Algebra

Assignment - 1

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1.) $\sqrt{7+\sqrt{7+\sqrt{7+\sqrt{7}}}}$

$$x = \sqrt{7+\sqrt{7+\sqrt{7+\sqrt{7}}}}$$

$$x = \sqrt{7+x}$$

$$x^2 = 7 + x$$

$$x^2 - x - 7 = 0$$

$$x = \frac{1}{2} \pm \frac{\sqrt{29}}{2}$$

since sq can't be (-) ve

$$x = \frac{1}{2} + \frac{\sqrt{29}}{2}$$

$$\approx 3.1926$$

2.) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$7x^2 + 11x^2 = 6y^2 - 4y^2$$

$$18x^2 = 2y^2$$

$$9x^2 = y^2$$

$$3x = y$$

$$\boxed{\frac{x}{y} = 1/3}$$

3. $x + 3y = 4$ — ①

$x^2 + y^2 - 4y = 12$ — ②

$x + 3y = 4$

$x = 4 - 3y$ — ③

Substituting eq ③ in eq ①

$(4-3y)^2 + y^2 - 4(4-3y) = 12$

$16 + 9y^2 - 24y + y^2 - 16 + 12y = 12$

$10y^2 - 12y - 12 = 0$

$5y^2 - 6y - 12 = 0$

$y = \frac{3}{5} - \frac{\sqrt{69}}{5}$

$y \approx -1.0613$

$\therefore x = 4 - 3(-1.0613)$

$x \approx 7.1839$

$y = \frac{3}{5} + \frac{\sqrt{69}}{5}$

$y \approx 2.2613$

$\therefore x = 4 - 3(2.2613)$

$x = -2.7839$

4. $d = 2r^2 - 16r + 34$

a) Density of smoke when $r = 3.5$

$d = 2(3.5)^2 - 16(3.5) + 34$

$= 2(12.25) - 56 + 34$

$\underline{d = 2.5}$

b) minimum r

$d = 2r^2 - 16r + 34$

$\frac{dd}{dr} = 4r - 16$

$r = 16/4 = \underline{4}$

$\frac{dd^2}{d^2r} = 4 > 0$

it is minimum at $r = 4$

No of revolutions for minimum output = 400 revolutions

c) $d = 100$

$100 = 2r^2 - 16r + 34$

$2r^2 - 16r - 66 = 0$

$r^2 - 8r - 33 = 0$

$r^2 - 11r + 3r - 33 = 0$

$r(r - 11) + 3(r - 11) = 0$

$r = 11 \quad / r = -3$

Since no of revolutions can't be negative, $r \neq -3$

$\therefore r = 11$

$\therefore$ Speed = 1100 revolution

5. a) For company 1

$C = 1500 + 25(n)$

b) For company 2

$C = 2100 + 22(n)$

c) For company 2 to be less expensive

$2100 + 22n < 1500 + 25n$

$600 < 3n$

d) $200 < n$

It is cheaper to rent the car from company if kms travelled are more than 200.

6. Bisection method accurate to 10^{-2}

$n^3 - 7n^2 + 14n - 6 = 0$

a) $[0, 1]$

let $f(n) = n^3 - 7n^2 + 14n - 6$

$f(0) = -6 < 0$

$f(1) = 2 > 0$

$\therefore a_0 = 0$

$b_0 = 1$

$\frac{a+b}{2} = 0.5$

$f(0.5) = -0.6250$

$\therefore a_1 = 0.5$

$b_1 = 1$

$p_1 = \frac{a+b}{2} = 0.75$

$f(p_1) = 0.9843750$

n	p_n	$f(p_n)$
0	0.5	− 0.625
1	0.75	+ 0 984375
2	0.625	+ 0.2597656
3	0.5625	− 0.1618652
4	0.59375	+ 0 0540466
5	0.578125	− 0 0526237
6	0.5859375	+ 0 0010313

$\therefore$ Ans ≈ 0.5859375 accurate to 10^{-2}

b) $a = 1, b = 3\,2$

$f(1) = 2 > 0$

$f(3\,2) = -0.112 > 0$

The function has atleast 1 real root in this interval

n	p_n	$f(p_n)$
1	2.1	+ 1.791
2	2 65	+ 0 552125
3	2 925	+ 0 0858
4	3.0625	− 0.054
5	2.99375	+ 0 00632 788
6	3.02813	− 0 02 65207
7	5.01094	− 0.0106969
8	3.00235	

∴ Ans = 3.00235 accurate to 10^{-2}

c) $a = 3.2$
$b = 4$

n	p	f(p)
1	3.6	+0.336
2	3.4	−0.016
3	3.5	+0.125
4	3.45	+0.046121
5	3.425	+0.0130156
6	3.4125	−0.00199805
7	3.41875	

Ans = 3.41875 accurate to 10^{-2}

8 a)

Mathmatic mean $= \frac{70 + 86 + 81 + 83 + x}{5} = \frac{320 + x}{5}$

b)

$$80 \leq \frac{320 + x}{5} < 90$$

$$400 - 320 \leq x < 450 - 320$$

c) $80 \leq x < 130$ → for B grade

9. NPV = 8.54 @ 4% dis rate

NPV = -11.81 @ 20% dis rate

∴ By linear interpolation,

10. $R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

Profit = Revenue − cost

$= (32x - 0.21x^2) - (195 + 12x)$

$P(x) = 20x - 0.21x^2 - 195$

for no of orders to ~~maximize~~ maximize profit we minimize profit

$P'(x) = -0.42x + 20$

$-0.42x = -20$

$x = \frac{20}{0.42}$

$x = 47.61$

$P''(x) = -0.42 < 0$

$P(x)$ is minimum at 47.61

∴ atleast 48 orders are needed to remain profitable

7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

Assuming $x_0 = 0.4$

Simulating the equation

$x_1 = -0.0927$

$x_2 = -0.0408$

$x_3 = -0.04066$

One root is -0.04066

Assuming $x_0 = 0.7$

$x_1 = 1.4326$

$x_2 = 1.1599$

$x_3 = 1.0138$

$x_4 = 0.9671$

$x_5 = 0.9624$

$x_6 = 0.9624$

Ans $= -0.04066$ & 0.9624

9. At $0.04 \Rightarrow 8.54$

at $0.02 \rightarrow -11.81$

$\therefore$ at 0

$$y = \frac{-11.81 - 8.54}{0.2 - 0.04}(0 - 0.04) + 8.54$$

$$= 0.136275$$