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Numerical Methods And Linear Algebra

Assignment 2

Q.1]
Answer Given that =

$$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix}\begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3-8 & b+2 \\ 3a & ab \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

Since, the matrices have equal orders.
∴ By comparing,

$b+2 = 6$ — (1)
$3a = 12$ — (2)
$\therefore a = 4$ — (3)
$ab = 16$ — (4)

From 3 and 4

$4b = 16$
$\therefore b = 4$

$\boxed{a = 4}$ and $\boxed{b = 4}$

Q.2]

Answer Given :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A \times A = A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For A^3 the a_{22} element would be (-1)
since, in A^{20}, the power 20 is even,

$$\therefore \boxed{A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I}$$

Q.3]

Answer A matrix is not invertible if and only if
$|\text{matrix}| = 0$

Let $$A = \begin{bmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

A is not invertible if $|A| = 0$

$$|A| = \begin{vmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\therefore 0 = \begin{vmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\therefore 0 = 1(2-0) + \lambda(0-3) + 0$$

$$\therefore 0 = 2 - 3\lambda$$

$$\therefore 3\lambda = 2$$

$$\boxed{\lambda = \frac{2}{3}}$$

Q.4]
Answer The sales matrix for A-Plus auto parts

$$S = \begin{array}{c} \\ \text{Mumbai} \\ \text{Pune} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Wiper blades} & \text{Fluid bottles} & \text{Floor mats} \end{array} \\ \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix}_{2\times 3} \end{array}$$

The price matrix for A-Plus auto parts

P =

$$\begin{array}{l} \\ \text{Wiper blades} \\ \text{Fluid bottles} \\ \text{Floor mats} \end{array} \begin{array}{c} \begin{array}{cc} \text{Undiscounted} & \text{Discounted} \end{array} \\ \left[\begin{array}{cc} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{array}\right]_{3\times2} \end{array}$$

The revenue matrix for A-Plus auto parts

$R = S \times P$

$$= \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix}_{2\times3} \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix}_{3\times2}$$

$$\boxed{R = \begin{bmatrix} 26,600 & 22000 \\ 18,900 & 15,400 \end{bmatrix}}$$

$$\begin{array}{l} \\ \text{Mumbai} \\ \text{Pune} \end{array} \begin{array}{c} \begin{array}{cc} \text{Undiscounted} & \text{Discounted} \end{array} \\ \left[\begin{array}{cc} 26,600 & 22000 \\ 18,900 & 15,400 \end{array}\right] \end{array}$$

Q.5]

Answer: For an arithmetic progression

Given: $t_4 = 3a$

$t_7 = 2t_3 + 1$

To find: a and d

Solution:

For an AP

$t_n = a + (n-1)d$

For $n = 4$

$t_4 = a + 3d$

$3a = a + 3d$

$2a = 3d$ —— ①

$t_7 = 2t_3 + 1$

$a + 6d = 2(a + 2d) + 1$

$a + 6d = 2a + 4d + 1$

$0 = a - 2d + 1$

$\therefore a = 2d - 1$

Multiplying above equation by 2

$2a = 4d - 2$ —— ②

From 1 and 2,

$3d = 4d - 2$

$2 = 4d - 3d$

$$\boxed{\therefore\ d = 2} \quad \text{--- (3)}$$

From 1 and 3,

$$2a = 3(2)$$
$$2a = 6$$
$$\boxed{a = 3}$$

Q.6]
Answer For an arithmetic progression,

$$7t_7 = 11t_{11}$$
$$7(a + 6d) = 11(a + 10d)$$
$$7a + 42d = 11a + 110d$$
$$-68d = 4a$$
$$-17d = a$$
$$\therefore a + 17d = 0$$
$$\therefore a + (18-1)d = 0$$
$$\therefore t_{18} = 0$$

Hence Proved

Q.7]

Answer: For an arithmetic series,

$$S_{20} = S_{22}$$

and $d = -2$

$$\frac{20}{2}[2a + 19(-2)] = \frac{22}{2}[2a + 21(-2)]$$

$$10[2a - 19(2)] = 11[2a - 21(2)]$$

$$10[2a - 38] = 11[2a - 42]$$

$$20a - 380 = 22a - 462$$

$$462 - 380 = 2a$$

$$82 = 2a$$

$$\boxed{a = 41}$$

Q.8]

Answer

$$S_n = 2n^2 + 5n$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$2n^2 + 5n = \frac{n}{2}[a + a + (n-1)d]$$

$$n(2n+5) = \frac{n}{2}[a + t_n]$$

$$2n + 5 = \frac{1}{2}[a + t_n]$$

$$4n + 10 = a + t_n$$

$$t_n = 4n + 10 - a$$

The above is the general form to find the n^{th} term.

Q. 89]
Answer:

$$111\ldots1 \text{ (91 times)} = 1 + 10 + 100 + \ldots + 10^{90}$$

This is a GP with $r = 10$

$$S_{91} = 1\left(\frac{10^{91} - 1}{10 - 1}\right) = \frac{(10^7)^{13} - 1}{10 - 1}$$

$$= \frac{(10^7)^{13} - 1}{10^7 - 1} \times \frac{10^7 - 1}{10 - 7}$$

$$= (1 + 10^7 + 10^{14} + \ldots + 10^{91}) \times (1 + 10 + 10 \ldots + 10^7)$$

Since both terms are natural numbers
It is a composite and not a prime number

Q. 10]

Answer Let the three numbers in GP be a, ar and ar^2

$$a + ar + ar^2 = 31$$

$$a(1 + r + r^2) = 31 \quad \text{—— } 1$$

$$a^2 + (ar)^2 + (ar^2)^2 = 651$$

$$a^2(1 + r^2 + r^4) = 651 \quad \text{—— } 2$$

Squaring equation 1,

$$a^2(1 + r^2 + r^4 + 2r + 2r^3 + 2r^2) = \cancel{31}\ 961$$

$$a^2(1 + r^2 + r^4) + a^2(2r + 2r^3 + r^2 2) = \cancel{31}\ 961 \quad 3$$

From 2 and 3

$$651 + 2ra^2(1 + r + r^2) = \cancel{31}\ 961$$

$$651 + 2ar(31) = \cancel{31}\ 961$$

$$2ar(31) = 310$$

$$2ar = 10$$

$$ar = 5$$

$$a = \frac{5}{r}$$

substituting in ①,

$$\frac{5}{r}(1 + r + r^2) = 31$$

$$\frac{5}{r} + 5 + 5r = 31$$

$$\frac{5}{r} + 5r = 26$$

$$5 + 5r^2 = 26r$$

$$5r^2 - 26r + 5 = 0$$

$$5r^2 - 25r - r + 5 = 0$$

$$5r(r-5) - 1(r-5) = 0$$

$$(r-5)(5r-1) = 0$$

$r = 5$ Or $r = \frac{1}{5}$

If $r = 5$
$a = 1$

If $r = 1/5$,
$a = 25$

$\therefore$ The numbers are 1, 5, 25