ARYAN TRIPATHI

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PRLI → Assignment 1

① i) The constant force of mortality is an assumption to model fractional ages. It says the force of mortality b/w the age x & $x+1$ is constant.

ii) In CFM,

$$p_x = e^{-\mu_x}$$

$$\therefore \mu = -\ln(p_x)$$

$$\mu_{55} = -\ln(1-q_{55})$$

$$\mu_{55} = 0.00470103\underline{3}$$

② i) A select mortality is death rate of an individual who has recently undergone some sort of medical test. It ~~has~~ generally lower mortality rates as the individual has passed certain medical tests

ii)

$$_{20}p_{[25]:[30]} = {}_{20}p_{[25]} \times {}_{20}p_{[30]}$$

$$= \frac{l_{45}}{l_{[25]}} \times \frac{l_{50}}{l_{[30]}}$$

$$= \frac{9801.3123}{9951.993} \times \frac{9712.0728}{9923.7497}$$

$$= 0.9638520314$$

③ i) $$_{1.4}q_{54.5} = 1 - {}_{1.4}p_{54.5}$$

$$= 1 - {}_{0.5}p_{54.5} \times {}_{0.9}p_{55}$$

$$= 1 - \left[\left(\frac{1-0.5\times q_{54}}{1-0.5\,q_{54}}\right)\left(1-0.9\,q_{55}\right)\right]$$

$$= 1 - \left[\left(\frac{1-0.5\times 0.00714}{1-0.5\times 0.00714}\right)\left(1-0.9\times 0.00197\right)\right]$$

$$= 0.010730091$$

(4) $P\ddot{a}_{35:\overline{30}|} = 5000000\, A_{35:\overline{30}|} + 30\, P A_{35:\overline{30}|}$

$\ddot{a}_{35:\overline{30}|} = 14.35 \longrightarrow$ from tables

$A_{35:\overline{30}|} = A_{35} - v^{30}\,{}_{30}p_{35}\, A_{65}$

$= 0.09452 - A \times 0.40177 = 0.03251$

$A_{35:\overline{30}|} = v^{30}\,{}_{30}p_{35}$

$= (1.06)^{-30} \left(\frac{8021.2612}{9894.4299}\right)$

$= 0.1352258197 \ (A)$

$\therefore P(14.35 - 30 \times A) = 5000000 \times B$

$P = 16771.98381$

(5) i) $\bar{a}_x$

ii) $PV = a_{\overline{T_x}|}, \quad T_x > 0$

$\Rightarrow PV = \bar{a}_x = \int_{-\infty}^{\infty} v^s\,{}_sp_x\, ds$

(6) $P = 5{,}000{,}000\, A_{45:\overline{20}|}$

(i)

$A_{45:\overline{20}|} = A_{45:\overline{20}|} - A_{45:\overline{20}|}$

$= 0.46998 - (1.04)^{-20} \left(\frac{8821.2612}{9801.3123}\right)$

$P = 296140.0863$

$\therefore$ Total single premium $= 10000 \times P$

$= 2961400863$

(ii) $V = 5000000\, A_{45:\overline{19}|} - 296140.0863$

$= 5000000\left(0.34599 - (1.06)^{-19}\left(\frac{21.2612}{9786.9534}\right)\right)$

$= -2916400863$

$= -55694.26417$

$BSAR = 5000,000 - (-55694.26417)$

$= 5055694.26417$

$PDS = 1000\ \ddot{a}_{45} \times BSAR$

$= 74065920.97$

$ADS = 10 \times PSAR$

$= 50559442.641$

$MP = DS - AD$

$= 23508978.33$

(7) $\epsilon PV = 100{,}000\ A_{40:\overline{20|}} + 200{,}000\ A_{60}\ V^{2}\ {}_{20}P_{40}$

$= 100{,}000\left(A_{40} - V^{20}\ {}_{20}P_{40}\ A_{60}\right) + \left(200{,}00\ V^{20}\ {}_{20}P_{40}\ A_{60}\right)$

$V^{20}\ {}_{20}P_{40} = (1.06)^{-20}\left(\frac{9287.2164}{9856.2863}\right)$

$= 0.2938021365$ — (1)

$A_{40} = 0.2313$

$A_{60} = 0.3292$

$\epsilon PV = 21917.97945$

$\epsilon(PV^2) = 100{,}000^2\left({}^2A_{40:\overline{20|}}\right) + 200{,}000^2\ {}^2A_{60}\ V^{20}\ {}_{20}P_{40}$

${}^2A_{40:\overline{20|}} = {}^2A_{40} - V^{40}\ {}_{20}P_{40}\ {}^2A_{60}$

$= 0.0187207 - (1.04)^{-40}(0.1408)\left(\frac{9287.2164}{9856.28}\right)$

$= 0.01415$

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$E(PV^2) = 1798358788$

$SD = \sqrt{E(PV^2) - [(E(PV))^2]}$

$= 363303.73211$

⑧ $A_{65} = 0.5412$

$A_{66} = 0.5591$

$A_{65} = q_{65} + (1 - q_{65}) A_{66} v$

$A_{65} = q_{65} + A_{66} v - A_{66} v q_{65}$

$A_{65} - A_{66} v = q_{65}(1 - A_{66} v)$

$$\frac{0.5412 - 0.5591(1+0.08)}{1 - 0.5591(1+0.08)^{-1}} = q_{65}$$

$\Rightarrow q_{65} = 0.048754$

$$A[65] = 0.75 q_{65} + \frac{(1 - 0.75 q_{65})0.5591}{1.8}$$

$A[65] = 0.535321$

$P = 100,000 \times 0.535321$

$P = 53532.12963$

⑨ (i) $P = 120,000 \, a_{70}$

$P = 120,000 (\ddot{a}_{70} - 1)$

$= 120\,000 (10562)$

$= 1267440$

(ii) $P(L_0 > 0) = 1$

$P(L_0 \leq 0 \mid q = q_n) = P(K_{90} \geq n)$

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(10) $PV = \bar{a}_{\overline{T_x}|}$

(i) $EPV = \int_0^\infty \bar{a}_{\overline{t}|} \cdot {}_tp_x \cdot \mu_{x+t}\, dt = \bar{a}_x$

(ii) we know that,

$$\int_0^\infty e^{-(\mu+\delta)t}\, dt = 16$$

$$= \frac{1}{-(\mu+0.04)} \cdot \left[e^{-(\mu+\delta)t}\right]_0^\infty = 16$$

$$= \frac{-1}{(\mu+0.04)} (0-1) = 16$$

$$= 1 = 16\mu + 0.64$$

$$= \mu = \frac{1-0.64}{16}$$

$$\Rightarrow \mu = 0.0225$$

$$V_x = \frac{1}{\delta^2}\left({}^2\bar{A}_x - (A_x)^2\right)$$

$$\bar{A}_x = 1 - \delta\bar{a}_x$$

$$\bar{A}_x = 1 - 0.64$$

$$= 0.36$$

$$^2\bar{A}_x = \int_0^\infty e^{-2\delta t} e^{-\mu t} \times \mu\, dt$$

$$= 0.0225 \left[\frac{e^{-(2\delta+\mu)t}}{-(2\delta+\mu)}\right]_0^\infty$$

$$= \frac{-0.0225}{2(0.0225+0.04)} (0-1)$$

$$= \frac{9}{34}$$

$$V_{SD} = \sqrt{\frac{1}{0.04^2}\left(\frac{9}{34} - (0.36)^2\right)} = 9.1898882$$

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(ii) (i) In term assurance, the premiums are paid annually in advance for the term where if a policy holder fails to pay the premium amount, he/she will no longer be in the force with the contract where as the annuity contract is a single premium contract, the policy holder would have paid the premium at the start and as in ~~the~~ force till he/she dies

ii) term assurance.

$P\ddot{a}_{40:\overline{25|}} = 2,500,000\, A_{40:\overline{25|}}$

$A_{40:\overline{25|}} = A_{40:\overline{25|}} - A_{40:\overline{25|}}$

$= 0.38907 - (1.04)^{-25} \left(\frac{8821.2612}{9856.2863}\right)$

$= 0.05334485019$ — (1)

$P = \frac{2,500,000 \times (1)}{15.814}$

$P = 8396.002296$ (2)

$_{10}V = 2,500,000\, A_{50:\overline{15|}} - P\ddot{a}_{50:\overline{15|}}$

$A_{50:\overline{15|}} = A_{50:\overline{15|}} - A_{50:\overline{15|}}$

$= 0.56719 - (1.04)^{-15} \left(\frac{8821.2612}{9712.0728}\right)$

$= 0.062885$

$_{10}V = 87334429.64$ (A)

Annuity: $_{10}V = 25000\, \ddot{a}_{60}$

$= 398000$ (B)

Total portfolio reserve: $8500 \times A + 9900 \times B$

$= 39431542050$

(iii) . Premiums of term assurance

. Investments in diff securities

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(12) $S_{55:\overline{10|}} = a_{55:\overline{10|}} (1+i)^{10} \; {}_{10}p_{55}$

(i) $= \left(\frac{88\ 21.2613}{955\ 7.8179}\right)(1.06)^{10} \times 7.61$

$= 12.57810568$

(ii) $a^{(4)}_{[40:\overline{20|}} = \ddot{a}^{(4)}_{[40]} \cdot v^{20} \; {}_{20}p_{[40]} \; \ddot{a}^{(4)}_{60}$

$= \ddot{a}_{[40]} - \frac{3}{8} - v^{20} \; {}_{20}p_{[40]} \left(\ddot{a}_{60} - 3/8\right)$

$= \left(15.494 - 3/8\right) - (1.06)^{-20} \left(11.891 \cdot \frac{3}{8}\right) \left(\frac{9287.2164}{9854.3064}\right)$

$= 14.67350635$

(13) $P\ddot{a}_{45:\overline{15|}} = 100{,}000\, A_{45:\overline{20|}} + 100{,}000\, A_{45:\overline{20|}}$

$\ddot{a}_{45:\overline{15|}} = 1.386$

$A_{45:\overline{20|}} = 0.046991$

$A_{45:\overline{20|}} = (1.04)^{-20} \; {}_{20}p_{45}$

$= 0.4107519827$

$P = 7734.603748$

(ii) ${}_{13}V = 100{,}000\, A_{58:\overline{7|}} + 100{,}000\, A_{58:\overline{7|}} - P\ddot{a}_{58:\overline{2|}}$

$A_{58:\overline{7|}} = 0.76516$

$A_{51:\overline{7|}}' = (1.04)^{7} \times {}_{7}p_{58}$

$= 0.712085$

$\ddot{a}_{56:\overline{2|}} = 1.955$

${}_{13}V = 132603.4289$

$DSAR = 100{,}000 - {}_{13}V$

$= -32603.42895$

$EDS = 19 \; 0.00565 \times DSAR$

$= -36657.66534$

$ADS = 4 \times DSAR$

$= -130413.7158$

$MP = \frac{EDS \times ADS}{}$

$= 93756.05046$

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(i) $5000 \, \bar{a}_{\overline{T_{68}-2|}} \, V^2$, $T_{63} < 2$, $T_{66} > 2$

(ii) $5000 \times 100 \times {}_2p_{63} \times V^2 \, \bar{a}_{65}$

$= 500\,000 \, (14.871 - 0.5)(1.04)^2 \left(\frac{7703.705}{7775.883}\right)$

$= 6594347.317$

(iii) $V_{a3} = 5000^2 \times 100 \times \frac{1}{\delta^2} \Big[\left(V^{2n} {}_nq_x + V^{2n} {}_np_x \, {}^2\bar{A}_{x+n} \right) - \left(V^n {}_nq_x + V^n {}_np_x \, \bar{A}_{x+n} \right)^2 \Big]$

$= 5000^2 \times 100 \times \frac{1}{\delta^2} \Big[\left(V^{2\times2} {}_2q_{63} + V^{2\times2} {}_2p_{63} \, {}^2\bar{A}_{65} \right) - \left(V^2 {}_2q_{63} + V^2 {}_2p_{63} \, \bar{A}_{65} \right)^2 \Big]$

$\bar{A}_{65} = 1 - \delta \, \bar{a}_{65}$

$\bar{A}_{65} = 0.436359$ (B)

${}^2\bar{A}_{65} = (1.04)^{0.5} (0.20847)$

$= 0.212598$ (C)

${}_2p_{63} = 0.9926$ (D)

${}_2q_{63} = 0.007313$ (E)

$$Var = 5000^2 \times 100 \times 13.53$$
$$= (183937.6804)^2$$

E

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a) $({}_tV + P)(1+i) - x(1+i)^{11/12}$
$- x(1+i)^{10/12} - Sq_{x+t} = {}_{t+1}V\, p_{x+t}$

b) $({}_tV + P)(1+i) - Sq_{x+t} - R\,0.5\,p_{x+t}$
$= {}_{1+t}V \times p_{x+t}$