Roll no: 74

Assignment -1

① $d^{(4)} = 8\%$

(i) $\left(1 + \frac{i^{(4)}}{4}\right) = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$

$= \left(1 + \frac{i^{(4)}}{4}\right) = \left(1 - \frac{d^{(4)}}{4}\right)^{(-1)}$

$= i^{(4)} = 4\left[\left(1 - \frac{d^{(4)}}{4}\right)^{(-1)} - 1\right]$

$= 8.16\%$

(ii) $(1+i) = \left(1 + \frac{i^{(4)}}{4}\right)^4$

$= \left[\left(1 + \frac{i^{(4)}}{4}\right)^{(4)} - 1\right]$

$= 8.42\%$

(iii) $\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= \frac{d^{(12)}}{12} = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}$

$$= d^{(12)} = 12\left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}\right]$$

$$= 8.0539339\%$$

$$= \underline{8.05\%}$$

(2) $\delta(t) = 0.004t + 0.0002t^2$

i) $PV_{10} = C \times v(10)$

$$= C \times v^{10}$$

$$= C \times e^{-\left[\int_0^{10} 0.004t + 0.0002t^2\right]}$$

$$= 1000 \times e^{-[(0.266667)]}$$

$$= 765.92883383 \text{ Rs}$$

$$= \underline{765.93 \text{ Rs}}$$

ii) $(1+i) = e^{\int_0^{10} \delta_t dt}$

$$= i = e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}} - 1$$

$$= 0.30560517 2$$

$$= \underline{30.56\%}$$

$d = i \times \frac{1}{1+i}$

$d = iv$

③ $\frac{1}{1+i} = 1-d$

$d = 1 - \frac{1}{1+i}$

$d = \frac{1+i-1}{1+i}$

$d = \frac{i}{1+i}$

$= d = i \times \frac{1}{1+i}$

$\boxed{d = vi}$

∴ Hence, proved.

b)

ii)

a) $\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^4$

$\therefore 1 - \frac{d^{(2)}}{2} = \left(1 - \frac{d^{(4)}}{4}\right)^2$

$= 1 - d\, d^{(2)} = 2\left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^2\right]$

$= 2\left[1 - (1 - 2.25\%)^2\right]$

$= 0.088987500$

$= 8.899\%$

b) $d^{(1/2)} = 5\%$

$= \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2}$

$= 1 - \frac{d^{(2)}}{2} = (1 - 2d^{(1/2)})^{1/4}$

$= 1 - d$

$= d^{(2)} = 2\left[1 - \{1 - 2(5\%)\}^{1/4}\right]$

$= 0.05199250 7$

$= 5.20\%$

4) $v(0,t) = \frac{1}{A(0,t)}$

$= 1 - d = \frac{1}{1+i}$

$= d = 1 - \frac{1}{1+i}$

$= d = \frac{1+i-1}{1+i}$

$= d = \frac{1}{1+i} = i\left(\frac{1}{1+i}\right)$

$\therefore \boxed{d = iv}$ Hence, proved.

ii)

a) for $d^{(12)}$

$$= \left(1 - \frac{d^{(12)}}{12}\right)^{-12} = 1+i$$

$$= \left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{(-1)}$$

$$= 1 - \frac{d^{(12)}}{12} = +it\,(1+i)^{(-1/12)}$$

$$= d^{(12)} = 12\left[1 - (1+i)^{(-1/12)}\right]$$

$$= 12\left[1 - (1+0.08)^{(-0.83333)}\right]$$

$$= 0.07671$$

$$= 7.67\%.$$

b) for $i^{(365)}$

$$= \left(\frac{1 + i^{(365)}}{365}\right) = 1+i$$

$$= i^{(365)} = 365\left[(1+i)^{1/365} - 1\right]$$

$$= 365\left[(1+0.08)^{1/365} - 1\right]$$

$= 0.0769$

$= 7.696\%$

$= 7.70\%$

c) $\delta = \ln(1+i)$

$= \ln(1+0.08)$

$= 0.07696$

$= 7.696\%$

$= 7.70\%$

d) for $i^{(0.5)}$

$= \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5} = (1+i)$

$= \left(1 + \frac{i^{(0.5)}}{0.5}\right) = (1+i)^{1/0.5}$

$= i^{(0.5)} = 0.5\left[(1+i)^2 - 1\right]$

$= 0.5\left[(1+0.08)^2 - 1\right]$

$= 8.32\%$

5

a) $\left(1 + \frac{i^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$

$= 1 + \frac{i^{(2)}}{2} = \left(1 - \frac{d^{(4)}}{4}\right)^{-2}$

$= i^{(2)} = 2\left[\left(1 - \frac{d^{(4)}}{4}\right)^{(-2)} - 1\right]$

$= 0.061377516$

$= 6.14\%$

b) $1 - \frac{d^{(12)}}{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}$

$= \frac{1 - d^{(12)}}{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}$

$= d^{(12)} = 12\left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}\right]$

$\therefore d^{(12)} = 0.0603025$

$= 6.03\%$

ii) -

(12) (6) $C = 7,049.$

$n = 2yrs.$

$i^{(12)} = 6\%$ p.a.

$= AV_2 = C \times A(0,2)$

$= C\left[\left(1 + \frac{i^{(12)}}{12}\right)^{12}\right]^2$

$= 7049\left[\left(1 + \frac{6\%}{12}\right)^{12}\right]^2$

$= 7945.34926$

$= 7945.35$

→ $n = 2.5$ yrs
i is I = 1.5 % per quarter

$C = 7945.349262$

$AV_{5.5} = C \times A(5.5)$
$= 7945.349262 [1 + 1.5\% \cdot (5.5)]$
$= 8600.84$

→ $n = 1.5$ yrs.
$d(12) = 6\%$ p.a.
$C = 8600.84$.

$= A_{(1.5)} = 9412.94$

(13) (i) $i = 10\%$ p.a
$n = ?$

$= AV = C.A(0,t)$
$\therefore 2x = x(1+i)^n$
$= 2 = (1+6\%)^n$
$=$ log on both sides.

$\log 2 = n \log(1.06)$

$n = 11.8956610$

$n = 11.9$ yrs

ii) for $d^{(12)} = 10\%$ p.a

$$i = \left[\left(1 - \frac{d^{(12)}}{12}\right)^{12} - 1\right]$$

$$= \left[\left(1 - \frac{10\%}{12}\right)^{12} - 1\right]$$

$$= 0.105634\,0$$

$$= 10.56\%$$

$= AV = C \times A(0, n)$

$2x = x(1+i)^n$

$2 = (1 + 10.56\%)^n$

$= \log 2 = n \log 1.1056$

$$n = \frac{\log 2}{\log 1.1056}$$

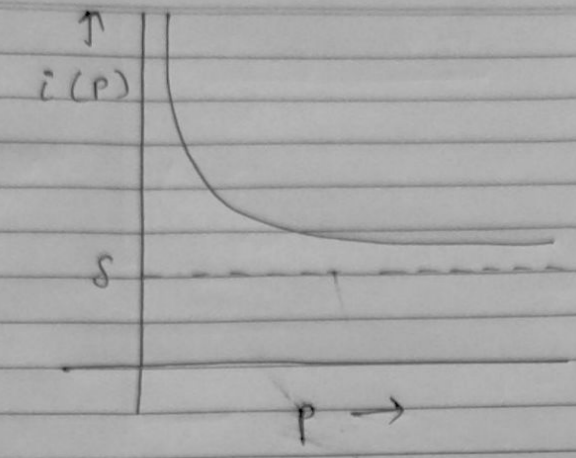
$= 6.905255041$ yrs

(8) $i = 8\%$

$i^{(p)} = p$

$i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$

$i^{(2)} = 7.85\%$	$i^{(10)} = 7.73\%$	$i^{(1000)} = 7.70\%$
$i^{(3)} = 7.80\%$	$i^{(25)} = 7.71\%$	
$i^{(4)} = 7.77\%$	$i^{(100)} = 7.70\%$	
$i^{(5)} = 7.76\%$	$i^{(365)} = 7.70\%$	



$= \lim_{p \to \infty} i^{(p)} = i^{(\infty)} = \delta \rightarrow$ force of interest.

(10) $$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 05 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

→ for time $t: 0 \le t \le 5$,

$$v(t) = e^{-\delta}$$
$$= e^{-0.08}$$
$$= 0.923116346$$

→ for time t, $5 \le t < 10$, $v(t) = e^{(-\delta)} e^{(-\delta)}$

$$= (e^{(-0.06)})(e^{-0.06})$$
$$= 0.87$$

→ for t, $t \ge 10$, $v(t) = (e^{-0.06})(e^{-0.06})(e^{-0.04})$

$$= 0.84$$