(X > 3100) = 1 - \left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$$

$\therefore$ Power of test depends on value of H_1's parameter.

iv) $\hat{\mu} \sim N(\mu, \hat{\mu}/n)$

$$\frac{\hat{\mu} - \mu}{\sqrt{\hat{\mu}/n}} \sim N(0,1)$$

$$P\left(-2.5758 < \frac{2.9 - \mu}{\sqrt{2.9/1000}} < 2.5758\right)$$

$$= 0.99$$

$$\mu \in (2.7613, 3.0387)$$

11) Revised Sample Mean $= \dfrac{213200 - 11000 - 48000 + 27500 + 31500}{12}$

$= 17,767$

$\Sigma X^2 = 4919860000 - 121000000 - 2304000000 + 756250000 + 992250000$

$= 4,243,360,000$

$\therefore$ Revised S.D $= \left[\dfrac{1}{11}\left(4,243,360,000 - \dfrac{213200^2}{12}\right)\right]^{1/2}$

$= 6435$

There has been no change in sample mean how S.d has reduced significantly from 10,144 to 6435.

The mean remains unaltered as the total salary of temporary employees who were being replaced = total salary of permanent employees who replaced them. The reason for reducⁿ in s.d is that salaries of permanent employees who replaced temporary employees are closer to sample mean.

12) i) The C.R.L.B results holds under very general conditions except where range of distⁿ involves the parameter, such as unif. distⁿ in this case. This is due to a discontinuity so the derivative in the formula doesn't make any sense.

ii)

$Y = Max_i\ X_i$

Since X_i lies b/w 0 & θ, the support for Y will be 0 & θ. For $0 < y < \theta$

$P(Y < y) = P[Max\ X_i < y]$

$= P\left[\bigcap_{i=1}^{n}(X_i < y)\right]$

$= \prod_{i=1}^{n} P[X_i < y]$ [X_i's are indep]

$= \prod_{i=1}^{n}\left[\int_0^y \dfrac{dx}{\theta}\right] = \left(\dfrac{y}{\theta}\right)^n$

Thus P.D.F of Y will be $g_Y(y) = \dfrac{d}{dy}P[Y < y]$

$= \dfrac{n}{\theta^n}\cdot y^{n-1}$ for $0 < y < \theta$.

Now for $k \geq 0$

$$E[Y^k] = \int_0^{\theta} y^k \cdot \frac{n}{\theta^n} \cdot y^{n-1} \, dy$$

$$= \frac{n}{n+k} \int_0^{\theta} \frac{n+k}{\theta^n} \cdot y^{n+k-1} \, dy$$

$$= \frac{n}{n+k} \cdot \frac{\theta^{n+k} - 0}{\theta^n}$$

$$= \frac{n\theta^k}{n+k}$$

iii) Bias of estimator $\hat{\theta}(c)$ is given as →

$$\text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}(c) - \theta]$$

$$= E[cY - \theta]$$

$$= cE[Y] - \theta$$

$$= \frac{c \cdot n\theta}{n+1} - \theta$$

$$= \left(\frac{cn}{n+1} - 1\right)\theta$$

M.S.E of estimator $\hat{\theta}(c)$ is given as:

$$M.S.E[\hat{\theta}(c)] = E[[\hat{\theta}(c) - \theta]^2]$$

$$= E[(cY - \theta)^2]$$

$$= E[c^2Y^2 - 2c\theta Y + \theta^2]$$

$$= c^2E(Y^2) - 2c\theta E(Y) + \theta^2$$

$$= \frac{c^2 \cdot n\theta^2}{n+2} - \frac{c \cdot 2n\theta^2}{n+1} + \theta^2$$

iv) For $\hat{\theta}(c)$ to be an unbiased estimator of θ, we need $\text{Bias}[\hat{\theta}(c_u)] = 0$

$$\Rightarrow \left(\frac{c_u \cdot n}{n+1} - 1\right)\theta = 0$$

$$\Rightarrow c_u = \frac{n+1}{n}$$

v) In order to minimize $MSE[\hat{\theta}(c)]$, we need,

$$0 = \frac{d}{dc} MSE[\hat{\theta}(c)] = \frac{d}{dc}\left[\frac{c^2 \cdot n\theta^2}{n+2} - \frac{2n\theta^2}{n+1} \cdot c + \theta^2\right]$$

$$= \frac{2cn\theta^2}{n+2} - \frac{2n\theta^2}{n+1}$$

Thus $C_m = \frac{2n\theta^2}{n+1} \cdot \frac{n+2}{2n\theta^2} = \frac{n+2}{n+1}$

$$\frac{d^2}{dc^2}\left[MSE\left[\hat{\theta}(c)\right]\right] = \frac{d}{dc}\left[\frac{2c.n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}\right]$$

$$= \frac{2n\theta^2}{n+2} > 0 \Rightarrow min^m$$

vi) In order to minimize error in estimation, it is preferred to opt for an estimator which has lower M.S.E among diff. competing estimators. So here $\hat{\theta}(C_m)$ will be preferred over $\hat{\theta}(C_u)$

As n becomes large $\hat{\theta}(C_u) = \frac{n+1}{n} Y \to Y$

$|||^{ly}$, $\hat{\theta}(C_m) = \frac{n+2}{n+1} Y \to Y$, the two estimators become one & same as $n \to \infty$.