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YOUVA

Calculus Assignment

1) $\frac{1}{50}\int^{2} \frac{e^{2/w}}{w^2}\, dw$

Let $e^{2/w} = t$

$\therefore e^{2/w}\left(\frac{-2}{w^2}\right)dw = dt$

$\therefore \frac{e^{2/w}}{w^2}\, dw = -\frac{dt}{2}$

At $w = 2$, $t = e$

$w = \frac{1}{350}$, $t = e^{1/25}$

$\therefore \int_{e^{1/25}}^{e} -\frac{dt}{2}$

$= \left[-\frac{t}{2}\right]_{e^{0.04}}^{e} = -\frac{e}{2} + \frac{e^{0.04}}{2} = 0.83874$

2) $\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

Let $t^4 + 2t = x$

$\therefore (4t^3+2)\, dt = dx$

$2(2t^3+1)\, dt = dx$

$\therefore \int \frac{1}{2x}\, dx$

$= \frac{\log 2x}{2} + c$

3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)\, dx$

$= \int x^4 + 3x - 9 \; dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - \frac{9x^{\cancel{2}}}{\cancel{2}} + c$

$\therefore f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

4) $\int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$= \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$

$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= 1 - (-2)^3 + (6(3)) - (6)$

$= 1 + 8 + 18 - 6$

$= 21$

5. $\int 3(8y-1)\, e^{4y^2-y}\, dy$

$= \int 3(8y-1)\, e^{(4y-1)y}\, dy$

∴ Let $4y^2 - y = t$

∴ $8y - 1 = \frac{dt}{dy}$

∴ $\int 3(e^t)\, dt$

$= 3e^t\, dt$

$= \frac{3e^{(4y^2-y)}}{4y^2-y}$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int f''(x)\, dx = \int 15\sqrt{x} + 5x^3 + 6$

$= \frac{(2)15x\sqrt{x}}{3} + \frac{5x^4}{4} + 6x$

$f'(x) = 10x\sqrt{x} + \frac{5}{4}x^4 + 6x + c$

$f(x) = \int f'(x)\, dx$

$= \int 10(x)^{3/2} + \frac{5}{4}x^4 + 6x + c\, dx$

$= \frac{(2)10x^{5/2}}{5} + \frac{5x^5}{20} + \frac{6x^2}{2} + cx$

$f(x) = 4x^2\sqrt{x} + \frac{x^5}{4} + 3x^2 + cx + c'$

$f(1) = 4 + \frac{1}{4} + 3 + c = \frac{16+1+12+4c+4c'}{4}$

$-\frac{5}{4} = \frac{29+4(c+c')}{4}$

∴ $-34 = 4(c+c')$

$f(4) = \frac{\cancel{34}}{\cancel{4}}$ 404

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$f(4) = 432 + 4c + c'$

$404 = 432 + 4c + c'$

$4c + c' = -28$

$4c + 4c' = -34$

$3c' = -6$

$c' = -2$

$\therefore 4c - 2 = -28$

$4c = -26$

$c = \frac{-13}{2}$

$\therefore f(x) = 4x^2\sqrt{x} + \frac{x^5}{4} + 3x^2 + \frac{-13x}{2} - 2.$

7. $f(x+iy) + g(x-iy) = z$ —(1)

differentiating w.r.t x

$f'(x+iy) + g'(x-iy) = \frac{\partial z}{\partial x}$ —(2)

differentiating w.r.t. y

$f'(x+iy)(i) + g'(x-iy)(-i) = \frac{\partial z}{\partial y}$ —(3)

differentiating (3) w.r.t. y

$f''(x+iy)(i^2) + g'(x-iy)(-i)^2 = \frac{\partial^2 z}{\partial y^2}$

differentiating ~~(4)~~ (2) w.r.t. x

$f''(x+iy) + g''(x-iy) = \frac{\partial^2 z}{\partial x^2}$

$i^2\left(\frac{\partial^2 z}{\partial x^2}\right) = \frac{\partial^2 z}{\partial x^2}$

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(4) = 2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

Integrating on both sides

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$-\frac{1}{r} = \ln\theta + c$

$\cancel{\frac{1}{r}}\ \frac{1}{r} + \ln\theta + c = 0$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\int \frac{dv}{(9.8 - 0.196v)} = \int dt$

$\frac{\log(9.8 - 0.196v)}{-0.196} = t + c$

$\log(9.8 - 0.196v) = -0.196t + c'$

$\log[(9.8)(1 - 0.02v)] = -0.196t + c$

$\log(1 - 0.02v) = -0.196t - \log 9.8 + c$

10. $ty' + 2y = t^2 - t + 1$

$y(1) = \frac{1}{2}$

12) $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

Taylor's Series

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{(x-a)^2 f''(a)}{2!} + \frac{(x-a)^3 f'''(a)}{3!} \dots$$

$x^3 - 10x^2 + 6 = f(x)$ $\qquad f(3) = -57$

$3x^2 - 20x = f'(x)$ $\qquad f'(3) = -33$

$6x - 20 = f''(x)$ $\qquad f''(3) = -2$

$6 = f'''(x)$ $\qquad f'''(3) = 6$

$$\therefore x^3 - 10x^2 + 6 = -57 + \frac{(-33)(x-3)}{1!} + \frac{(x-3)^2(-2)}{2!} + \frac{(x-3)^3(6)}{3!}$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

13) $(1+x)^4$ Find Maclaurin series

$f(x) = (1+x)^4$ $\qquad f(0) = 1$

$f'(x) = 4(1+x)^{4-1}$ $\qquad f'(0) = 4$

$f''(x) = 4(4-1)(1+x)^{4-2}$ $\qquad f''(0) = 4(4-1)$

$f'''(x) = 4(4-2)(4-1)(1+x)^{4-3}$ $\qquad f'''(0) = 4(4-1)(4-2)$

Maclaurin series

$$f(x) = f(0) + \frac{(x)^1 f'(0)}{1!} + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!}$$

$$(1+x)^4 = 1 + (x)(4) + \frac{x^2 4(4-1)}{2} + \frac{x^3 (4)(4-1)(4-2)}{6}$$

$$(1+x)^4 = 1 + x4 + \frac{x^2 4(4-1)}{2} + \frac{x^3 4(4-1)(\cancel{x}\ 4-2)}{6}$$

14) $f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $\qquad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\qquad f'''(0) = \frac{3}{8}$

Maclaurin series

$$f(x) = f(0) + \frac{x f'(0)}{1!} + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \frac{x^2}{2!}\left(\frac{1}{4}\right) + \frac{x^3}{3!}\left(\frac{3}{8}\right)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$$

15. $f(x) = e^{-6x}$ $\qquad x = -4$

Taylor series

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{(x-a)^2 f''(a)}{2!} + \frac{(x-a)^3 f'''(a)}{3!}$$

$f(x) = e^{-6x}$ $\qquad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\qquad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\qquad f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ $\qquad f'''(-4) = -216e^{24}$

$$e^{-6x} = e^{24} + \frac{(x+4)}{1!}e^{24}(-6) + \frac{(x-a)^2(36e^{24})}{2!}$$

$$+ \frac{(x-a)^3(-216e^{24})}{3!}$$

$$= e^{24}\left(1 - 6(x+4) + (18)(x-a)^2 - (36)(x-a)^3\right)$$

16. $f(x) = \ln(3+4x)$ at $x = 0$

Taylor series.

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$f(x) = \ln(3+4x)$ $\qquad f(0) = \ln 3$

$f'(x) = \frac{1(4)}{3+4x}$ $\qquad f'(0) = 4/3$

$f''(x) = -(3+4x)^{-2}(16)$ $\qquad f''(0) = -16/9$

$f'''(x) = 2(3+4x)^{-3}(64)$ $\qquad f'''(0) = 128/27$

$$\ln(3+4x) = \ln 3 + \frac{4}{3}\frac{(x)^1}{1!} + \frac{(-16)(x)^2}{9(2!)} + \frac{128(x)^3}{27(3!)}$$

$$\ln(3+4x) = \ln 3 + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64}{81}x^3$$