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Calculus Assignment - 1

1) $$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$$

$$\lim_{h \to 0} \frac{2(9+h^2-6h)-18}{h}$$

$$\lim_{h \to 0} \frac{\cancel{18}+2h^2-12h-\cancel{18}}{h}$$

$$\lim_{h \to 0} \frac{2h^{\cancel{2}}}{\cancel{h}} - \frac{12\cancel{h}}{\cancel{h}}$$

$$\lim_{h \to 0} 2h - 12$$

$$= 0 - 12$$

$$= -12$$

$$\therefore \lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = -12$$

2) $$g(x) = \begin{cases} 2x & x<6 \\ x & x \geq 6 \end{cases}$$

a) $x = 4$

$$f(x) = 2x$$

$$f(4) = 2 \times 4$$

$$= 8$$

$$\lim_{x \to 4^-} = 2x$$

$$= 2 \times 4$$

$$= 8$$

$$\lim_{x \to 4^+} = 2x$$

$$= 2 \times 4$$

$$= 8$$

$\therefore$ The function is continuous at $x = 4$

2) b) $x = 6$

$f(x) = x - 1$

$= 6 - 1$

$= 5$

L.H.L

$\lim_{x \to 6^-} = 2x$

$= 2 \times 6$

$= 12$

R.H.L

$\lim_{x \to 6^+} = x - 1$

$= 6 - 1$

$= 5$

$\therefore$ Function is discontinuous at $x = 6$

as L.H.L $\neq$ R.H.L.

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3) $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x\to 9} \frac{(\sqrt{x})^2 - 3^2}{(3-\sqrt{x})}$

$= \lim_{x\to 9} \frac{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)}{-\cancel{(\sqrt{x}-3)}}$

$= -(3+3)$

$= -6$

4) $f(x) = \frac{4x+5}{9-3x}$

a) $x = -1$

$f(x) = \frac{4x+5}{9-3x}$

$f(-1) = \frac{4(-1)+5}{9-3(-1)}$

$= \frac{1}{12}$

$\lim_{x\to 1} = \frac{4x+5}{9-3x}$

$= \frac{-4+5}{9-3(-1)}$

$= \frac{1}{12}$

$\therefore$ Graph is continuous at $x = -1$

b) $x = 0$

$\lim_{x\to 0} \frac{4x+5}{9-3x}$

$= \frac{4\times 0+5}{9-0} = \frac{5}{9}$

$\therefore$ Graph is continuous at $x = 0$

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c) $x = 3$

$$\lim_{x \to 3} \frac{4x+5}{9-3x}$$

$$= \frac{4 \times 3 + 5}{9 - 3 \times 3}$$

$$= \frac{12+5}{9-9}$$

= Not defined

$\therefore$ The function is discontinuous at $x = 3$.

5) $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \lim_{x \to \infty} \left(\frac{6e^{4x} - e^{-2x}}{e^{4x}} \right) \left(\frac{e^{4x}}{8e^{4x} - e^{2x} + 3e^{-x}} \right)$$

$$= \lim_{x \to \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

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6) $g(y) = \begin{cases} y^2+5 & \text{if } y<-2 \\ 1-3y & \text{if } y\geq -2 \end{cases}$

a) $\lim_{y\to 6} g(y) = \lim_{y\to 6} (1-3y)$

$= 1-3\times 6$

$= -17$

b) $\lim_{y\to -2} g(y)$

L.H.L

$\lim_{x\to -2^-} y^2+5$

$= (-2)^2+5$

$= 4+5$

$= 9$

R.H.L

$\lim_{x\to -2^+} 1-3y$

$= 1-3\times(-2)$

$= 1+6$

$= 7$

7) $f(t) = (4t^2-t)(t^3-8t^2+12)$

$\frac{df(t)}{dt} = (4t^2-t)\frac{d}{dt}(t^3-8t^2+12) + (t^3-8t^2+12)\frac{d}{dt}(4t^2-t)$

$= (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$= 12t^4 - 64t^3 - 3t^3 - 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$

$= 20t^4 - 132t^3 + 24t^2 - 12 + 96t$

8) $R(w) = \frac{3w+w^4}{2w^2+1}$

$\frac{d(R(w))}{dw} = \frac{(2w^2+1).\frac{d}{dw}(3w+w^4) - (3w+w^4)\frac{d}{dt}(2w^2+1)}{(2w^2+1)^2}$

$= \frac{4w^5 - 6w^2 + 4w^3 + 3}{4w^4 + 4w^2 + 1}$

$f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log_e 8$

~~$y = x^5$~~

$y = z^5 - e^z \ln(z)$

$\frac{dy}{dz} = 5z^4 - \left[\frac{e^z}{z} + \ln(z) e^z\right]$

$= 5z^4 - \frac{e^z}{z} - e^z \ln(z)$

$f(x) = (6x^2 + 7x)^4$

$f'(x) = 4(6x^2 + 7x)^3 (12x + 7)$

$f(t) = 5 + e^{4t + t^7}$

$f'(t) = e^{4t + t^7} (4 + 7t^6)$

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12) a) $z = \ln(7 - x^3)$

$$\frac{dz}{de} = \frac{-1}{7-x^3}(3x^2)$$

$$\frac{d^2z}{dx^2} = -\left[\frac{(7-x^3)(6x) - (3x^2)(-3x^2)}{(7-x^3)^2}\right]$$

$$= -\left[\frac{42x - 6x^4 + 9x^4}{(7-x^3)^2}\right]$$

$$= \left[\frac{-42x - 3x^4}{(7-x^3)^2}\right]$$

b) $$Q(v) = \frac{2}{(6+(2v)-v^2)^4} = y$$

$$\frac{dy}{dv} = \frac{2(2-2v)}{(6+2v-v^2)^5}$$

$$= \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$\frac{d^2y}{dv^2} = -16\left[\frac{(6+2v-v^2)^5(-1) - (1-v)(6+2v-v^2)}{(6+2v-v^2)^{10}}\right]$$

$$= \frac{-16\cancel{(6+2v-v^2)^4}}{\cancel{(6+2v+v^2)^4}}\left[\frac{(v^2-2v-6) - (1-v)(2-2v)}{6+2v-v^2}\right]$$

$$= -16\left[\frac{(v^2-2v-6) - 2(1-v)^2}{(6+2v-v^2)^6}\right]$$

$$= \frac{(2(1-v)^2 - v^2 + 2v + 6)16}{(6+2v-v^2)^6}$$

c) $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2}(2t)$

$f''(t) = \frac{(1+t^2)2 - (2t)2t}{(1+t^2)^2}$

$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$

$= \frac{2-2t^2}{(1+t^2)^2}$

$= \frac{2(1-t^2)}{(1+t^2)^2}$

13) $f(x) = y = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

$f''(-1) = -6 - 6$

$= -12$

$f''(3) = 18 - 6 = 12$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x+1)(x-3) = 0$

$x = -1, \quad x = 3$

$f(-1) = -1 - 3 + 9 + 12$

$= 17$ (maximu

14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$f''(x) = 24x - 36$

$x^2 - 3x + 2 = 0$

$x^2 - x - 2x + 2$

$= x(x-1) - 2(x-1)$

$= (x-2)(x-1)$

$x = 2 \,;\, x = 1$

$f''(2) = 48 - 36$

$= 12$

$f''(1) = 24 - 36$

$= -12$

$f(1) = 4 - 18 + 24 - 7$

$= 3$

(Maximum value)

$f(2) = 32 - \overset{112}{\cancel{72}} + 48 - 7$

$= 80 - 119$

$= -39 + 40$

$= 1$ (Minimum value)

15) $f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$f'(x) = 3x^2 + 6x = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \,;\, x = -2$

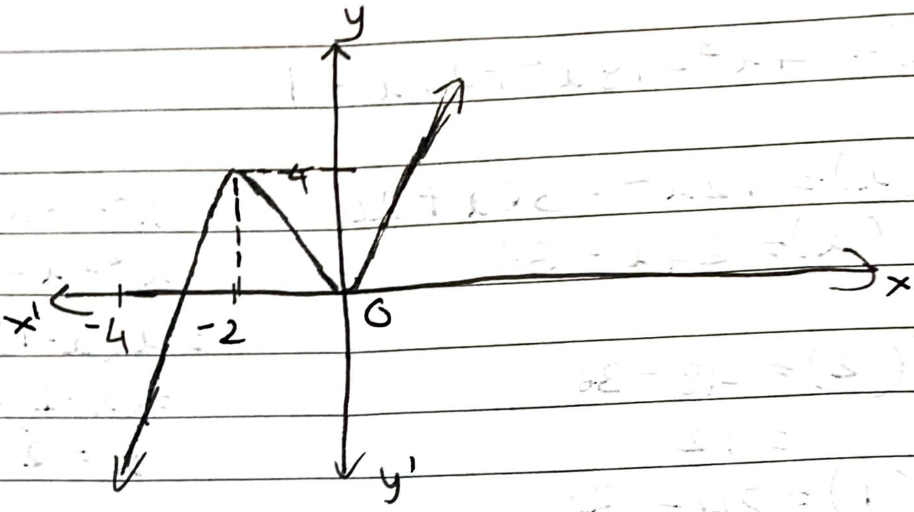
$f''(x) = 6x + 6$

$f''(0) = 6$

$f''(-2) = -6$

$\therefore$ [At 0, $f(x)$ has minimum value]

[At (-2), $f(x)$ has maximum value]



15) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6 \quad (x = 1)$

$f''(x) = 6$

$\therefore$ At $x = 1$, $f(x)$ attains minimum value.

$f(1) = -3$

$f'(0) = -6, \; f'(-2) = 6$