

SALONI TAYSHETE

B-76

NUMERICAL METHODS AND ALGEBRA

ASSIGNMENT-1

I] 1. x 7 8 9 10
 y 3 1 1 9
 x_0 x_1 x_2 x_3
 Find value of y at $x=9.5$

$$x_k = 9.5$$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} (3) +$$

$$\frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} (1) +$$

$$\frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} (1) +$$

$$\frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} (9)$$

$$= \frac{1.5 \times 0.5 \times -0.5}{-1 \times -2 \times -3} (3) +$$

$$\frac{2.5 \times 0.5 \times -0.5}{1 \times -1 \times -2} (1) +$$

$$\frac{2.5 \times 1.5 \times -0.5}{2 \times 1 \times -1} (1) +$$

$$\frac{2.5 \times 1.5 \times 0.5}{3 \times 2 \times 1} (9)$$

$$= 0.1875 - 0.3125 + 0.9375 + 2.8125$$

$$= \boxed{3.625}$$

2.	x	3	7	9	10
	y	168	120	72	63

Find y when $x=6$

$$x_k = 6$$

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} (168) + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} (120) + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} (72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} (63)$$

$$= \frac{-1 \times -3 \times -4}{-4 \times -6 \times -7} (168) + \frac{3 \times -3 \times -4}{4 \times -2 \times -3} (120) + \frac{3 \times -1 \times -4}{8 \times 2 \times -1} (72) + \frac{3 \times (-1) \times -3}{1 \times 3 \times 1} (63)$$

$$= 12 + 180 - 72 + 27$$

$$= \boxed{147}$$

3	x	1	2	7	8
	y	4	5	5	4

Find y when $x = 6$

$$x_k = 6$$

$$\begin{aligned}
 f(6) = & \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} (4) + \\
 & \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} (5) + \\
 & \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} (5) + \\
 & \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} (4)
 \end{aligned}$$

$$= \frac{4 \times -1 \times -2}{-1 \times -6 \times -7} (4) +$$

$$\frac{8 \times -1 \times -2}{1 \times -8 \times -83} (5) +$$

$$\frac{8 \times 4 \times -2}{38 \times 8 \times -1} (5) +$$

$$\frac{5 \times 4^2 \times -1}{7 \times 83 \times 2} (4)$$

$$= \frac{-16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{20}{21}$$

$$= \frac{25}{3} - \frac{36}{21}$$

$$= \frac{175 - 36}{21} = \frac{139}{21} = \boxed{6.619047619}$$

4.	x	1	2	3	4	7
	y	2	4	8	16	128

Find y when $x=5$

$$x_k = 5$$

$$\begin{aligned}
 f(5) &= \cancel{5} \times \cancel{2} \times 1 \times \cancel{2} (2) + \frac{4 \times 2 \times 1 \times \cancel{2}}{1 \times \cancel{2} \times \cancel{2} \times 5} (4) \\
 &+ \frac{4 \times 3 \times 1 \times \cancel{2}^1}{\cancel{2} \times 1 \times \cancel{1} \times 4} (8) + \frac{4 \times \cancel{3} \times \cancel{2} \times \cancel{2}}{\cancel{3} \times \cancel{2} \times 1 \times 3} (16) \\
 &+ \frac{4 \times 3 \times 2 \times 1}{3 \times 5 \times 4 \times 3} (128)
 \end{aligned}$$

$$= \frac{-2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{3}$$

$$= \boxed{67.0667}$$

5.	x	10	20	14	16
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f(x)	0.25	0.2	0.15	0.10
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Find $f(x)$ when $x=15$

$$x_k = 15$$

$$\begin{aligned}
 f(15) &= \frac{-5 \times \cancel{1} \times \cancel{1}}{-10 \times -4 \times -6} (0.25) + \frac{5 \times 1 \times \cancel{1}}{10 \times 6 \times 4} (0.2) \\
 &+ \frac{5 \times -5 \times \cancel{1}}{4 \times -6 \times -2} (0.15) + \frac{5 \times -5 \times 1}{6 \times -4 \times 2} (0.10)
 \end{aligned}$$

$$= \frac{-25}{192} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96}$$

$$= \boxed{0.120833}$$

II] 1. $x^3 + x - 1 = 0$

① x $y = x^3 + x - 1$

0.5 -0.375

1 1

$$\bar{x} = \frac{0.5(1) - 1(-0.375)}{1 - (-0.375)} = \frac{0.5 + 0.375}{1.375} = \frac{1}{1.1} = \underline{0.6364}$$

$$\begin{aligned}\bar{y} &= (0.6364)^3 + 0.6364 - 1 \\ &= -0.105854843 \\ &= -0.1059\end{aligned}$$

② x y

0.6364 -0.1059

1 1

$$\bar{x} = \frac{0.6364(1) - 1(-0.1059)}{1 - (-0.1059)} = \frac{0.6364 + 0.1059}{1.1059}$$

$$= 0.6712$$

$$\begin{aligned}\bar{y} &= (0.6712)^3 + 0.6712 - 1 \\ &= -0.0264\end{aligned}$$

③ x y

0.6712 -0.0264

1 1

$$\bar{x} = \frac{0.6712(1) - 1(-0.0264)}{1 - (-0.0264)} = \frac{0.6712 + 0.0264}{1.0264}$$

$$= 0.6797$$

$$\bar{y} = (0.6797)^3 + 0.6797 - 1 = -0.006283976 = -0.0063$$

x	y
0.6797	-0.0063
1	1

$$\bar{x} = \frac{0.6797(1) - 1(-0.0063)}{1 - (-0.0063)} = \frac{0.6797 + 0.0063}{1.0063}$$

$$= 0.6817$$

$$\bar{y} = (0.6817)^3 + 0.6817 - 1$$

$$= -0.0015$$

x	y
0.6817	-0.0015
1	1

$$\bar{x} = \frac{0.6817(1) - 1(-0.0015)}{1 - (-0.0015)} = \frac{0.6817 + 0.0015}{1.0015}$$

$$= 0.6822$$

$$\bar{y} = (0.6822)^3 + 0.6822 - 1$$

$$= -0.0003$$

$\therefore$ Root of $x^3 + x - 1 = 0$ is approximately 0.6822

2. $x^3 - 2x + 0.5 = 0$

① x $y = x^3 - 2x + 0.5$
 0 0.5
 1 -0.5

$$\bar{x} = \frac{0(-0.5) - 1(0.5)}{-0.5 - 0.5} = 0.5$$

$$\bar{y} = (0.5)^3 - 2(0.5) + 0.5 = -0.375$$

② x y x y
 0.5 -0.375 0 0.5
 1 -1 0.5 -0.375

$$\bar{x} = \frac{0(-0.375) - 0.5(0.5)}{-0.375 - 0.5} = 0.2857$$

$$\bar{y} = (0.2857)^3 - 2(0.2857) + 0.5 = -0.04808$$

③ x y
 0 0.5
 0.2857 -0.04808

$$\bar{x} = \frac{0(-0.04808) - 0.2857(0.5)}{-0.04808 - 0.5} = 0.2606$$

$$\bar{y} = (0.2606)^3 - 2(0.2606) + 0.5 = -0.0035$$

$\therefore$ Root of $x^3 - 2x + 0.5 = 0$ is approximately 0.2606

3. $x^3 + 2x - 20 = 0$

①	x	$y = x^3 + 2x - 20$
	0	-20
	1	-17
	2	0
	2.5	0.625

Ans. The root of $x^3 + 2x - 20 = 0$ is 2

4. $3x - \cos x - 1 = 0$

①	x	$y = 3x - \cos x - 1$
	0	-2
	1	1.0002

$$\bar{x} = \frac{0(1.0002) - 1(-2)}{1.0002 - (-2)} = 0.6666$$

$$\bar{y} = 3(0.6666) - \cos(0.6666) - 1 = -0.00013$$

Ans. The root of $3x - \cos x - 1 = 0$ is approximately 0.6666

5. $x^3 - 3x - 5 = 0$

①	x	$y = x^3 - 3x - 5$
	2	-3
	2.5	3.125

$$\bar{x} = \frac{2(3.125) - 2.5(-3)}{3.125 - (-3)} = 2.2449$$

$$\bar{y} = (2.2449)^3 - 3(2.2449) - 5 = -0.4214$$

②

x	y
2.2449	-0.4214
2.5	3.125

$$\bar{x} = \frac{2.2449(3.125) - 2.5(-0.4214)}{3.125 - (-0.4214)} = 2.2752$$

$$\bar{y} = (2.2752)^3 - 3(2.2752) - 5 = -0.0479$$

③

x	y
2.2752	-0.0479
2.5	3.125

$$\bar{x} = \frac{2.2752(3.125) - 2.5(-0.0479)}{3.125 - (-0.0479)} = 2.2786$$

$$\bar{y} = (2.2786)^3 - 3(2.2786) - 5 = -0.0053$$

④

x	y
2.2786	-0.0053
2.5	3.125

$$\bar{x} = \frac{2.2786(3.125) - 2.5(-0.0053)}{3.125 - (-0.0053)} = 2.279$$

$$\bar{y} = (2.279)^3 - 3(2.279) - 5 = -0.0002$$

Ans. : The root of $x^3 - 3x - 5 = 0$ is approximately 2.279

6. $x \log_{10} x - 1.2 = 0$

①	x	$y = x \log_{10} x - 1.2$
	2	-0.5979
	3	0.2314

$$\bar{x} = \frac{2(0.2314) - 3(-0.5979)}{0.2314 - (-0.5979)} = 2.72097$$

$$\bar{y} = 2.72097 \log_{10}(2.72097) - 1.2 = -0.01713$$

②	x	y
	2.72097	-0.01713
	3	0.2314

$$\bar{x} = \frac{2.72097(0.2314) - 3(-0.01713)}{0.2314 - (-0.01713)} = 2.7402$$

$$\bar{y} = 2.7402 \log_{10}(2.7402) - 1.2 = -0.0004$$

Ans. : The root of $x \log_{10} x - 1.2 = 0$ is approximately 2.7402

7. $x^3 - 4x + 1 = 0$

①	x	$y = x^3 - 4x + 1$
	0	1
	1	-2
	$\frac{1}{2}$	$\frac{1}{8}$
	$\frac{3}{4}$	$-\frac{1}{8}$

$$\bar{x} = \frac{0(-2) - 1(1)}{-2 - 1} = 0.3333$$

$$\bar{y} = (0.3333)^3 - 4(0.3333) + 1 = -0.2962$$

$$\textcircled{2} \quad \begin{array}{cc} x & y \\ 0 & 1 \end{array}$$

$$0.3333 \quad -0.2962$$

$$\bar{x} = \frac{0(-0.2962) - 0.3333(1)}{-0.2962 - 1} = 0.2572$$

$$\bar{y} = (0.2572)^3 - 4(0.2572) + 1 = -0.0118$$

$$\textcircled{3} \quad \begin{array}{cc} x & y \\ 0 & 1 \end{array}$$

$$0.2572 \quad -0.0118$$

$$\bar{x} = \frac{0(-0.0118) - 0.2572(1)}{-0.0118 - 1} = 0.2542$$

$$\bar{y} = (0.2542)^3 - 4(0.2542) + 1 = -0.0004$$

Ans. The root of $x^3 - 4x + 1 = 0$ is approximately 0.2542

8. $x^3 + 2x^2 - 8 = 0$

①

x	$y = x^3 + 2x^2 - 8$	
0	-8	
1	-5	-ve
2	8	+ve

$$\bar{x} = \frac{1(8) - 2(-5)}{8 - (-5)} = \frac{8 + 10}{8 + 5} = \frac{18}{13} = 1.3846$$

$$\bar{y} = (1.3846)^3 + 2(1.3846)^2 - 8 = -1.5113$$

②

x	y
1.3846	-1.5113
2	8

$$\bar{x} = \frac{1.3846(8) - 2(-1.5113)}{8 - (-1.5113)} = 1.4824$$

$$\bar{y} = (1.4824)^3 + 2(1.4824)^2 - 8 = -0.3474$$

③

x	y
1.4824	-0.3474
2	8

$$\bar{x} = \frac{1.4824(8) - 2(-0.3474)}{8 - (-0.3474)} = 1.5039$$

$$\bar{y} = (1.5039)^3 + 2(1.5039)^2 - 8 = -0.0752$$

x	y
1.5039	-0.0752
2	8

$$\bar{x} = \frac{1.5039(8) - 2(-0.0752)}{8 - (-0.0752)} = 1.5085$$

$$\bar{y} = (1.5085)^3 + 2(1.5085)^2 - 8 = -0.0162$$

x	y
1.5085	-0.0162
2	8

$$\bar{x} = \frac{1.5085(8) - 2(-0.0162)}{8 - (-0.0162)} = 1.5095$$

$$\bar{y} = (1.5095)^3 + 2(1.5095)^2 - 8 = -0.0033$$

Ans. $\therefore$ The root of $x^3 + 2x^2 - 8 = 0$ is approximately 1.5095

29. $x^3 - 9x + 1 = 0$

x	y = x ³ - 9x + 1
0	1
1	-7

$$\bar{x} = \frac{0(-7) - 1(1)}{-7 - 1} = \frac{-1}{-8} = 0.125$$

$$\bar{y} = (0.125)^3 - 9(0.125) + 1 = -0.123$$

②	x	y
	0	1
	0.125	-0.123

$$\bar{x} = \frac{0(-0.123) - 0.125(1)}{-0.123 - 1} = 0.1113$$

$$\bar{y} = (0.1113)^3 - 9(0.1113) + 1 = -0.0003$$

Ans. : The root of $x^3 - 9x + 1 = 0$ is approximately 0.1113

10. $x^3 - x - 4 = 0$

①	x	y = $x^3 - x - 4$	
	0	-4	
	1	-4	-ve
	2	2	+ve

$$\bar{x} = \frac{1(2) - 2(-4)}{2 - (-4)} = \frac{2+8}{2+4} = \frac{10}{6} = 1.25$$

$$\bar{y} = (1.25)^3 - 1.25 - 4 = -3.2969$$

②	x	y
	1.25	-3.2969
	2	2

$$\bar{x} = \frac{1.25(2) - 2(-3.2969)}{2 - (-3.2969)} = 1.7168$$

$$\bar{y} = (1.7168)^3 - 1.7168 - 4 = -0.6567$$

x	y
1.7168	-0.6567
2	2

$$\bar{x} = \frac{1.7168(2) - 2(-0.6567)}{2 - (-0.6567)} = 1.7868$$

$$\bar{y} = (1.7868)^3 - 1.7868 - 4$$
$$= -0.0822$$

x	y
1.7868	-0.0822
2	2

$$\bar{x} = \frac{1.7868(2) - 2(-0.0822)}{2 - (-0.0822)} = 1.7952$$

$$\bar{y} = (1.7952)^3 - 1.7952 - 4$$
$$= -0.0097$$

x	y
1.7952	-0.0097
2	2

$$\bar{x} = \frac{1.7952(2) - 2(-0.0097)}{2 - (-0.0097)} = 1.7962$$

$$\bar{y} = (1.7962)^3 - 1.7962 - 4$$
$$= -0.001$$

Ans. $\therefore$ The root of $x^3 - x - 4 = 0$ is approximately 1.7962.