Algebra Assignment

Sol ①

$r = 12.5\ cm.$

$r' = 12.65$

Actual area $= \pi r^2 = \frac{22}{7}(12.5)^2 = 490.874\ cm^2$

Calculated area $= \pi r'^2 = \pi(12.65)^2 = 502.725\ cm^2$

(i) Absolute error $=$ Measured value $-$ Actual.

$= 502.725 - 490.874$

$= 11.851$

(ii) Relative error $= \frac{\text{Absolute error}}{\text{Actual value}}$.

$\Rightarrow \frac{11.851}{490.874} = 0.02414$

(iii) % error $=$ Relative error $\times 100$

$= 2.2414\ \%$

Sol ②

x	$f(x) = \log x$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513

(a) $y = y_1 + (x - x_1)\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

$f(5) = 0.60206 + (1)\left(\frac{0.7781513 - 0.60206}{2}\right)$

$\boxed{f(5) = 0.69010565}$

(b) $y = 0.6532125 + (5 - 4.5)\left(\frac{0.7403627 - 0.6532125}{5.5 - 4.5}\right)$

$\boxed{y = 0.696788}$

$y(a) = -6.05$
$y(b) = 20.$

Sol (3) $y = x^4 - x - 10$, $4 \rightarrow a = 1.5$
$b = 2$

$\Rightarrow x_1 = \frac{a+b}{2} = \frac{3.5}{2} = 1.75$

$y_1 = (1.75)^4 - (1.75) - 10 = -2.371094$

$\Rightarrow x_2 = \frac{x_1 + b}{2} = \frac{1.75 + 2}{2} = 1.875$

$y_2 = (1.875)^4 - (1.875) - 10 = 0.48462$

$\Rightarrow x_3 = \frac{x_1 + x_2}{2} = \frac{1.75 + 1.875}{2} = 1.8125$

$y_3 = (1.8125)^4 - (1.8125) - 10 = -1.020249$

$\Rightarrow x_4 = \frac{x_2 + x_3}{2} = \frac{1.875 + 1.8125}{2} = 1.84375$

$y_4 = (1.84375)^4 - (1.84375) - 10 = -0.2877$

$\Rightarrow x_5 = \frac{x_2 + x_4}{2} = \frac{1.875 + 1.84375}{2} = 1.859375$

$\boxed{y_5 = 0.093378}$

$\Rightarrow x_6 = \frac{x_4 + x_5}{2} = \frac{1.84375 + 1.859375}{2} = 1.8515625$

$y_6 = -0.098$

So, $x = 1.859375$ is the required solution.

Sol (4) (using newton Raphson's method)

$f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	$f(x)$	$f'(x)$
0	+1	−1
1	−1	2
2	5	11

So, $\boxed{x_0 = 1}$

So, $f(x_0) = -1$

$\Rightarrow x_1 = x - \frac{f(x)}{f'(x)} \Rightarrow 1 - \frac{(-1)}{2} \Rightarrow \frac{3}{2} = 1.5$

$y_1 = (1.5)^3 - (1.5) - 1 \Rightarrow 0.8750$

$f'(x_1) = 5.75$.

$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 1.5 - \frac{0.875}{5.75} = 1.3478$

$y_2 = (1.3478)^3 - (1.3478) - 1 \Rightarrow 0.10056$

$f'(x_2) = 4.4496$

$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3478 - \frac{0.10056}{4.4496}$

$= 1.3252$.

$y_3 = (1.3252)^3 - (1.3252) - 1 = 0.002056$

$f'(x_3) = 4.268$.

$\Rightarrow x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \Rightarrow 1.3252 - \frac{0.002056}{4.268}$

$\Rightarrow 1.3247$

$y_4 = (1.3247)^3 - (1.3247) - 1 \Rightarrow -0.00007658$

So, $x_4 = 1.3247$ is req. solution

Sol ⑤ $G \rightarrow \quad A^2 = A$

$T.F \rightarrow \quad 7A - (1+A)^3.$

$\Rightarrow 7A - (1^3 + A^3 + 3I\bar{A} + 3A^2 1)$

$\Rightarrow 7A - (1 + A^3 + 3A^2 + 3A^2)$

$\Rightarrow -I.$

Sol ⑥ $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$ To find $\Rightarrow A^{-1}$.

$|A| = 1(0-6) + 1(-4) + 2(4)$

$\Rightarrow -6-4+8 \Rightarrow \boxed{-2}$

Adj of A: $\begin{array}{ccc} + & - & + \end{array}$ $\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 1 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{\text{adj } A}{|A|} = \begin{vmatrix} 3 & -1/2 & 3 \\ -2 & -1/2 & -1 \\ 2 & -1/2 & -2 \end{vmatrix}$

29×5

$A^{-1} = \begin{matrix} 3 & -1/2 & 3 \\ -2 & -1/2 & -1 \\ -2 & -1/2 & -2. \end{matrix}$

Sol ⑦ $A = 8i - 9j \quad ; \quad B = 7i - 9j$ Ea.

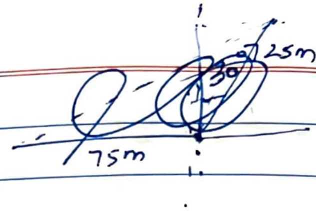
$|\vec{A}| = \sqrt{64 + \cancel{81}} = \cancel{2\sqrt{..}} \quad \sqrt{145}$

$|\vec{B}| = \sqrt{49 + 81} = \sqrt{130}$

$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos Q \Rightarrow \cos Q = \dfrac{137}{\sqrt{145}\sqrt{130}}$

$\theta = \cos^{-1} \dfrac{137}{\sqrt{145}\sqrt{130}}$ $\Rightarrow$

~~Sol 8~~



Sol 8

$\vec{R} = \vec{a} + \vec{b}$ $\Rightarrow$ $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\vec{b}$

$= 75^2 + 25^2 + 2(75)(25)\sqrt{3}/2$

$\boxed{\vec{R} = 97.46\ m}$

Sol 9

$a = 101$

$a_n = 998$

$d = 3$

$a_n = a + (n-1)d$

$101 + (n-1)3$

$\boxed{n = 300}$

$S_n = \frac{n}{2}(a + a_n) = \frac{300}{2}(101 + 998)$

$\Rightarrow \boxed{164850}$

Sol 10

$ar^5 = 32$

$ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$\boxed{r = 2}$

Sol 11

$f(x) = (4+3x)^5$

Using binomial theorum :

${}^5C_0 (3x)^5 + {}^5C_1 (4)^1 (3x)^4 + {}^5C_2 (3x)^3 (4)^2 + {}^5C_3 (3x)^2 4^3 + {}^5C_4\, 3x\, 4^4$
$+ {}^5C_5\, 4^5$

$\Rightarrow 243x^5 + 1620x^4 + 4320x^3 + 7560x^2 + 3840x + 1024.$

Sol (12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Using $|A - \lambda I| = 0$

$\begin{matrix} \cancel{2}-\lambda & \cancel{-3}-\lambda & \cancel{\lambda} \\ 2-\lambda & -5-\lambda & -\lambda \\ & & \cancel{3}-\lambda \end{matrix}$ $\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = 0$

$-(2-\lambda)(5+7)(3-\lambda) - (-3)[(3-\lambda)(2)-0] = 0$

$\boxed{\lambda^3 - 13\lambda + 12 = 0}$

if $\lambda = 1$

then $1^3 - 13 + 12 = 0$

So, $\lambda = 1$ is one of the roots.

	1	0	−13	12
1		1	1	−12
	1	1	−12	0

$\lambda^2 + \lambda - 12 = 0$

$(\lambda+4)(\lambda-3) = 0$

So, ~~others~~ $\boxed{\lambda = -4, 3, 1}$ sol^n

sol (13) $f(x) = x^3 - 7x^2 + 8x - 3$ if $x_0 = 5$

$x_0 = 5$

$f(x_0) = -13$ $f'(x_0) = 13$

$x_1 = 5 - \frac{(-13)}{13} = 6$

$f(x_1) = 9$ $f'(x_1) = 32$

$x_2 = 6 - \frac{9}{32} \Rightarrow 5.71875$

$\boxed{x_2 = 5.71875}$

sol (14)

$(1-x+x^2)^4$

let $1-x = y$.

$(y+x^2)^4$.

Using binomial theorem 3

${}^4C_0 y^4 + {}^4C_1 y^3(x^2) + {}^4C_2 (x^2)^2 y^2 + {}^4C_3 y^1 (x^2)^3 + {}^4C_4 x^2 y$

$y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

sol $(1-x)^4 + 4(1-x)^3(x^2) + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

~~1e4x4+6e3~~

sol (15) $e^{-x} = 3\log(x)$; $f(x) = e^{-x} - 3\log(x)$ ~~=0~~ .

x	$f(x)$
0.5	1.50962
1	0.36757
1.5	-0.305143

$\Rightarrow x_1 = \frac{1+1.5}{2} = 1.25$

$y_1 = -0.0042252$

$\Rightarrow x_2 = \left(\frac{1+1.25}{2}\right) = 1.125$

$y_2 = 0.1711949$

$\Rightarrow x_3 = \frac{1.125+1.25}{2} = 1.1875$

$y_3 = 0.0810219$

$\Rightarrow x_4 \quad \frac{1.1875+1.25}{2} = 1.21875$

$y_4 = 0.0378565$

So, $x = 1.21875$ is required sol^n.